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TEXT BOOK

OF

GUNNERY.

1887.

BY

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PREFACE TO THE EDITION OF 1887.

(SECOND EDITION.)

THE previous edition of the Text Book of Gunnery, 1883, was chiefly written for the use of the Gentlemen Cadets R.M. Academy, but, as it was afterwards adopted for use in the Royal Artillery and has also had a considerable circulation elsewhere, the arrangement has now been altered in order to make the work more generally useful. Besides a general revision, the mathematical parts of the subject have been placed in Part II: additional gunnery tables and fresh Chapters on Steel, the Penetration of Earth and Masonry, and on the Strength of Guns, have been introduced.

The many changes and improvements in artillery matériel during the past few years have produced a complication which tends to confusion in the minds of many; this book aims rather to supply a key to these changes by investigating the broad principles upon which gunnery rests, than to enter into mechanical details in particular cases; it is intended to assist in the formation of a sound judgment, and to enable an intelligent use to be made of means at disposal.

The author gratefully acknowledges the assistance he has received from many Officers and others.

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PART II.

GUNNERY TABLES.

ABBREVIATIONS.

B.L.	Breech-loading (applied to new type ordnance).
C.	Centigrade.
cm.	Centimètre.
D. of A.	Director of Artillery and Stores.
E.O.C.	Elswick Ordnance Company, or Sir W. Armstrong and Co.
F.	Fahrenheit.
f.s.	Feet per second.
G.D.	Gravimetric density.
G.O.	General orders.
Inst. C.E.	Institution of Civil Engineers.
Kg.	Kilogram.
L.G.	Large-grain powder.
m.	Mètre.
M.H.	Martini-Henry rifle.
mm.	Milimètres.
M.V.	Muzzle velocity.
P.	Pebble powder.
P.P.R.E.	Professional Papers of the Corps of Royal Engineers.
Phil. Trans. R.S.	Philosophical Transactions of the Royal Society.
Pro. R.A.I.	Proceedings of the Royal Artillery Institution.
R.B.L.	Rifled breech-loading (applied to old type guns).
R.C.D.	Royal Carriage Department.
R.G.F.	Royal Gun Factory.
R.L.	Royal Laboratory.
R.L.G.	Rifled large-grain powder.
R.M.L.	Rifled muzzle-loading.
R.U.S.I.	Royal United Service Institution.
R.V.	Remaining velocity.
S.B.	Smooth bore.
U.S. Ord. Notes.	United States Ordnance Notes.

TEXT BOOK OF GUNNERY, 1887.

PART I.

CHAPTER I.—DEFINITIONS AND UNITS.

In the following pages of Part I (after the definitions have been given), the subject of gunnery is considered in the order which naturally suggests itself: beginning with the forces which act in the interior of a piece of ordnance when the charge is ignited, we are led on to the amount of work developed; hence comes a comparison of the power of different guns, a sketch of the principles of construction follows; sighting and laying are then considered; afterwards the course of the projectile is followed through the air, and the probable accuracy of fire is estimated; while finally, the various effects produced on striking the target are reviewed. The more mathematical parts of gunnery are placed in Part II, after which, for convenience of reference, are the tables necessary for working out gunnery problems; and at the end of the book is the Index.

The chapters are to a great extent independent of each other, and most of them may be read separately if desired. The first few chapters, particularly No. II, deal with somewhat difficult matters; those who are new to the subject may prefer to begin at Chapter IX, read to the end of Part I, and then take up the subject of internal ballistics and gun construction.

Lines, Planes, Area, and Angles in Gunnery.

(*Vide* G.O., 1st January, 1878. See Fig. 1,)

The **axis of the piece** is a straight line passing down the centre of the bore. Lines.

The **axis of the trunnions** is a straight line passing through the centre of the trunnions, at right angles to the axis of the piece.

The **line of sight** is a straight line, passing through the sights of the piece and the point aimed at.

The **line of fire** is a straight line from the muzzle of the piece to the point aimed at. This term would be used instead of the preceding, if the ordinary sights cannot be used.

The **line of departure** is the direction in which the projectile is moving on leaving the piece; or in other words, a tangent to the trajectory at the muzzle.

The **trajectory** is the curve described by the projectile in passing from the muzzle of the piece to the first point of impact.

The **calibre** is the diameter of the bore measured in inches. In rifled pieces it is measured across the lands.

Clearance is the linear distance between the lower part of the body of the projectile and the bore of the gun, or it may be defined as the difference between the height of the stud and the depth of the groove.

Range is the distance from the muzzle of the piece to the (second intersection of the trajectory with the line of sight, but on the practice ground the distance of the target from the gun (in yards) is called the range.

Drift is the constant deflection of the projectile from the plane of departure due to the rotation imparted by the rifling of the piece. It is sometimes termed derivation.

Lateral deviation is the perpendicular distance of the point of impact of the projectile right or left of the plane of sight.

Planes. The **planes of sight and departure** are vertical planes passing through the lines of sight and departure respectively.

Area. **Windage** is the difference between the sectional area of the gun through its grooves, and of the body of the projectile through its studs.

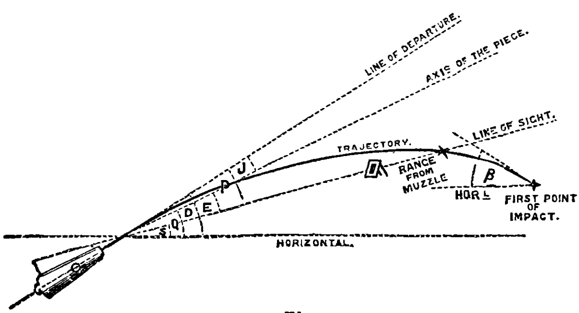
Windage is sometimes expressed in linear units; it is then the difference between the diameter of the projectile and the calibre of the gun, irrespective of studs or grooves.

Angles. The **angle of sight** is the angle which the line of sight makes with the horizontal plane (S, Fig. 1).

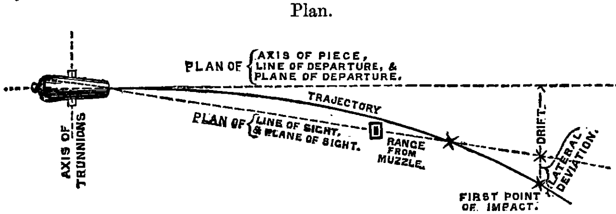
The **quadrant angle** is the angle which the axis of the piece, when laid, makes with the horizontal plane. It is termed quadrant *elevation* or *depression*, according as the piece is laid above or below the horizontal plane (Q, Fig. 1); the term depressed fire means that a piece is fired at a quadrant angle of depression.

Fig. 1.

Elevation.



Plan.



NOTE.—The lines of sight, departure, &c., do not really intersect at the muzzle; but it is assumed that they do so, and this is very near the truth. The drift and lateral deviation are exaggerated in the diagram.

The **angle of departure** is the angle which the line of departure makes with a horizontal plane (D, Fig. 1).

The **angle of elevation** is the angle between the axis of the piece and the line of sight (E, Fig. 1).

The **angle of projection** is the angle between the line of departure and the line of sight (P, Fig. 1).

Jump is the angle between the line of departure and the axis of the piece before firing (J, Fig. 1).

The **angle of descent** is the angle which a tangent to the trajectory at the first point of impact makes with the horizontal plane (β, Fig. 1).

The following example may help to define these angles :—

If the angle of elevation is 2° 20′, quadrant angle 4° 15′, and jump 7′, find the angles of departure, sight, and projection. Example 1.

From definitions and from Fig. 1—

$$
\begin{aligned}
\text{Angle of departure} &= \text{quadrant angle} + \text{jump} \\
&= 4^\circ\,15' + 7' \\
&= 4^\circ\,22' \\
\text{Angle of sight} &= \text{quadrant angle} - \text{angle of elevation} \\
&= 4^\circ\,15' - 2^\circ\,20' \\
&= 1^\circ\,55' \\
\text{Angle of projection} &= \text{angle of elevation} + \text{jump} \\
&= 2^\circ\,20' + 7' \\
&= 2^\circ\,27'.
\end{aligned}
$$

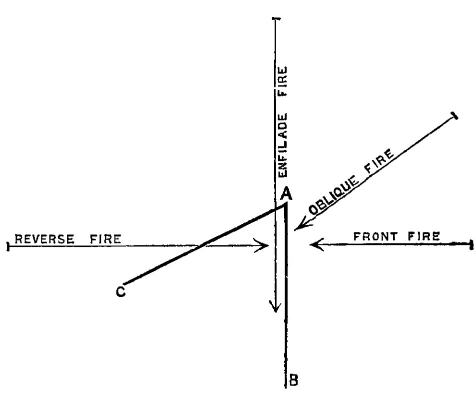
Fire with reference to the Horizontal and Vertical Planes.

In **front, oblique,** and **enfilade fire,** the line of fire is respectively perpendicular, inclined, and parallel (or nearly so) to the front of the object fired at. The term flank fire is sometimes used instead of enfilade fire.

With **reverse fire** the object is fired at from the rear.

Fig. 2 represents the face (AB) of a work (ABC) being fired at in plan.

Fig. 2.

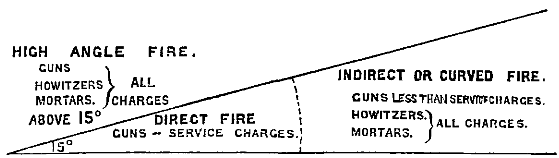


Direct fire is from guns, with service charges (including reduced charges) at all angles of elevation not exceeding 15°. (Fig. 3.)

Indirect or curved fire is from guns, with less than service charges, and from howitzers and mortars, at all angles of elevation not exceeding 15°.

High angle fire is from guns, howitzers, and mortars, at all angles of elevation exceeding 15°.

Fig. 3.



In the Naval Service these three terms have a different meaning, being applied when the angle of descent is less than 8°; between 8° and 20°; and above 20° respectively (*see* Manual of Gunnery for H.M. Fleet, 1886).

Velocity.

Muzzle velocity is the velocity of a projectile on leaving the piece in feet per second.

Remaining velocity is that at any point of the trajectory.

Final or **striking velocity** is that at the point of impact.

Terminal velocity is the maximum and uniform velocity attainable by a body falling through the air: this term is seldom required.

Physical Definitions.

Definitions of a few physical terms are necessary, as until lately many of them have been used in a vague and general way in ordinary books; of late years endeavours have been made to attach a definite meaning to each of the following terms.

Force is that which produces, tends to produce or prevents motion.

The statical or gravitation unit of force employed in gunnery is the attraction of the earth on one pound or one ton.

Stress is the action of balancing forces.

Pressure is a stress tending to separate bodies.

Tension is a stress tending to draw bodies together.

The intensity of stress, whether pressure or tension, is generally measured in tons on the square inch, occasionally in atmospheres (each being 15 lbs. on the square inch); but the resistance of the air to projectiles in flight is expressed in pounds on the square inch.

Total pressure P is the intensity of stress p multiplied by the area A in square inches over which it acts, or $P = pA$.

Strain is deformation produced by stress.

Compression is the strain produced in a body which is exerting pressure.

Extension is the strain produced in a body which is exerting tension.

The **limit of elasticity**, or shortly the **elasticity** of a substance, is the least stress producing permanent strain. For any stress less than the elastic limit, the ratio of stress to strain is practically found to be constant; it is called the **modulus of elasticity** (E).

$$\text{Thus E} = \frac{\text{stress}}{\text{strain}}$$

$$= \frac{\text{pressure}}{\text{compression}}, \text{ or } \frac{\text{tension}}{\text{extension}},$$ according as the stress is a pressure or a tension. Under these circumstances, when the stress is removed the strain disappears, and the body returns to its original dimensions; this is sometimes called Hooke's law. The stresses here referred to are intensities of stress; and the strains are estimated per unit of length.

The **tenacity** of a substance is the least breaking tension. Elasticity and tenacity are expressed in tons on the square inch.

Elongation is the permanent extension of a body strained beyond its limit of elasticity; it is measured by the percentage which it bears to the original length.

For further definitions *see* Clerk Maxwell's "Matter and Motion," Alexander's "Elementary Applied Mechanics," &c.

WORK.

Work is performed when a force P pounds or tons, acts on a body through a distance s feet in any direction, and it is expressed by the product Ps, called foot-pounds or foot-tons, according as P is in pounds or in tons. Work.

The **unit of work** called the **foot-pound** is that amount which is performed in raising a weight of 1 lb. through a distance of 1 foot against gravity; but for artillery purposes the **foot-ton** is the unit generally employed, *i.e.*, the amount required to raise 1 ton 1 foot high; a foot-ton contains 2240 foot-pounds. Units of work.

Work can be *represented graphically* by a rectangular area in which one side is proportional to the pressure, and the other to the distance through which the force acts.

Thus, if AD is proportional to P, and AB to s (Fig. 4), the area DB must be proportional to the work done. Suppose at some point C on AB (Fig. 5) the pressure suddenly alters; if at C we erect a perpendicular and mark off on it CF, proportional to the new pressure, and complete the rectangle FB, the work must be proportional to the two areas DC and FB.

Fig. 4.

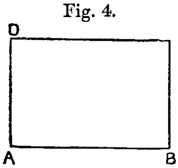


Fig. 5.

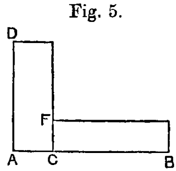


Fig. 6.

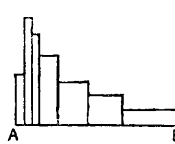
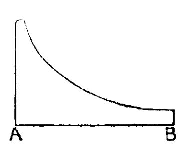


Fig. 7.



If the pressure changes more than once we must take a greater number of rectangles (Fig. 6), and the sum of the areas is then proportional to the work done by a pressure P, which has suddenly changed in magnitude several times, in acting on the body, over the distance AB.

Now, suppose the pressure to change in magnitude, but to do so *gradually*, in this case the number of rectangles becomes indefinitely increased, and the work done is represented by the area enclosed by a curved line (the locus of the corners of an indefinite number of rectangles). In Fig. 7 we have such a figure where the pressure (as in the bore of a gun) begins from nothing, soon rises to a high maximum; then falls off, and ceases soon after the projectile leaves the muzzle. We thus see how the work done by a variable pressure can be represented by an area.

If a force has acted on a weight or mass over a given distance, and has caused it to move at a certain velocity, work is *stored up;* the question now comes, how can this stored-up work be expressed in terms of the velocity and weight of the body which is in motion?

Considering only the magnitude of the work stored up, it is immaterial *how* the velocity was obtained, or in what direction it is. Suppose it was produced by the weight falling from a certain height h feet, under gravity, until it had attained the same velocity v feet per second. We can tell the relation between v and h under these circumstances from the elementary dynamical formula—

$$v^2 = 2gh$$
$$\text{or } h = \frac{v^2}{2g}$$

If there are w lbs. in the weight, by definition the work done must be wh foot-pounds; substituting the value of h just obtained this is equal to

$$\frac{wv^2}{2g} \text{ ft. lbs.}$$
$$\text{or } \frac{wv^2}{2g \times 2240} \text{ ft. tons.}$$

This expression is a measure of the work contained in a moving body in terms of its weight and velocity; in this form it is called kinetic energy, or shortly **energy.**

The energy in ft. lbs. due to the rotation of a rifled projectile is expressed by $\frac{w}{2g}(k\omega)^2$, in which k is the radius of gyration in feet, and ω is the angular velocity; it is, however, small in proportion to the energy due to translation, and is generally neglected, when considering the total work impressed on the projectile.

The work done by the powder pressures in the bore on the projectile must equal the energy contained in the projectile. Thus, if P is the mean total pressure in pounds exerted over a length of bore s feet on the whole base of the projectile, we must have $Ps = \frac{wv^2}{2g}$; where v is the muzzle velocity of the projectile in feet per second and w its weight in pounds.

The following examples will illustrate these definitions of work and energy:—

Example 2. On firing the 9·2-inch B.L. gun, Mark V, the powder pressure acts on the base of the shell as it moves over a distance of 246 inches in the bore of the gun; the weight of the projectile is 380 lbs., and its M.V. 2065 f.s., what must be the mean total pressure on the base of the shell?

$$P \times \frac{246}{12} = \frac{380 \times (2065)^2}{2 \times 32{\cdot}19},$$

$$\text{whence } P = 1{,}229{,}000 \text{ lbs.,}$$
$$\text{or } 548{\cdot}7 \text{ tons;}$$

some additional pressure is also given to compress the driving-band into the grooves. If the mean pressure p per square inch is required, we must divide the result just obtained by the area of the base of the shell in square inches $\frac{1}{4}\pi(9{\cdot}2)^2$, and we obtain $p = 8{\cdot}25$ tons per square inch; the *maximum* pressure, however, greatly exceeds this.

What must be the speed of a vessel of 2000 tons, that she may ram another with an energy equal to that of the projectile in the last example? Example 3.

Here $$\frac{2000 \times 2240v^2}{2g} = \frac{380 \times (2065)^2}{2g},$$

$$\text{whence } v = 19{\cdot}02 \text{ f.s.,}$$
$$\text{or } 12{\cdot}97 \text{ miles an hour.}$$

It should be noticed that the amount of work increases as the *square* of the velocity; thus if the weight of the projectile is constant, and its velocity is doubled, the energy becomes four times as great as before.

Units.

The **units** employed in gunnery are unfortunately numerous; this leads to possible mistakes in calculations.

The *units of length* are—

Yards for ranges at practice and for Maitland's graphic diagram.

Feet in Bashforth's tables, and in expressing the height of any point of the trajectory above plane, as, for instance, the position of the burst of a shell in the air. The height of the 50 per cent. probable zone is in some range-tables given in feet, in others in yards. *See* Table XIV; compare (A) and (B), pp. 308 and 309.

Inches are used for calibres of ordnance, diameters of projectiles, and for the distances apart of the marks on the chronometer of Boulengé's instrument.

Thousandths of an inch are employed in denoting the shrinkages given to the parts of a gun.

The *units of weight* are—

Tons, cwts., and *lbs.*, for ordnance.

Lbs. and *ozs.* for projectiles, powder charges, and bursting charges of shells.

Grains troy, for bullets and powder charges of machine guns and small arms. 7000 grains troy = 1 lb. avoirdupois.

Angles are measured in *degrees* and *minutes*, but *angular velocity* in units of *circular measure revolved through per second.*

The decimal notation of mètres and kilogrammes is universally employed in Continental works on gunnery. The calibres of foreign guns are expressed in centimètres; an approximation to their value in inches can be obtained by multiplying by 0·4. Thus: calibre 32 centimètres, multiply by 0·4, and we have 12·8 inches; the real equivalent is 12·599 inches.

For the English equivalent of foreign measures, see Table XVIII, p. 316.

Use of Symbols.

In order to prevent confusion, the following symbols in this list will only be used as indicated. It is suggested that in future works on gunnery this plan may be adhered to, and that this list may be consulted before appropriating any symbol to a fresh meaning. The need for some system in the employment of symbols has been much felt; for instance, the diameter of a projectile has been variously expressed as D, d, and $2r$, and range by R and r: in some mathematical books pressure is expressed by p, in others by q: all such variations tend to confusion, and should be avoided.

A. Area.
a. Mathematical constants (pp. 128, 246, and 256).
b. Mathematical constant (p. 255).
C. Coefficient for recoil (p. 106)—Axis of suspension in diagram (p. 238).
c. Coefficient for armour piercing (p. 212).
D. Drift in yards (p. 84)—Angle of departure only in diagram (p. 8).
D_v. Reduced angle, tabulated (p. 293).
d. Diameter of projectile in inches.
E. Total energy in foot tons (p. 213)—Modulus of elasticity (p. 11)—Angle of elevation only in diagram (p. 8).
e. Energy of a projectile in foot tons per inch of circumference (p. 211).
f. Acceleration or retardation in f.s. (p. 141)—Factor of safety of a gun (p. 67).
G. Position of centre of gravity in diagrams except on p. 242, where O is employed, because it is more convenient for the diagram on the succeeding page.
g. Acceleration of gravity in f.s.
H. Height when considerable, as that of vertex of trajectory (p. 155).
h. Height generally (p. 156)—Distance of centre of gravity from axis in ballistic pendulum (p. 239).
I. Moment of inertia.
J. Angle of jump only in diagram (p. 8).
K. Coefficient for resistance of the air (p. 255).
k. Radius of gyration (p. 12)—Coefficient for armour piercing (p. 214).
L. Length of part of bore of a gun rifled with increasing twist (p. 241).
l. Length generally—Of simple pendulum (p. 238)—Of arm of whirling machine (p. 240).
m. Number of revolutions per second (p. 129)—A number generally (p. 86).
n. Number of calibres in which one turn is made (p. 127)—A number generally (p. 86).
P. *Total* pressure (p. 13)—Angle of projection only in diagram (p. 8).
P_a. Mathematical abbreviation (p. 267).
p. Horizontal component of velocity at a definite point (p. 265)—Pressure in tons per square inch, or intensity of pressure (p. 59). In considering the strength of guns, p is used for radial stress, which is always a pressure; t is employed for hoop stress, which is most often a tension, a hoop *pressure* is conveniently expressed by $-t$ (p. 250); p is also used for pressure on the circular inch (p. 141).
Q. A physical constant (p. 237)—Quadrant angle of elevation only in diagram (p. 8).
q. Horizontal component of velocity at a definite point (p. 266)—Longitudinal stress on a gun (p. 252).
R. Total resistance (p. 141)—Range (p. 84)—A constant (p. 236).
ΔR. Small increase of range (p. 191).
r. Radius generally.
S. The angle of sight only in diagram (p. 8).
S_v. Reduced distance, tabulated (p. 288).
s. Distance in feet.
T. Considerable time (p. 156)—*Total* tension.
T_v. Reduced time, tabulated (p. 283).
t. Time in seconds—Temperature—Penetration of wrought iron in inches (p. 211)—Tension in tons per square inch or intensity of tension (p 59); $-t$ is conveniently employed for hoop pressure (p. 250); *see* also above, note on symbol p.
U. Velocity of gun on recoil (p. 106)—Velocity of wind (p. 311).
u. Horizontal component of velocity generally (p. 266).
u_0. Horizontal velocity of vertex (p. 267).

V. Muzzle velocity of projectile in f.s.
v. Velocity generally—Volume of a gas (p. 237).
W. Weight in lbs. of a comparative heavy body, such as gun and carriage (p. 106)—Pendulum (p. 239).
w. Weight in lbs., generally that of a projectile.
X, Y, Z. Axes at right angles to each other.
x, y, z. Mathematical co-ordinates.
${}_{a}t_{\beta}$, ${}_{a}x_{\beta}$, ${}_{a}y_{\beta}$. Time and co-ordinates when ϕ changes from a to β (p. 266).

a. Angle of departure (p. 152)—Angle of rifling (p. 128)—Fraction of volume of charge (p. 236).
Δa. Small increase of elevation (p. 191).
β. Angle of descent (p. 152).
γ. Mathematical abbreviation (p. 267)—Mean error (p .274).
δ. A small angle in degrees (p. 293)—A fractional part of explosion chamber (p. 236).
ΔD, ΔS, ΔT. Differences between tabulated values D_v, S_v, and T_v (p. 297).
ϵ. Angle of elevation (p. 84).
θ. Angle of deflection of sight (p. 84)—Angle of strike with armour (p. 219)—Angle of opening of shell (p. 186) —Half angle of swing of pendulum (p. 239)—An angle generally.
ϕ. Angle of inclination of trajectory (p. 265). *Note.*—Continental writers employ θ for this angle.
$\bar{\phi}$. Mean value of ϕ over a definite range.
π. Ratio of circumference to diameter.
σ. Coefficient for steadiness in flight (p. 140).
ρ. Radius of curvature (p. 266).
τ. Small time (Boulengé) (p .124)—Coefficient for density of the air (p. 141)—Intensity of tension (only at p. 250).
ω. Angular velocity.

CHAPTER II.—INTERNAL BALLISTICS.

(*See* also Part II, Chapter I, and Tables I and II, pp. 278, 279).

Before considering the actual case of a powder charge exploding in the bore of a gun, it may be well to refer to important experiments for determining the **pressure** and **temperature of fired gunpowder**: this problem has been attempted by many experimenters, with widely different results; but of late years, when great attention has been given to the subject by our own and by foreign military committees, a good deal of definite information has been obtained. The best results, however, have probably been gained by Captain A. Noble, F.R.S., and Sir Frederick Abel, C.B., F.R.S., who conducted long and careful experiments on this subject with great success; they proceeded as follows:—Charges of powder whose different gravimetric densities were known were consecutively exploded in a very strong chamber of mild steel, and the pressure each time was noted by means of an enclosed crusher gauge, and recorded;* the permanent gases were afterwards drawn off and examined, and the volumes at 0° Centigrade and 760 mm. barometric pressure were found to be about 280 times that of the original charge (of unit gravimetric density, *see* p. 22); the residue, which was liquid at the moment of explosion, then occupied about 0·6 of the volume of the original charge, but on cooling down and solidifying, it shrank to about half that bulk.

The *pressure* of fired gunpowder of unit gravimetric density was found to be about 42 tons on the square inch, and the *temperature* on explosion was calculated from the data obtained to be about 2223° Centigrade (4033° F.)

Instruments.

Explosion chamber.

The principal apparatus used by Captain Noble and Sir F. Abel for their experiments on fired gunpowder held some $2\frac{1}{4}$ lbs. of gunpowder, and is best described in their own words as follows:—

"A (*see* Figs. 1 and 2) is a mild steel vessel of great strength, carefully tempered in oil, in the chamber of which (B) the charge to be exploded is placed.

The main orifice of the chamber is closed by a screwed plug (C), called the firing-plug, which is fitted and ground into its place with great exactness.

In the firing-plug itself is a conical hole, which is stopped by the plug D, also ground into its place with great accuracy. As the firing-plug is generally placed on the top of the cylinder, and as, before firing, the conical plug would drop into the chamber if not held, it is retained in position by means of the set-screw S, between which and

* *See* Phil. Trans. R.S., "Fired Gunpowder," 1875 and 1880. Experiments with Prism brown powder and with gun-cotton have since been made; the temperature of explosion of gun-cotton was found to be much higher than that of gunpowder.

the cylinder a small washer (W) of ebonite is placed. After firing, the cone is, of course, firmly held, and the only effect of internal pressure is more completely to seal the aperture. At E is the arrangement for letting the gases escape; the small hole F communicates with the chamber where the powder is fired, and perfect tightness is

Fig. 1.

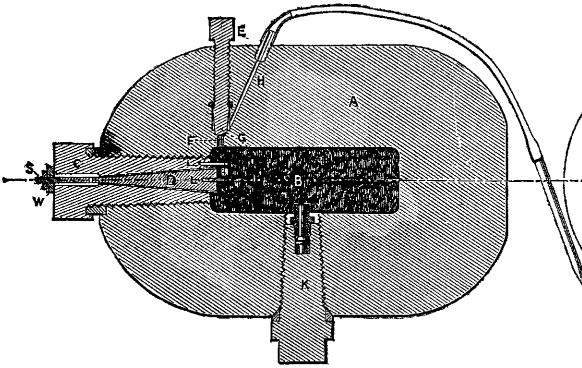
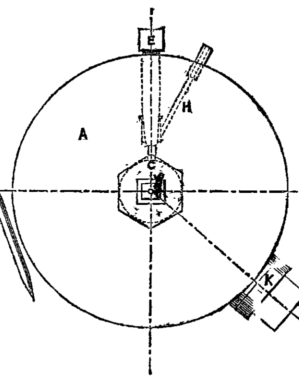


Fig. 2.



secured by means of the mitred surface (G). When it is wished to let the gases escape, the screw (E) is slightly withdrawn, and the gas passes into the passage H.

At K is placed the crusher-apparatus for determining the pressure at the moment of explosion.

When it is desired to explode a charge, the crusher-apparatus, after due preparation, is first carefully screwed into its place, and the hole F closed. The cone in the firing-plug is covered with the finest tissue-paper, to act as an insulator.

The two wires L,L, one in the insulated cone, the other in the cylinder, are connected by a very fine platinum wire passing through a small glass tube filled with mealed powder. Upon completing connexion with a Daniell's battery the charge is fired.

The only audible indication of the explosion is a slight click; but frequently, upon approaching the nose to the apparatus, a faint smell of sulphuretted hydrogen is perceptible."

Great care was necessary in exploding the powder in this chamber, and any looseness of screws at once gave an exit to the gas, which washed away the metal of the threads in its rapid rush. When such a state of things occurred, the metal had apparently been fused.

The use of improved carefully tempered mild steel gave these experimenters an advantage over their predecessors, as it enabled them to explode larger charges and obtain higher pressures without risk of breaking the apparatus.

The method of deducing the temperature of explosion from the data obtained by experiment is explained in Part II, p. 236; the calculations were roughly verified by the following observed facts:—

(1.) The explosion chamber was put into a water calorimeter, and the quantity of heat developed on firing was determined in the usual manner. The composition of the gases and residue being found from analysis, and the specific heats of all the constituents being known, a calculation of the temperature of explosion was made, which, how-

14

ever, gave a much higher result than that previously obtained. But the experimenters explain that (judging from analogy) the specific heat of the solid residue, which they examined when cold, would probably be greatly increased when it assumed the liquid form under the heat of explosion; they had no means of determining this point with certainty. Taking this into consideration, the agreement seemed good.

(2.) Thin platinum wire and foil were put into the chamber, and after explosion small parts showed signs of the beginning of fusion; but there was no appearance of volatilization, which can be effected by the blowpipe at about 3700° C. (6692° F.). Platinum melts at about 2000° C. (3632° F.).

The crusher gauge.

The pressure was found by means of the **crusher gauge**, which has been very widely employed in the Service for finding the powder pressures in the bores of guns. It consists (*see* Figs. 3 and 4) of a removable steel cylinder of the same diameter as a vent bush; at the inner end of it is a nozzle (G), which, when unscrewed and taken off, exposes an interior chamber (B). In the chamber is placed a hard steel anvil (E), which fills one end of it; a small cylinder of copper (A) (accurately made 0·5 inch long, and $\frac{1}{12}$th of a square inch in sectional area) is then put in; this is lightly held in the middle of the chamber by a piece of watch-spring (F), which per-

THE CRUSHER-GAUGE.

Fig. 3.

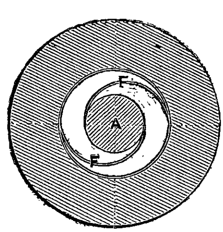
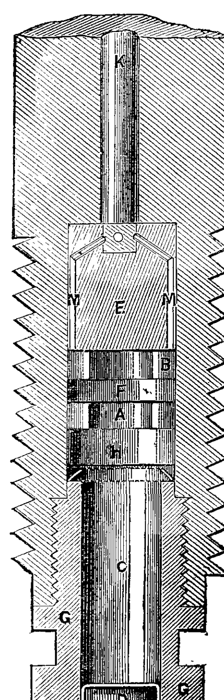


Fig. 4.



mits free lateral expansion. One end of the cylinder rests against the anvil, and when the nozzle (G) is screwed on again, the other end of the copper cylinder is held by the head (H) of a piston (C), which can move through the nozzle. A small brass cup (D) is put below the piston; so that when the pressure of the powder gas comes, it may expand and prevent leakage of gas into the chamber of the gauge. In Fig. 4 channels K and MM allow exit to any powder gas which may find its way into the gauge;—but they are not employed in the more modern instruments in which the gas-check cup D acts efficiently.

When the charge is fired, the piston of the gauge is driven against the copper cylinder, which is shortened by the pressure to which it has been subjected. The gauge is then taken out, nozzle unscrewed, copper removed, and the amount of pressure is found by carefully measuring the amount of compression of the copper cylinder with a micrometer; on reference to a table, the corresponding pressure is read off at once. The table is prepared on the assumption that the pressures are the same as those which produce the same compression in a statical pressing machine.

Greater accuracy is obtained by previously compressing the cylinders in a statical pressing machine to a little less than that expected in the bore of the gun, in order to allow for variations in the softness of the copper: thus if the cylinder is initially compressed statically by 12 tons on the square inch, and shortens as much as the copper of standard hardness would do under $12\frac{1}{2}$ tons: it will evidently register too high unless a suitable allowance is made. For an *apparent* pressure of $12\frac{1}{2}$ tons, $\frac{1}{2}$ a ton too much is indicated, and $\frac{1}{2} \div 12\frac{1}{2}$ or $\frac{1}{25}$ should be deducted.

If the gun is fired, and the copper in the gauge gives an *apparent* pressure of 17 tons on the square inch, the *corrected* amount will be $17 - \frac{17}{25} = 16{\cdot}32$ tons on the square inch. Though this plan cannot pretend to absolute accuracy, it furnishes most valuable comparative results; and much use has been made of this gauge, and many necessary data have been obtained by its means. Pressure gauges of a similar kind have been screwed radially into a few experimental guns to determine the pressures at various points in the bore; some gauges, which are much shorter, have been attached to the breech screws of guns, and others are fitted to the bases of projectiles, while at the proof of powder it is usual to place one inside the cartridge at its rear end; it is generally found in the bore somewhere near the truninons after firing, and it is so hot that it cannot be held in the hand.

Noble's chronoscope.

The pressures in the bore may also be *calculated* from the records of **Noble's chronoscope**, which tells the times of the projectile breaking electric currents at plugs distant only a few inches from each other in the bore of a gun. Hence the velocity of the projectile at any point in the bore of the gun can be found, and also the varying rate of the increase of the velocity, or the acceleration; from these data, as the weight of the projectile is known, the pressures, which produced the acceleration, can be found; the crusher gauge, however, is much more generally employed.

Products of explosion.

The chemical composition of the **products of explosion** of fired gunpowder are found to vary greatly from a variety of causes; not only do differences in mechanical treatment in manufacture, causing differences in hardness, density, &c., modify the results, but the same powder gives different proportions of products when exploded under different pressures; consequently Noble and Abel stated that "no value whatever can

be attached to any attempt to give a general chemical expression of the metamorphosis of gunpowder of normal composition." However, the pressures produced and the work capable of being done by gunpowder appear to be tolerably uniform, even though the powders compared differ a good deal in chemical composition. Rather less than half the weight (about 0·43) of the powder becomes permanent gas.

Explosion in the Bore of a Gun.

We will now consider the **action of powder gas in the bore of a gun**, when the question becomes complicated by the additional circumstance that the gases are allowed to expand and perform work, and in this case the *time* of explosion is a very important consideration.

All gases lose pressure in expanding, but in addition to this, when they perform work, the temperature falls (the heat lost being an exact measure of the work done), and consequently the pressure becomes still less.

This occurs with the expanding powder gases; but the heated residue, which is probably disseminated in a very finely divided state, somewhat counteracts the loss of pressure; for as the temperature tends to fall, a supply of heat is drawn from it, and the gases are kept at a higher state of expansion, and much more pressure is maintained than if no heated residue were present.

Time of explosion.

Let us now consider the influence which the **time of explosion** has upon the pressure in the gun.

If all the cartridge were instantaneously converted into gas, we should have an immediate and very high pressure; but this is not the case practically, as the charge takes a very small interval of time (calculated to be about 0·005 of a second with pebble powder in a 10″ R.M.L. gun) to become gas. This period, though so small, is of great importance, as it gives time for the projectile to move forward while the charge is exploding, and thus the space for the gas is increased, and a dangerous high maximum pressure on the gun is avoided.

The slower burning the powder, the further down the bore will the projectile move before all the gas is produced, and the maximum pressure is diminished to such an extent that larger charges can safely be employed, but as the total quantity of gas produced is more than before, the pressure is more sustained, even though the maximum may be less than with the smaller charge of quicker burning powder.

This is well illustrated in the diagram (Fig. 5) of the pressures and velocities in a 10″ gun, which was fired—

(1) with a charge of 70 lbs. pebble powder,
(2) „ 60 lbs. R.L.G.

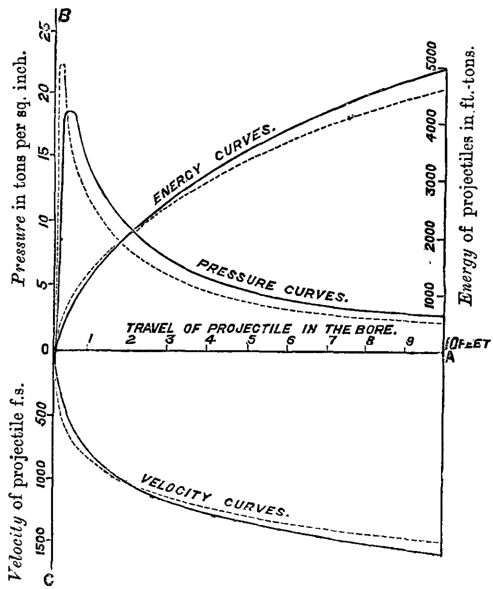
The projectiles were of the same weight in each case.

For purposes of comparison, the figure referring to one experiment is superposed over that belonging to the other; the continuous curved lines refer to the pebble powder, and the dotted ones to the R.L.G. charge.

OA is a scale of feet representing the travel of the projectile down the bore. One of the upper scales at right angles to it on the left is of *pressures*, and refers to the pressure curves; the other upper scale on the right is of *energies* in ft.-tons, and refers to the energy curves:

the lower scale is of *velocities*, referring to the velocity curves. The meaning of the diagram is, that at any spot—say at 3 feet from the starting point—the pressure of the powder gas is represented by the vertical ordinate drawn to the pressure curve from 3 on the horizontal scale, equal on the pressure scale, for pebble powder, to about 7 tons per square inch; the energy of the projectile at the same point

Fig. 5.



is shown by the vertical ordinate to the energy curve equal on the scale at the right to about 2450 ft.-tons for the same charge; the velocity at the same spot is represented by the vertical ordinate drawn to the continuous curve below, and this referred to the scale at the side indicates about 1130 f.s. Thus, after the charge of pebble powder has moved the projectile 3 feet the pressure *at that instant* is 7 tons on the square inch, the energy of the projectile is 2450 ft.-tons, and its velocity is 1130 f.s.

From what has already been said, it will be seen that the areas bounded by the pressure curves and the line OA represent the work done by the two charges respectively; the greater area belongs to the charge which gives the greater velocity to the projectile.

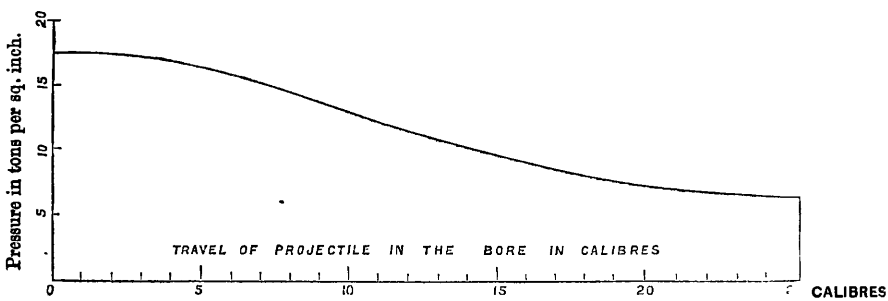
The pebble powder charge takes about four times as long to explode as the R.L.G. With the former the projectile has moved a much greater distance (about 12 times as far) before the maximum pressure is attained, and then the pressure is less than with the other charge; but as more gas is produced with the 70-lb. charge than with that of 60 lbs., the pressure with the larger amount is more sustained, and a greater total of work is given in the whole length of the bore.

It will be seen from the diagram that at about 1·7 feet travel of the projectile the velocity given by both charges is the same (about 1000 f.s.). It will also be noticed that long guns are most suitable for use with the slow burning powder, as an equal addition of length produces a larger addition to the area (or work) with the pebble than with the R.L.G. charge.

Great alterations have been made in the powder employed for gun cartridges, since the experiments above referred to were made. Fig. 6 (drawn to the same scale as Fig. 5) represents the sustained pressures in a modern, high velocity, long 9·2-inch B.L. gun, and it shows (from its greater area) the proportionate increase of work given by the cartridge, although the maximum pressure is by no means excessive.

Fig. 6.

Curve of Pressure. Ratio of weight of charge to projectile about ·6, derived from experiments with 9·2-inch B.L. gun.



Note.—These figures of the work done in the bores of the guns suggest at once the indicator diagrams of a high pressure, quick cut off, engine, such as Corliss', where great expansion is given; and one working at a lower pressure with more steam, slower cut off, less expansion, and consequently more sustained pressure. With the steam engine the former is the economic method; but with the gun, economy in powder gives way to the necessity for high velocity, which can only with safety be attained by large charges of slow-burning power.

As an instance of the qualities now wished for in a gun charge, we may quote from Colonel Maitland's lecture (R.U.S.I., 1884), that powder for the new breech-loaders was asked for which, with the usual charge, would only give a M.V. of 800 f.s., and 5 tons pressure in the 38-ton R.M.L. proof-of-powder gun; this was supplied, and under the altered conditions of a breech-loader with a holding band on the projectile, which prevented motion till a pressure of one or two tons per square inch was developed, the M.V. was 1900 f.s., and the pressure rose to 15 tons on the square inch.

Gravimetric Density.

The **gravimetric density** of a charge of powder in the chamber of a gun is the ratio of its weight to the weight of that volume of water which would fill the space behind the projectile in the gun. It is the mean density of the grains of powder and of all the interstitial and other spaces. A gun charge is expressed thus: $75\ P^2 \frac{33}{0{\cdot}840}$, which means a charge of 75 pounds of P^2 powder; with 33 cubic

inches allotted to each pound of gunpowder when in the chamber, and a consequent gravimetric density of 0·840.

If the gravimetric density is unity, it occupies the same volume as an equal weight of water at standard temperature, viz., 27·73 cubic inches per pound.

Hence in the above-mentioned gun charge the gravimetric density is obtained from this proportion—

33 : 27·73 :: 1 : gravimetric density .... (i)

whence gravimetric density = 0·840—*a result less than unity.*

Table I (p. 278) is calculated by making this proportion, and gives the G.D. corresponding to different cubic spaces per lb. of charge.

Captain Noble has calculated a table of **Expansions** (*see* Table II, p. 279) giving the amount of work which 1 lb. of powder of unit gravimetric density will perform when allowed to expand to different numbers of volumes. **Expansions.**

If the gravimetric density is less than unity, the expansion from unit density must be supposed to have taken place without performing work. The work due to this expansion must therefore be subtracted from the work due to the total number of expansions in the bore, which is—

$$\frac{\text{volume of the whole of the bore in cubic inches}}{\text{number of pounds of powder in the charge} \times 27{\cdot}73.}$$

To find, for instance, how many times the gas of powder of unit gravimetric density must expand to fill a space of 33 cubic inches to the pound, this proportion must be made—

27·73 : 33 :: 1 : No. of vols. of expansion for which work is lost ...... (ii)

whence No. of vols. required = 1·190—*a result greater than unity.*

The proportions made in (i) and in (ii) resemble each other, but a moment's consideration will make it plain which of the two to use in any case, as the one result is *less* and the other *greater* than unity.

If the gravimetric density of the charge 0·840 is given, the number of vols. of expansion for which work is lost is obtained from this proportion—

0·840 : 1 :: 1 : No. of vols. for which work is lost .... (iii)

whence No. of vols. required = 1·190—the same result of course as in (ii).

This calculation need not be made in practical examples, since the first column in Table II, headed "Number of volumes of expansion," gives the number of volumes whose expansive work is practically lost, when the gravimetric density is equal to a number expressed in the column headed "Corresponding density of the products of explosion," in the same table.

It has been found, that when the gravimetric density of a charge is decreased, the pressures and the velocity of the projectile both fall off. When a certain additional quantity of powder is added (keeping to the same decreased gravimetric density of charge), the velocity of the projectile becomes as great or greater than before, but the maximum pressure in the bore is found to be less than with the smaller charge of greater gravimetric density. This is advantageous, and great use has been made of this method, which is termed "air-spacing." But with the more modern powders, now being introduced, this device may be no longer necessary; the proof charges of some guns have a G.D. greater than unity.

The methods of using Table II are best illustrated by examples:—

Example 1. Suppose the volume of the bore of a gun to be 1386·5 cubic inches, and the charge 10 lbs. of powder (grav. density 0·8), the latter if compressed to unit gravimetric density would fill a space of $27{\cdot}73 \times 10 = 277{\cdot}3$ cubic inches; thus the charge can expand $\frac{1386{\cdot}5}{277{\cdot}3} = 5$ times.

Table II, p. 279, states that each pound of powder will realise 91·385 foot-tons of work for this expansion; but from this must be deducted the loss (19·226 foot-tons per lb.), due to the expansion (1·25) from a supposed gravimetric density of unity to that of 0·8 (see Table II); therefore the amount becomes $91{\cdot}385 - 19{\cdot}226 = 72{\cdot}16$ foot-tons per lb. of powder.

This must be multiplied by the number of pounds in the charge to obtain the total theoretical work which can be put into the projectile; in this case it is $72{\cdot}16 \times 10 = 721{\cdot}6$ foot-tons.

Only a proportion of this, called the **factor of effect**, is, however, really obtained. Loss of energy results from heat being communicated to the bore and other causes; this is generally more the case with the smaller than with the heavier natures.

Suppose in the case we are considering the factor of effect is 0·7, the total work realised is 0·7 of $721{\cdot}6 = 505{\cdot}12$ foot-tons.

The exact determination of the factor of effect depends on several circumstances, and it can only be approximated to beforehand. It is influenced by the quality of the gunpowder, amount of air-space, windage, cooling surface of the bore, frictional resistance of the studs or gas check, and also by the weight of the projectile, as the heavier it is the more slowly will it take up velocity at starting, and thus more time is given for the development of high powder pressures, and the factor of effect is increased. A similar effect has been produced by the use of copper rotating rings in some of the new type B.L. ordnance; these prevent the ready passage of the projectile, and increase the powder pressures.

To take a practical illustration, a 10-in. gun was fired with charges of 130 lbs. and 140 lbs. with gravimetric densities of 0·792 and 0·840 respectively. It was easily found, as above, that one charge could expand 4·294 times, the other 4·050 times.

From Table II (p. 279) taking differences, we have as the work attained by the projectile per lb. of charge—

for the 130 lbs. charge $85{\cdot}13 - 20{\cdot}00 = 65{\cdot}13$ ft. tons.
for the 140 lbs. charge $82{\cdot}37 - 15{\cdot}40 = 66{\cdot}97$ „

The total theoretical amount of work would therefore be in each case—

$$\left.\begin{array}{l}\text{for the 130 lbs. charge } 65{\cdot}13 \times 130 = 8466{\cdot}9 \text{ ft. tons.}\\ \text{for the 140 lbs. charge } 66{\cdot}97 \times 140 = 9375{\cdot}8 \quad ,,\end{array}\right\} \ldots \text{(iv.)}$$

The velocities with each charge were found from experiment, and the weights of the projectiles being known, the energy actually developed in each was found from the expression $\frac{wV^2}{2g \times 2240}$ to be—

$$\left.\begin{array}{r}\text{for the 130 lbs. charge} = 7158 \text{ ft. tons;}\\ \text{and for the 140 lbs. charge} = 8092 \quad ,,\end{array}\right\} \ldots \text{(v.)}$$

dividing the realized work in (v) by the theoretical work in (iv) in each case, factors of effect of a little under and a little over 0·85 are obtained.

Knowing the work put into the projectile, we can calculate its velocity from the relation—

$$\frac{wV^2}{2g} = \text{energy in ft. lbs.,}$$

$$\text{or } \frac{wV^2}{2g \times 2240} = \text{energy in ft. tons,}$$

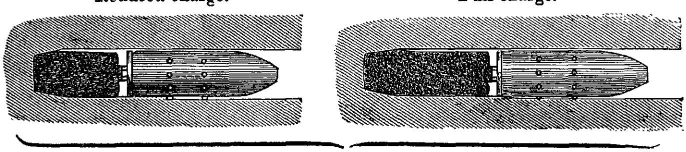
and thus the probable velocity in experimental guns has been estimated beforehand with fair accuracy.

The question of gravimetric density is of the greatest practical importance, as muzzle velocity and range depend upon it; the employment of gas-checks in heavy R.M.L. guns has brought this out in a striking manner. When they were first used the ranges were considerably increased, as leakage of the expanding powder gas was prevented; but when Mark II gas-checks with projections were issued, the ranges decreased greatly; thus when the elevation for 1200 yards was given, the projectiles fell 150 yards short, and there was a proportionate loss of range at other elevations. A little consideration makes the reason plain; with the reduced charge for, say, a 10-inch gun, the studs (Fig. 7) go nearly to the ends of the grooves, and the

Fig. 7.

10-INCH R.M.L. GUN.

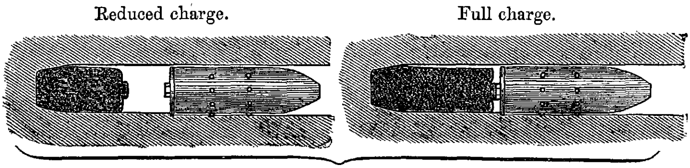
Reduced charge. Full charge.



Mark I Gas-Check (without projections).

Scale $\frac{1}{36}$th.

Reduced charge. Full charge.



Mark II Gas-Check (with projections).

base of the projectile, or rather the smooth Mark I gas-check attached to it, just touches the cartridge; but with the Mark II gas-check the projections on it cause the projectile to be stopped some 10 inches short of its former position, and thus there is an empty space between the cartridge and the projectile, altering the gravimetric density of the charge considerably. Calculations by Table II, page 279, of the work lost by the expansion of the charge when the increased air space is given, and the consequent effect on muzzle velocity and range, agree fairly well with the results actually attained; but it is evident at a glance

that there must be *some* falling off in muzzle velocity, for the pressure of the powder gas only acts in the bore for about 9 feet 2 inches, whereas in the former case it acted over a distance of 10 feet. With the full charge, however, scarcely any difference in range is noticed, whether Mark I or Mark II gas-check is used; and this, too, is evident from Fig. 7, in which it appears that the long cartridge itself stops the projectile in each case at about the same spot. As increased charges have been used with the heavy muzzle-loaders, it is evident that their range-tables have fluctuated a good deal of late years from several causes.

Some matters in internal ballistics, as might be expected, still present difficulties; occasionally, at experimental practice, abnormally high pressures are obtained, as on one occasion when a very small quantity of powder of a finer grain was mixed with a large charge of pebble powder*: this is said to be due to "wave action;" but it is not thoroughly understood, though it is known that certain circumstances, such as very long cartridges, tend to produce this result.

With all the newly invented explosives of late years, it is somewhat strange that gunpowder has held its place as a propelling agent, and only lately, when cocoa or prism brown powder has been adopted, has any alteration been made in its chemical composition: the necessary changes in rate of burning being effected by modifications in the mechanical treatment in manufacture and size of grain. Chemical explosive compounds, such as gun-cotton and nitroglycerine, are too violent in their action: their work is to blast and tear, and not to act as propelling agents. Gun-cotton in different forms has been used for small arms; but when experiments were made some years ago in the Royal Arsenal, it was found that now and then a barrel would tear open: this was probably due to detonation of the cartridge set up by a rather larger quantity of detonating composition in the cap than usual. Detonation causes great maximum pressure, and it is a very rapid action; the velocity of its transmission in gun-cotton has been found from both English and French experiments to be about 17,000 f.s. for dry, and about 20,000 f.s. for wet gun-cotton.

A few problems on work and expansion are appended:—

Example 2. At what velocity must a projectile move to have half the energy which it had when travelling at 1000 f.s.?

Let v be the required velocity.

$$\text{Consequently } \frac{w\,(1000)^2}{2g} = \frac{2wv^2}{2g},$$

$$\text{whence} \qquad v^2 = \frac{1000^2}{2},$$

$$\text{or} \qquad v = 707{\cdot}1 \text{ f.s.}$$

Example 3. Suppose a 64-pr. projectile has $M.V.$ of 1000 f.s., and a 32-pr. „ „ 2000 f.s. compare their energies.

	For 64-pr.		For 32-pr.
	$\frac{64(1000)^2}{2g}$	:	$\frac{32(2000)^2}{2g}$,
or	1	:	2

---

* This mixture has since been adopted for the bursting charges of shells, *see* p. 233.

Hence the 32-pr. has twice the energy of the 64-pr.

If the *M.V.* of a filled common shell of 64 lbs. weight is 1383 f.s., what will be the *M.V.* if by accident the shell is fired empty? Weight of bursting charge 7 lbs. 2 ozs. Example 4.

The weight of empty shell = 64 lbs. − 7 lbs. 2 ozs.
= 56·875 lbs.

Assume the amount of work given to the projectile to be the same in each case as the expansions are the same (this is only an approximation, the lighter shell really receives a little less work, as it moves off more readily than the other).

$$\therefore \frac{64 \times (1383)^2}{2g} = \frac{56{\cdot}875 \times V^2}{2g},$$

whence V = 1467 f.s.—an increase of 84 f.s.

A certain charge with an experimental field gun gave a muzzle velocity of 1670 f.s. to its 12-lb. projectile, when the calibre was 3 inches; but when the calibre was increased to 3·2 inches (with the same weight of projectile and charge) the *M.V.* was 1700 f.s. Why was this increase? Example 5.

The capacity of the bore was enlarged, and the number of expansions of the powder charge increased: hence more work, involving a greater velocity, was given to the projectile.

If a charge of 10 lbs. of unit gravimetric density is allowed 9 expansions in the bore of a gun, what must be the *M.V.* of the projectile if its weight is 64·4 lbs. and the factor of effect of the gun is 0·9? Example 6.

From Table II, p. 279, 9 expansions per lb. of charge give .... 113·937 ft.-tons.
∴ for 10 lbs. charge .... 1139·37 ,,
The factor of effect is 0·9.
∴ the work realised is 0·9 of 1139·37 = 1025·43 ,,

Now the energy of the projectile $= \dfrac{wV^2}{2g}$ ft. lbs.

$$\therefore 1025{\cdot}43 = \frac{64{\cdot}4V^2}{2g \times 2240} \text{ ft.-tons.}$$

whence V = 1516 f.s.

Suppose, in the last example, that the projectile was not rammed home, and that consequently the space for the cartridge was doubled: find the *M.V.* to be expected. Example 7.

As before, for 9 expansions, the work per lb. of charge is	113·937	ft.-tons.
In this case the work lost in expanding from 1 to 2 volumes must be deducted as practically lost: from Table II it is per lb. of charge	49·050	,,
∴ work per lb. of charge becomes..	64·887	,,
for 10 lbs.............	648·87	,,
Taking account of the factor of effect 0·9, it becomes...................	583·983	,,

$$\text{The energy of projectile} = \frac{wV^2}{2g}\ \text{ft.-lbs.}$$

$$\therefore\ 583{\cdot}983 = \frac{64{\cdot}4V^2}{2g \times 2240}\ \text{ft.-tons.}$$

$\therefore\ V = 1144$ f.s., — a considerable decrease of velocity to that attained in Example 6 with the same weight of projectile and charge; this is an effect similar to that produced by firing shells with reduced charges and Mark II gas checks, see p. 25.

Example 8. An experimental gun is to be designed to fire a projectile of 46·34 lbs. with *M.V.* of 1500 feet. How can the charge and length of bore be determined?

$\dfrac{wV^2}{2g \times 2240}$ can be found; it is 723 ft.-tons.

Assume a factor of effect from previous experience with other guns of about the same calibre with the same powder, suppose it is 0·8.

Then $723 \div 0{\cdot}8$ is the *theoretical* amount of work furnished by the charge $= 903{\cdot}8$ foot-tons.

Now, suppose it is assumed that five expansions shall be given to the charge (consult Table II, p. 279), we find that if the gravimetric density is unity each lb. of powder then gives 91·385 ft. tons of work.

$$\therefore\ \frac{903{\cdot}8}{91{\cdot}385} = 9{\cdot}89 \text{ lbs. will be required.}$$

The length of bore of course follows: and if this is found to be inconvenient a different number of expansions must be assumed and fresh calculations made until the necessary conditions are fulfilled.

If the gravimetric density is not unity; suppose it is 0·847, this is the same as losing expansions to 1·18 volumes (see Table II),

$$\text{for } 0{\cdot}847 : 1 :: 1 : 1{\cdot}18.$$

Therefore, the theoretical amount for each pound of powder is (see Table II) $91{\cdot}385 - 14{\cdot}725 = 76{\cdot}660$.

And the total amount required is—

$$\frac{903{\cdot}8}{76{\cdot}66} = 11{\cdot}8 \text{ lbs.}$$

CHAPTER III.—GENERAL COMPARISON OF GUNS.

A **steam engine** and a **gun** can fairly be **compared** as machines for obtaining work in a useful form out of a source of power: the difference being that one produces it almost continuously, the other intermittently. Comparison of a steam engine and gun.

Each pound of ordinary mixed Welsh and North Country coal (such as is used on board Her Majesty's ships) contains about 5118 foot-tons of work, according to Joule's heat equivalent. The furnaces of H.M.S. "Serapis" consume, on an average, about 64·4 lbs. of coal per minute when working at ordinary half-speed on long voyages, or $5118 \times 64{\cdot}4 = 329{,}600$ foot-tons of work are drawn upon.

During the same time the engines give some 17,325 foot-tons of work to the ship by maintaining its velocity in opposition to the resistance of the water, thus only $\frac{17{,}325}{329{,}600}$ or 0·0526 of the total work in the coal is realised by the ship.

Gunpowder only affords some 486 ft.-tons per lb.: it consists largely of saltpetre for the purpose of supplying oxygen for the combustion of the charcoal and sulphur, whereas coal draws its supply of oxygen from the atmosphere, and a certain amount of work is employed in causing the oxygen in the saltpetre to assume the gaseous form.

A charge of 295 lbs. of prism brown powder exploded in the bore of a 45-ton gun produces 18,060 foot-tons of work or energy in the projectile; the proportion here realised is $\frac{18{,}060}{486 \times 295} = 0{\cdot}1260$,—very much better than with the steam engine.

Thus, supposing it were possible to continue discharging a 45-ton gun at the rate of one round per minute, it would produce rather more work per hour than the engines of the "Serapis" under ordinary circumstances.

A rough comparison is shown in the following:—

TABLE A.

Machine.	Source of Power.	Proportion of work realised from source of power.	Quantity of work in foot-tons realised from 1 lb. of each source of power.	Relative cost of work as far as source of power is concerned.
Engines of "Serapis".......	Coal.	0·0526	269·0	1
45-ton Gun......	Gunpowder.	0·1260	61·2	290

From which we see that the gun usefully realises a much larger proportion of work out of the gunpowder than the engine and boiler can obtain from the coal; in fact, the gun (in this view) is the less wasteful machine of the two. But as, in equal weights, the coal contains far more work than the gunpowder, the absolute amount realised per pound of fuel is considerably greater in the engine: and with regard to the expense of fuel consumed for equal quantities of work produced, the cost with the gun is nearly 300 times as much as with the engines.

Basis of Comparison of Guns with each other.

As it is generally a matter of importance to keep the weights as low as possible, **guns** may best be **compared with each other** by the amount of work realised by the projectile per ton of weight of the piece.

Engines may be compared with each other according to the proportion of work which they can realize out of each pound of coal; for instance, in the Cornish pumps for mines weight is of little account, the great object being to obtain as much work as possible out of each pound of coal for economic reasons.

The same comparison *could* be made among guns, but with them the economy resulting from saving gunpowder gives way to the necessity of keeping the weight of the gun at a minimum to allow of rapid laying, and with field and siege guns this is rendered still more important by the necessity of moving them from place to place as quickly as possible, often over rough ground.

Therefore with guns the amount of **work** producible with safety, **per ton weight of gun,** offers the best basis of comparison.

Thus, comparing the 12-pr. (screw) R.B.L. of 1860 and the 13-pr. R.M.L. of 1878, which are both of the same weight (8 cwt.), the former (with a charge of $1\frac{1}{2}$ lbs.) produces 118 foot-tons of work in the projectile, while the latter with a charge of 3 lbs. 2 ozs. realises 220 foot-tons —nearly double as much. But if the comparison is made according to the amount of work produced per lb. of powder in the charge, the advantage is with the older piece, the numbers being 78·7 and 70·4 foot-tons respectively: this latter consideration, however, is not of much importance practically, and the first named comparison is the better one to make. Of course (other things being equal) it *is* a drawback to use more powder for a given amount of work, but this gives way before the more important consideration of lightness in the gun, and considerable energy in the projectile.

Taking this view, the undermentioned table of field guns is of interest. Approximately, the same results are yielded by other natures; bronze pieces are, however, rather the most favourable specimens of smooth-bores.

TABLE B.

Name of piece.	Weight of piece.	Calibre in inches.	Weight of charge.	Weight of projectile.	Muzzle velocity of projectile.	Muzzle energy of projectile.	Muzzle energy of projectile per ton of gun.	Bursting charge in common shell.	Date of manufacture.	Remarks.
	cwts.		lbs. oz.	lbs. oz.	f.s.	ft.-tons	ft.-tons	lbs. oz.		
9-pr.	13½	4·2	2 8	9 5¾	1613	169	250	0 0	..	S.B. Combined in one battery, gun for hard hitting, howitzer for shell power.
24-pr. Howzr.	12½	5·7	2 8	16 11½	1252	182	290	1 0	..	
12-pr.	8	3·0	1 8	11 4	1239	118	295	0 8	1860	R.B.L. (screw)
9-pr.	8	3·0	1 12	9 1¾	1380	118	295	0 7½	1871	R.M.L.
16-pr.	12	3·6	3 0	16 4	1355	207	345	1 2	1872	,,
9-pr.	6	3·0	1 12	9 1¾	1390	122	408	0 7½	1874	,,
13-pr.	8	3·0	3 2	13 1	1560	220	541	0 10	1878	,,
12-pr.	7	3·0	4 0	12 5	1710	254	726	0 8	1882	B.L.

From this table it is seen that the first rifled guns were less powerful, *i.e.*, they produced less energy at each round than their smooth-bored predecessors: the projectiles with 169 foot-tons of energy from the 9-pr. and 182 foot-tons from the howitzer were replaced by one with only 118 foot-tons from the rifled gun, while the bursting charge of the howitzer's common shell of 1 lb. was superseded by half that amount in the 12-pr. R.B.L. (screw); the latter gun was, however, lighter than the older pieces.

The defects of the first rifled guns came into notice in the campaigns in which they were used.

In China, Armstrong field guns failed to set fire to junks, which 24-pr. S.B. howitzers, with their larger bursting charges, afterwards succeeded in doing. The effect of the small bursting charges of the shells of the rifled guns against earthworks was noticed to be slight, and at short ranges it was found that they were not so effective as the old smooth-bores. In New Zealand also the want of a shell with a large bursting charge for the field piece was much felt, and the effect on earthworks was inconsiderable.

The defect in muzzle energy was particularly noticeable with **case-shot**, as the velocities in the S.B. and early rifled systems were about as under (considering each case shot as a whole) :—

TABLE C.

Nature of piece.	Weight of piece.	Weight of case.	Velocity of case.
	cwts.	lbs. oz.	f.s.
9-pr. Gun, S.B.	13½	13 9	1340
24-pr. Howitzer, S.B.	12½	13 13	1380
12-pr., R.B.L. (screw)	8	11 8	1225

We notice here that the case-shot of the rifled gun was about 2 lbs. lighter, and its velocity was from 115 to 155 f.s. less than with the S.B. pieces, giving very much less energy. The rifling of the bore is a positive disadvantage when case-shot is employed, as it tends to make the bullets scatter too much from centrifugal action.

With the present service field guns, however, the case-shot is more effective than with the old smooth-bores, since the powder charges have been increased, and the total energy as seen from Table B, p. 31, is now greater than ever.

The above facts explain the reason for the adoption of 32-pr. S.B. guns converted into breech-loaders for rapidity of fire with case in flanking ditches.

Many have said that with long ranging and accurate guns and small arms, case-shot will be very seldom required, but this hardly seems to be borne out practically, for we find a Russian officer* writing of his experience at the battle of Dzuranli in the Russo-Turkish war, 1877–78, as follows:—

"Suddenly the artificer of No. 4 gun, who was on horseback, shouted out 'The Turks! the Turks! there they are!' and pointed with his finger to a maize field. There was no doubt of it: at 300 paces from the battery among the stalks of the maize, we clearly distinguished the red fez—the moment was critical.

Once more the order went forth 'Case-shot,' and hardly were we loaded when the Turks discharged a volley right in our faces, and we received a shower of bullets. . . . . One after another the guns belched forth case-shot against the Turks. . . . . This was the final and desperate effort that the Turks made against our centre: the most vigorous and energetic that took place in this action, according to official accounts. . . . . At this critical moment the Chasseurs reappeared on the right flank, and fell on the left of the Turks . . . . and the fate of the battle was decided.

. . . . It is worthy of remark that at a period, when in consequence of the improvements in firearms, artillery is being obliged to modify its system of fighting in order to operate at much longer ranges, and when many artillerists are considering case-shot as being hardly worth discussion, a battery of 4-prs. for some minutes at 200 to 300 paces resisted the enemy's infantry and was able to maintain itself and save its guns, expending all its case-shot, and suffering comparatively small losses.

This cannot be considered an exceptional circumstance, for in

* Pro. R. A. I., vol. xi. Trans. by Captain Dalton, R.A., from "Memorial de Artillería."

this campaign it was constantly happening—above all when acting on the defensive."

And on the 8th February, 1881, at the engagement at the Ingogo with the Boers, Lieut. Parsons, R.H.A., wrote—

"Eventually the guns commenced firing case upon the enemy, who were almost on the very crest of the plateau about 300 yards from us. All the case was soon expended, and we then used shrapnel shell inverted with indifferent result. However, it sufficed to prevent the enemy charging the guns."*

The question now arises, Why did these defects exist in the earlier rifled guns?

The following considerations help to answer the question:—The calibre of a rifled piece must of necessity be less than that of a S.B. for projectiles of the same weight.

Thus the (see Table B, p. 31) calibre of 9-pr. S.B. is 4·2 inches,
" " " " " 9-pr. rifled gun is 3 inches.

Hence it follows that if the bores are of equal lengths, the capacity of the S.B. must always be much greater than that of the other; and an equal charge has more room for the expansion of the gas produced on firing, and consequently it can impress a greater fraction of its total energy on the projectile. The rifled guns had smaller charges, and even then the amount of expansion allowed to the charge was often less than before.

Why, however, were the remarkably low charges of only one-eighth the weight of the projectile (see Table B, p. 31) employed with the early rifled guns?

An elongated projectile offers a less surface for the pressure of the powder gas than a spherical one of the same weight: and, therefore, its resistance to motion must be greater: hence the stress on the gun is increased. In the early rifled guns, which were breech-loaders, the lead coat was forced into the grooves, and a narrow part or "grip" just in front of the chamber hindered its first movement. These arrangements were favourable in enabling the projectile to receive considerable pressure, and in centering it for accurate shooting; but they imposed extra stress on the gun, and small charges of quickly burning powder were used. The system of giving rotation by means of lead-coated projectiles did not allow the use of high velocities, as it was then impossible to prevent stripping or tearing off of parts of the leaden envelope, which would fly forward in a destructive manner—a great disadvantage if troops are advancing in front of the guns.

Rifled pieces had been invented for more than 300 years, but it was only when guns of greater strength on the built-up principle were adopted that their general introduction became possible. Cast iron was quite out of the question for the increased stresses, and bronze was not suitable on account of its softness, as the grooves wore away, and it was difficult to obtain uniformity in the alloy.

This being so, why were rifled guns introduced, notwithstanding the defects we have noticed?

It was on account of their marvellous accuracy compared with the smooth-bores, and although the muzzle velocities were lower, yet, since the retardation due to the resistance of the air was less than with round shot, a rifled elongated projectile struck a distant object

* Pro. R. A. I., vol. xi.

with a greater energy than a smooth-bore, although it had started with less; ranges were also increased. This was borne out by the records of active service; as in China, New Zealand, Japan, and Cape Haytien, the practice at long ranges was found to be accurate and good.

Other **advantages of rifled guns** are—

(1.) The bursting charges of shells of the same weight are increased.

(2.) The pointed shape of the projectiles gives better penetration.

(3.) All the projectiles of one nature of ordnance can be made of about the same weight, and so range tables are simplified.

(4.) Since the head always travels first, the action of percussion fuzes can be simplified.

The first alteration of rifled guns in our Service was the adoption of the M.L. system for the sake of simplicity, as it happened that the particular plan of breech-loading adopted had several serious defects, and it could not be applied to very heavy ordnance. The first R.M.L. guns were not so accurate as the breech-loaders, and they showed no improvement in power; thus the 9-pr. R.M.L. of 8 cwt. and 12-pr. R.B.L. have just the same muzzle energy of projectile per ton of gun (see Table B, p. 31).

Guns were really not improved for some time after muzzle-loaders were introduced, and power was sought for by making heavier and heavier pieces, as this was rendered possible by the system of gun architecture adopted; the muzzle velocities of the projectiles remained about the same, or only slowly increased. The 16-pr., however, was a really powerful field gun with a heavy projectile and considerable velocity, giving an energy of 345 foot-tons per ton of gun, and having a bursting charge of 1 lb. 2 oz. in its common shell; it was said to be, for a short time, the most powerful field gun in Europe.

But progress was now being made in another direction: previous endeavours had been chiefly to strengthen the gun. It became evident that the action of the powder must be modified, and more room must be given for its expansion. This was effected in the long 9-pr. of 6 cwt., which in its turn is seen (Table B) to surpass all its predecessors in its proportions with 408 foot-tons of energy per ton of gun.

Other improvements are illustrated by the 13-pr., in which advantage is taken of air-spacing and chambering to prevent the maximum strains from being too severe, while at the same time the large charge gives a continuous pressure to the projectile all the way down the bore, and consequently a high velocity is attained. The 12-pr. B.L. of 7 cwt. surpasses all its predecessors, being the latest design.

But it is with the heavier guns that the highest velocities are attained, these are due to the large charges of the very slow burning powder now consumed in their long bores. Improvements in the carriages and platforms have led to the control of recoil which otherwise would be excessive.

The great advantage of the new high velocity heavy guns was well seen in the late war between Chili and Peru, when the "Angamos" bombarded Arica with destructive effect at a range of 8000 yards,* while the old type guns of the Peruvians were unable to reply, as their opponents were beyond their range.

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* See Journal R. U. S. I., vol. xxv, No. CXII. "Incidents of the War between Chili and Peru," Lieut. Madan, R.N.

Summary.

The progress of late years may be **summarised** as follows :—

S.B. Guns.—With cast-iron and bronze guns very good materials were chosen. Care was exercised in manufacture, and a rigorous proof was undergone by each piece before it was admitted into the Service.

R.B.L. (Screw) Guns.—In consequence of the knowledge of the principles of gun construction, increase in manufacturing skill, and advances in metallurgy, R.B.L. (screw) guns were introduced with their very moderate velocities, but with their great advantages of accuracy, range, and increased shell power.

Muzzle-Loaders and Heavy Guns.—It was found that the R.B.L. system adopted had many objections, and was unsuited to heavy guns. On the plea of simplicity, muzzle-loaders were adopted with hardly any increase of muzzle velocity, and attention was principally directed to the attainment of power by the manufacture of very heavy guns ; this was rendered possible by the methods of gun construction adopted.

Greater Energy without increase of Calibre.—Gradually it began to be recognised that the velocities of the projectiles must be increased : this was done by improvements in the strength of the guns, higher charges, as in the 16-pr., and longer bores, as in the 9-pr. of 6 cwt.

High Velocities.—During the last few years great attention has been paid to the quality of the powder ; by a combination of air-spacing, to diminish the maximum pressure in the bore of the gun, and chambering to shorten the cartridge, very large charges can be burnt in the heavy long B.L. guns, while the projectile receives a well sustained pressure during all the time it passes down the bore, and it leaves the muzzle with a very high velocity, ranging far and accurately.

Recent experiments point to the possibility of giving up air-spacing and filling the whole of the large chamber of a modern gun with a very slow-burning powder, which will give a well-sustained pressure without an undue maximum strain to the piece.

An illustration of the method of finding the muzzle energy per ton of gun is now given.

Compare the undermentioned pieces by the amount of muzzle energy per ton weight of gun realised by each— Example 1.

64-pr. R.M.L. gun of 64 cwt. firing a projectile of 67 lbs. 2 ozs. with *M.V.* of 1260 f.s.

6-in. B.L. gun of 80 cwt. firing a projectile of 80 lbs. with *M.V.* of 1880 f.s.

This is done by finding in each case the value of the expression—

$$\frac{\text{Muzzle energy } \textit{in foot-tons.}}{\text{Weight of gun in tons.}}$$

It is—

For 64-pr. gun. For 6-in. gun.

$$\frac{\dfrac{67\frac{1}{8}(1260)^2}{2g2240}}{\dfrac{64}{20}} = 231 \text{ ft.-tons} \qquad \frac{\dfrac{80(1880)^2}{2g2240}}{\dfrac{80}{20}} = 490 \text{ ft.-tons}$$

The 6-inch gun, which is of modern design, is thus shown to be far superior to the older 64-pr. gun.

CHAPTER IV.—STEEL FOR ORDNANCE.*

Materials. THE *materials* for gun making hitherto employed in our Service were steel tubes and wrought-iron coils; but the great improvements in the manufacture of mild steel of high tenacity and considerable elasticity, have now led to the adoption of ordnance made entirely of steel.

The hardness of steel and its power of resisting the corroding influence of fired gunpowder, and the wearing action of the studs on the grooves, added to its superior strength, long ago caused it to be universally employed for the bores of guns. It used to be said that the wrought-iron coils, being fibrous, would give way gradually, if the strength of a gun were overmatched; while a steel gun, under the same circumstances, would give no warning, but would shiver like glass on account of its brittleness. Wrought iron can, however, be broken to show a short crystalline fracture, if the breaking strain is very considerable and sudden, when the fragments will fly apart with great violence; and on the other hand, mild steel has of late been so much improved in manufacture, that it is now solely employed in gun making; hence some account of the progress of steel and it application to military purposes is important in gunnery.

Statistics. The **statistics of iron and steel** are very suggestive of the conditions of the times. In the middle of last century the amount made in England was about one five-hundredth part of what it is at present; the amount then imported was more than that manufactured at home, whilst now we annually export millions of tons; the last few years have shown a striking increase in the world's annual production, though great fluctuations have occurred; the amount produced has risen from some 10½ million tons in 1869 to about 21 millions in 1882—an enormous increase, due to the vast development in the use of iron and steel for ships, railways, bridges, buildings, machines, &c. Apart, however, from the increase in the total quantities, we notice (Fig. 1) that cast iron, wrought iron, and steel have not increased uniformly, but the last has far surpassed the others in the rate of its growth, having multiplied some 12 times in the 13 years under consideration.†

Although steel has been made from remote ages, it was only about 25 years ago employed on a comparatively small scale for such articles as tools, knives, swords, springs, &c., and from the expenditure of time and fuel on its manufacture, it was necessarily an expensive article; but the inventions of Bessemer, Siemens, and others have resulted in the production of a class of "mild" steel in large quantities, moderate in price, suitable for various purposes of construction, and surpassing wrought iron in all, or almost all, of its good qualities. Of late years competition in the trade has led to numberless improve-

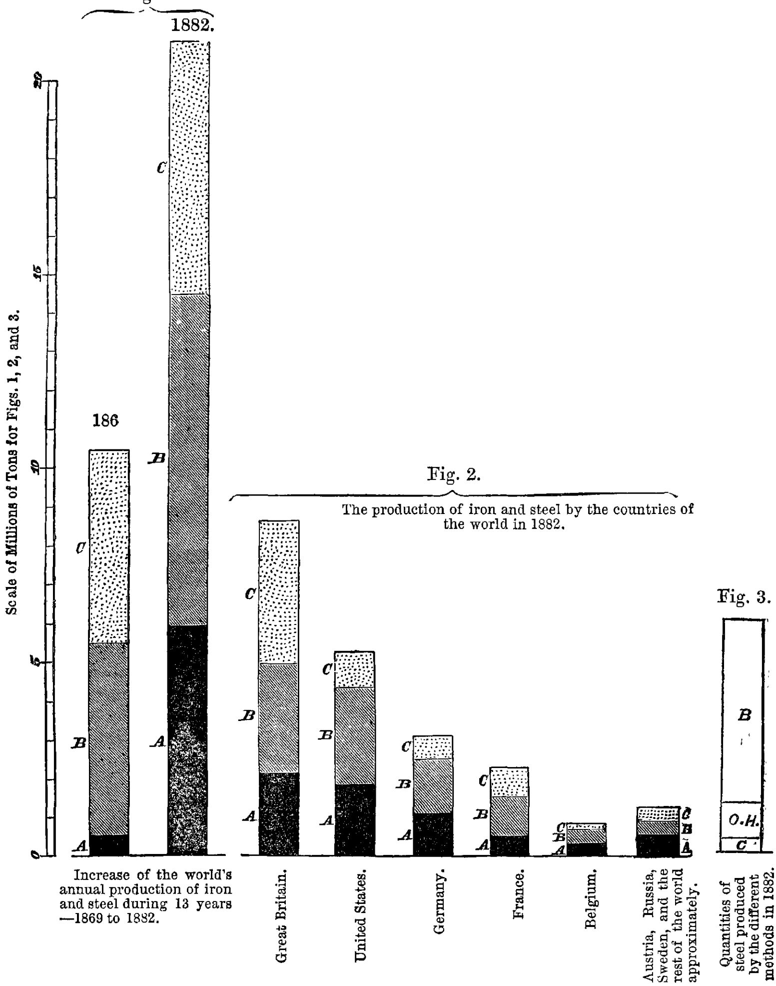
* This chapter is the substance of papers read at the R. A. Institution and the R. U. S. Institution by the author.

† *Vide* President's address, Iron and Steel Institute, 1883.

ments, and great economy in manufacture, especially in the amount of fuel consumed.

It may be interesting to notice the **proportion of iron and steel** annually produced (in 1882, for example) in each country of the world.* We are at once struck by the salient feature

Fig. 1.



Figs. 1 and 2: *A*, steel; *B*, wrought-iron; *C*, cast-iron. Fig. 3:—*B*, Bessemer; *C*, Crucible; *O.H.*, open hearth.

that **Great Britain** is far ahead of all others in *quantity;* in fact, excluding the United States, her production probably exceeds that of all the rest of the world put together. With regard to

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* *Vide* "Journal, Iron and Steel Institute," 1883. Fig. 1, and several others, are reproduced from Pro. R. A. I., vol. xii.

quality, the reputation of English iron and steel is deservedly very high; the great majority of modern improvements in manufacture are due to English inventors, practically developed by English makers, and many foreign firms are glad of English assistance. Iron ores and good coal, as well as materials for fire-bricks and fluxes, are found abundantly in our island, while our great carrying trade gives facilities for the plentiful supply of good ores from Spain, and pig iron from Sweden for modern steel making, as well as for the export of manufactured iron and steel to foreign customers. The resources of other countries are, however, being largely developed.

Second on the list comes the **United States,** whose rate of increase of manufacture is unexampled. The demand has been hitherto so great, to meet the wonderful development of the railway system, and other large works, that millions of tons have been sent from this country across the Atlantic, but the time has now come when the United States produces sufficient for its own wants. Its natural resources are very great, and it has excellent ores of great richness and abundance, vast coal-fields, calculated to last for centuries, and a people full of energy. At first sight it may seem strange that at the present moment steel for only a 6-inch gun can be produced there, but this is simply because until lately there has been no demand for it. The colossal plant needed for heavy gun manufacture does not yet exist in America, where the heaviest hammer is one of 17 tons, while France has one of 100 tons. Whitworth has supplied the Government of the States with steel for 10-in. guns. The United States have lately sent a Gun Foundry Board to make inquiries in Europe concerning the manufacture of steel for military purposes: their Report has been published, and contains most valuable information, and orders have been given for the erection of a large steel plant suitable for making masses for guns, but not in a Government factory.

Next on the list of producers comes **Germany,** which possesses the largest manufactory in the world—that of Krupp—remarkable for its excellent steel, which soon attained a wide reputation, as shown by the fact that in 1865 England ordered from this firm, for her own use and that of her Colonies, no less than 11,396 tyres and 564 axles for railway purposes.* Krupp early applied his steel to the manufacture of ordnance, and he has supplied several nations with guns. Germany depends greatly on this maker for her ordnance, but the American Officers do not consider this a wise arrangement, as the Government may find it difficult to deal with a single private firm in times of great emergency.

Fourth in magnitude comes **France,** with her well-known Le Creusôt, Terre Noire, and other works. The first, with its massive plant and 100-ton hammer, has produced steel armour-plates of excellent quality; while the second has, after many difficulties, attained great success in steel castings, and in the manufacture of steel projectiles. Although the loss of Alsace has told heavily on the production of iron and steel by France, and though half her ores are imported, her progress is certainly very good, especially in the steel required for warlike purposes, as after the war of 1870 the French Government encouraged private companies to such an extent that several can produce the largest steel ingots, others have the plant of a gun factory, and others again are able to manufacture excellent armour-plates. The American Officers consider that France has made better

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* *Vide* "Steel," by J. S. Jeans.

arrangements than other nations between the Government and the steel makers.

Belgium has good supplies of coal, but having exhausted her own ores, has to import for the manufacture of steel; nevertheless she has made good progress, especially considering the means at her disposal, and she enters keenly into competition with England in certain departments of the trade.

Austria, though formerly celebrated for metallurgical manufactures, and possessing excellent ores, has been unable to keep pace with other countries in the production of iron and steel. The want of coal, and of easy communications, and perhaps the lack of organisation and capital, have proved hindrances to development.

Russia possesses excellent ores in the mountains of Siberia and the Ural, but the want of good means of transport and of coal prevents a large manufacture; mild steel has been made since 1876, and the production for military purposes is now considerable; as the Government of Russia seems anxious to supply itself with steel, armour and guns up to 50 tons are now made in that country: formerly Krupp supplied ordnance.

The best **Swedish** pig iron is of capital quality, since the ores from which it is produced are very pure, and the charcoal employed as the fuel in smelting being free from sulphur, does not contaminate the cast iron; but the want of coal limits the supply.

Spain cannot be called a manufacturing country; but the Bilbao ore has lately been very largely used in steel making; 20 years ago hardly any foreign ore was used in England; now we annually import about 3,000,000 tons from Spain, and other nations also supply themselves largely. This country is consequently important to the steel makers. Works for the manufacture of steel ordnance are being established at Trubia: the steel to be made from Spanish ore.

Italy has few iron or steel works; the Elba ore has long been famous, but it is not so important as formerly. The huge armourclad ships of Italy, with their very heavy guns, have, however, called for large supplies of mild steel for warlike purposes from abroad, and they have contributed not a little to the development of some of our private firms for producing heavy war material: Sir W. Armstrong and Co. have obtained a concession for establishing large factories for making guns and armour-plates in the neighbourhood of Naples.

The other nations of Europe produce but little iron and steel, and they are chiefly supplied from those already named. In the other countries of the world considerable progress has been made, especially in some of our Colonies; and in many places large supplies of ores are available.

Definition of Steel.

Steel is by no means easy to define, since it is a complex body, and as all persons are not exactly agreed concerning the difference between it and iron, lawsuits have consequently arisen; but the definition of mild steel given by Holley, that it is "an alloy of iron, cast, while in a fluid state, into a malleable ingot," has, at any rate, the advantage of simplicity, if not of perfect exactness. "Homogeneous iron" and "ingot metal" are names sometimes used instead of mild steel; blister steel which has not been melted cannot be included. This **definition** serves to distinguish it from cast iron on the one hand, and from wrought iron on the other: as cast iron is not malleable, but crumbles to pieces if heated and struck by the hammer; this plan is actually

employed in the Royal Gun Factory, in breaking up parts of old cast-iron guns into pieces of convenient size for manufacture (*see* Fig. R, Plate I, p. 56). Wrought iron, though malleable, is produced, not as a fluid, but as a pasty mass, the temperature of the ordinary furnace being insufficient to melt it.

Properties of steel.

Wrought iron is more fibrous than mild steel, and it contains, from the method of its manufacture, an average of 3 per cent. by weight, or $7\frac{1}{2}$ per cent. by volume of slag, dispersed through its mass in fine filaments, which detracts from its strength: on the other hand, the process of fusion in the manufacture of mild steel gets rid of all traces of slag; and this is a great advantage. (*Vide* lecture by Sir W. Siemens, F.R.S., Journal of the R.U.S. Institution, 1870.) The absence of slag probably accounts for the fact that steel resounds much better than wrought iron when struck. Steel bells have come into use on a considerable scale; as a makeshift a burst open steel common shell was employed as a gong in the experimental practice camp at Lydd in 1883. Old definitions of steel stated that the proportion of carbon in it was more than in wrought iron, but less than in cast iron: but this hardly holds good at the present time, when some mild steel has no more carbon than some wrought iron. Sir W. Armstrong states that "steel is iron produced by a process of fusion instead of by one of adhesion, and in that sense it is independent of any particular sense of carbonization. Using the term in this sense, steel has the advantage over iron in being free from defects in welding. It generally contains more carbon than wrought iron, which renders it stronger. * * * The manufacture of steel continues to improve, while that of iron is stationary, and the time is probably near when the manufacture of iron, as now practised, will entirely merge into that of steel, as produced by the process of fusion."*

Taking Holley's definition, **carbon** is the essential substance which alloys with iron to form steel; it increases the hardness, elasticity, and tenacity, though it decreases the elongation before fracture: it also gives rise to the important property of tempering by rapid cooling, by which these advantages can be increased, though with a further loss of power to elongate, as shown in Whitworth's diagram of test pieces (Fig. 4). The presence of carbon, however, greatly adds to the difficulty of forging, which must be done at a lower temperature as the proportion of carbon increases; if too hot, the mass will crumble to pieces under the hammer, or if this does not actually take place, the result will be to give brittleness instead of strength in the operation of forging—producing what is technically called **burnt steel**: in explanation of this remarkable fact, it can only be stated that some physical and (possibly) chemical change takes place in steel at a certain temperature depending on the proportion of carbon present, which causes it to assume a coarsely crystalline structure, greatly decreasing its strength and reliability: the forging of steel is consequently more tedious than that of wrought iron, which can often be heated to a white heat when it becomes soft, and can then be quickly hammered into the required shape. At the beginning of modern steel making, the necessity for care about the temperature for forging was not fully recognized, and burnt steel was often produced, accounting probably for many of the mysterious fractures which happened, and which gave reason for so much distrust. At Terre Noire especially, the men who had been

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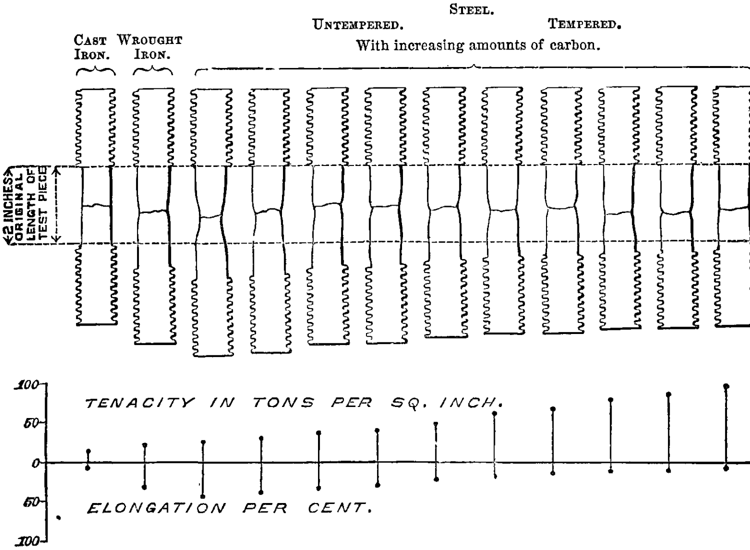
* *Vide* Presidential address, Min. of Proc. Inst. C.E., vol. lxviii, p. 46.

FIG. 4.

WHITWORTH'S TEST PIECES.

Showing how increase of tenacity is attended by loss of elongation before fracture.



Note.—The thick ends of the test-pieces have a rounded screw thread on them by which they are grasped in the testing machine.

accustomed to deal with wrought iron could not understand for a long time that steel must be treated differently, and experience was only gained at the cost of a good many failures. On one occasion, the ordinary workmen were turned away, and carpenters who had to learn the work, but who would do what they were told, were engaged instead.

So great, however, is the progress in some departments of steel-making, that notwithstanding these difficulties of working we find Mr. Denny writing that "steel for ships' plates has become so uniform as to have lost interest, while iron attracts attention from its deterioration and want of uniformity, and the men complain if they are put to work upon it on account of the amount of spoilt work involved;"* and this is not an isolated opinion. It cannot be said, however, that the large masses of steel for guns are as yet produced with such uniformity.

Mild steel can be **welded** like wrought iron, but the operation becomes more and more difficult as the carbon increases: the 13-pr. steel axletree is consequently made in one piece without any weld; but wrought-iron axletrees are most conveniently made in two pieces, which are afterwards welded together (*see* Plate I, Fig. *S*, p. 56. On the other hand, large quantities of excellent tubular steel are now produced by the process of lap welding.

It is more **difficult to cast mild steel** than cast iron, because a more uniform result is required, and since the temperature of fusion is so much higher (from the smaller proportion of carbon) much

* *Vide* "Journal, Iron and Steel Institute," 1884, p. 144.

greater contraction takes place on cooling, giving rise to a variety of intense molecular strains, which are sometimes sufficient to tear the metal apart in the mould, and often cause surface cracks, telling of a state of strain which must be allowed for and counteracted as far as possible by suitable methods of cooling and reheating. Krupp takes the greatest care on this point, making the cooling of his ingots very slow indeed, keeping them warm with hot ashes sometimes for weeks; and they are said to be remarkably free from cracks or flaws. Annealing for several days is often resorted to in the case of steel castings; it also has the effect of lessening the density or increasing the bulk: cast shells, low in gauge, when annealed for some days increase sensibly in diameter.

As small differences in the proportion of carbon make very great differences in the elasticity, tenacity, hardness, elongation, &c., it is most important to use every effort to control the quantity admitted into steel in manufacture, and it is one of the greatest aims in making steel for ordnance to obtain the desired proportion of carbon, and also to have the steel as uniform as possible; but with every care it is found that the percentage of carbon in different parts of the same ingot will vary to a certain extent; for ordnance it ranges from about 0·25 to 0·5: for the hard steel face of compound armour-plates, it may be as high as 0·8 per cent.

In all modern steel there is a considerable proportion of **manganese**; this is supposed to act in the same way as carbon in hardening, but its effect is not so marked. It is always added to melted steel in manufacture, in order (it is supposed) to take away ferrous oxide, which forms at a high temperature, and also, to a certain extent, it may take away other impurities which would impart red shortness or brittleness at the forging temperature; as manganese has an extraordinary affinity for oxygen, and cannot be kept alone in the metallic state under ordinary circumstances, it is prepared for use as an alloy with carbon and iron, termed spiegeleisen or ferro-manganese, according as the proportion of iron or manganese predominates; both of these alloys are now important articles of commerce. When the manganese mixture is put into the liquid steel, some of it immediately attacks the ferrous oxide, deprives it of its oxygen, and becoming itself oxidised, runs harmlessly into the slag, leaving a certain small quantity of metallic manganese to alloy with the steel: this is the last operation before pouring out in the Bessemer and "open hearth" processes, so that no time may be given for the formation of any more ferrous oxide. Mild steel should not have more than about 1·0 per cent. of manganese, or it will be brittle, but it is generally best to have less than this quantity. However, a steel having as much as 12 per cent. of manganese has lately been produced, which has shown remarkable qualities, and may perhaps become of practical utility in various ways; this alloy is very hard, but not brittle, and is scarcely affected if heated and plunged into water; but considerable variations have occurred in its properties.

Silicon has the property of rendering cast-steel ingots sound and free from blow-holes; small quantities of pig iron, rich in silicon, are often added for this purpose. It is supposed to de-oxidise the carbonic oxide gas which forms the bubbles, and thus silica is produced which enters the slag. Not more than about 0·3 per cent. of silicon should be present, and much less if the steel is to be of very mild quality (or brittleness will be produced).

Sulphur and **phosphorus** (especially the latter) are the enemies of the steel maker, for if present beyond a very small amount (say

0·04 and 0·06 per cent. respectively) they produce brittleness. As a very large proportion of the ore-deposits in the world contain too much phosphorus to allow them to be used in the manufacture of steel as it is generally carried on, it has long been a great problem to invent some process by which it could be eliminated, and it appears that this has at last been practically accomplished by the **basic process** invented by Messrs. Thomas, Gilchrist, and Snelus, in which freshly calcined magnesian limestone (dolomite) absorbs the phosphorus almost entirely when the metal is melted, and by this process good steel has been produced from highly phosphoric ores. The averages of analyses made in the North Eastern Railway Company's laboratory by Mr. Routledge of twenty steel rails made from hæmatite iron, and of twenty others made from phosphoric Cleveland iron by the basic process, give very nearly the same results.*

TABLE A.

	Carbon.	Silicon.	Sulphur.	Phosphorus.	Manganese.	Iron.
Hæmatite steel	0·452	0·105	0·121	0·052	1·178	98·092
Cleveland steel	0·450	0·065	0·095	0·051	1·201	98·134

As far as present experience goes, the physical qualities of the two steels are said to be similar: basic steel has not yet, however, been employed for ordnance.

Although this plan was invented by Englishmen, it has been more widely adopted in some parts of the Continent (where phosphoric ores abound) than in England, where good ores are generally procurable from abroad if not close at hand; and the practical question now seems to be generally one of economy, whether it is better according to local conditions, cost of carriage, &c., to employ an expensive ore and a comparatively cheap method of production, or a cheap ore and a somewhat more expensive process: as further experience, however, is obtained, the additional labour and expense of the basic lining plan may possibly be reduced. The results have been so good that it seems as if a new departure has occurred in the progress of steel making; already about a million tons of steel are manufactured annually in Europe from phosphoric pig, and the highest honours have been given to the inventors. It must, however, be stated that ores rich in silicon present some difficulty with this process, as the furnace lining is much corroded, and care must be taken to use freshly calcined limestone, as it soon absorbs moisture from the atmosphere.

The **power to resist abrasion** and rubbing, possessed by mild steel, is greater than that of wrought iron; this is apparently due, not only to greater hardness imparted by more carbon, but it also results from the greater uniformity of its structure: wrought iron, on the other hand, flakes off, from the presence of filaments of slag. The advantage of mild steel over wrought iron in this respect has been shown in a marked manner by the superior endurance and uniformity of steel rails: as, for instance, when the London and North Western Railway

* *Vide* "Principles of the Manufacture of Iron and Steel," by Sir I. Lowthian Bell, F.R.S.

Company some years ago made a careful experiment at Chalk Farm Station, at a spot where the traffic was specially heavy, the top side only of steel rails lasted eleven times as long as both sides of wrought iron ones on the other sides of the same line. Steel has consequently been widely adopted for rails, though it cannot be said that all now manufactured are of nearly such good quality as those made for that experiment. This same property was long ago recognized in gun making, when the bores of ordnance which have to resist the abrasion of studs and the erosive action of fired gunpowder, were made of steel. From experiments lately made to find the kind of steel least worn by the erosive action of the rush of gas in the bore of a gun on firing, it appears that the more steel has been forged the less it is eroded, and the less the carbon the better is *generally* the resisting powers, but this last rule does not always hold good. Effects somewhat similar to the erosive action of fired powder in the bore of a gun have been observed in pipes subjected to great hydraulic and steam pressures.

The process of **drawing into wire** increases the elasticity, tenacity, and elongation of steel to a degree unattainable in any other way, and advantage has of late years been taken of this fact in the manufacture of ordnance.

Many other physical and chemical properties of steel might be considered, but those just mentioned are probably the most important for the purposes under consideration.

Manufacture.

Manufacture. Excluding the older processes for the **manufacture** of hard tool steels, we find that mild steel for constructional purposes is produced by the three following methods (in about the proportions denoted in Fig. 3, p. 37), viz. :—

1. Crucible.
2. Open hearth.
3. Bessemer.

In each of these plans a very high temperature must be obtained in order to melt the steel, which must not be in contact with the solid fuel, because if it were so, the proportion of carbon in the steel would be too large; and means are always taken to render the product as uniform as possible.

1. The **crucible plan** (Fig. 5) is the oldest, but it is now the least employed: it has long been known that wrought iron enclosed in a crucible with carbonaceous matter is capable of combining with the carbon, and melting, to form steel, at a temperature insufficient to melt wrought iron alone. In the process now employed, carefully weighed proportions of wrought iron, with sometimes a little steel, or even good cast iron of known composition, powdered charcoal, and spiegeleisen are put into a crucible capable of holding some 60 to 100 lbs.,—this being about the limit which its strength will enable it to bear without risk of cracking. A number of such crucibles are placed in a furnace specially constructed for their reception; a very high temperature is obtained, and after some two and a half to three hours, according to the degree of carbonization required, the contents are melted, the carbon has alloyed with the iron to form steel, and the manganese in the spiegeleisen has reduced any ferrous oxide which may be present. A liquid slag formed at the top, and a fire-clay cover prevent oxidation, which might otherwise occur at the high temperature attained; the contents of several crucibles are then

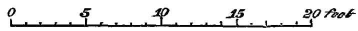
poured into the same ingot mould as rapidly as possible. Krupp has greatly developed this method, and he has poured from as many as 1,800 crucibles into one mould.

The advantages claimed for this plan are that the flame is not in contact with the steel, and therefore cannot contaminate it; and uniformity of quality is gained if the temperature and time of heating

CRUCIBLE PROCESS.

Fig. 5.

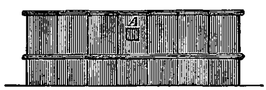
Approximate Scale for Figs. 5 to 9.



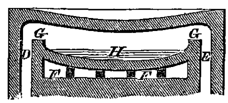
OPEN-HEARTH PROCESS.

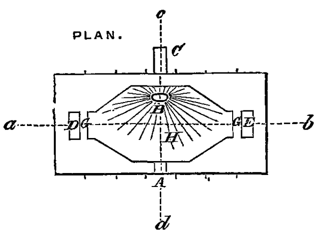
Fig. 6.

Elevation.

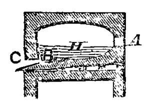


Sectional Elevation on *a b*.





Sectional Elevation on *c d*.



A Charging door.
B Tapping hole, which is at the lowest part of hearth.
C Spout.
D E To fuel and to chimney (generally reversible).
F Air spaces underneath to keep the outer parts cool.
G Fire bridges.
H Hearth.

BESSEMER PROCESS.

Fig. 7.

Charging.

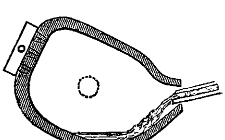


Fig. 8.

Blowing.

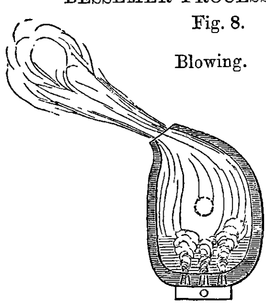
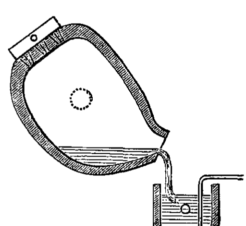


Fig. 9.

Pouring out into the ladle.



are the same, otherwise the reactions may differ in different crucibles if the heat varies in duration and intensity. The disadvantages are, the expense and the great care and arrangement necessary for very large ingots, when great numbers of men must be carefully trained to work together; the length of time taken in casting is also a drawback: but the product is often excellent, if all necessary conditions are observed, and it has been largely employed for gun tubes, as for instance, in our Service, in which Firth's crucible steel has been much used.

2. The **open hearth plan** (Fig. 6, p. 45) is perhaps the most recent method of steel making, and it owes its success to the inventions of Siemens and others, who have constructed furnaces of the regenerating type, capable of producing very high temperatures. The so-called hearth is somewhat spoon-shaped, sloping down to one point (B, Fig. 6, where the tapping hole is situated), and the bed is covered with partly fused sand; good cast iron is first thrust in, and the flame beating down, or reverberating from the very low roof, soon melts it, as the metal is readily fusible from the large quantity of carbon which it contains, and thus the so-called "bath" is formed. Sometimes only one kind of pig iron is used, but generally it is more economical to make a mixture; for instance, a very pure Swedish pig may be combined with a cheaper iron from English hæmatite ore, which has a good deal of sulphur, but not enough to make an injurious amount in the mixture, and sometimes steel is mixed at first with the cast iron. When the bath is sufficiently fluid, steel scrap, crop ends of rails or wrought iron are thrown in, after being warmed by the waste heat: the object being to obtain a mixture having less and less carbon; when these have been well melted down, a further reduction of carbon is effected by the addition of known quantities of good Spanish hæmatite ore—an oxide of iron. When this is added, a violent ebullition ensues, due to the combination of the carbon in the melted charge with the oxygen in the hæmatite, and the consequent production of carbonic oxide gas; this is technically called "boiling." The whole operation *might* be done by the use of pig iron and ore without any wrought iron or steel: this was a good deal practised at one time, but it was found to be very expensive, as more fuel was required, and the wear and tear of the furnace lining was considerable, from the corrosive action of the large quantities of slag which were produced. During the latter part of the process, the steel maker frequently tests the composition of the molten mass, by inserting a long iron rod with a spoon at the end of it, and taking out a little of the steel: this is judiciously cooled, hammered on an anvil, bent over and broken, and the fracture and general appearance are compared with other samples previously obtained which have given good results, and which have been chemically analyzed. If it is judged that the carbon is not low enough, more ore must be added if no "boil" is going on; but if, on the other hand, it is too low already, some good pig iron must be put in; and on leaving off the operation, the steel should be as quiet and free from bubbles as possible, to avoid blow-holes in the casting.

Great practical skill is required on the part of the steel maker; his tests must be taken rapidly, and a judgment must soon be made from the scanty and apparently rough data furnished by the comparison of the test pieces with the standards; but, nevertheless, the same results may be repeatedly attained by the same operator within comparatively narrow limits.

Just before the pouring out, comes the addition of the proper quantity of spiegeleisen or ferro-manganese; both of these necessarily

contain carbon, and consequently the melted mass should previously be more decarbonized than the finished steel is intended to be. The manganese is previously heated, carefully scattered over and stirred up in the steel, and allowed to remain a short time to become thoroughly incorporated and to ensure uniformity of composition: at the last moment two spoon tests are taken, one for chemical analysis of the carbon and the other for physical trial; the tapping hole at the bottom is knocked through by means of hammers and a long rod, and the liquid steel flows out along a gutter into a large iron ladle lined with fire-clay, and previously heated: if not heated, or if the temperature of the liquid steel is too low, a good deal of waste occurs from the formation of a considerable "skull," or metallic lining, caused by the solidifying of the outer part of the steel in the ladle. When all the steel has been poured out, two spoon samples are very generally taken for chemical and physical tests, the ladle is carried by a powerful crane to the ingot mould it is desired to fill, and a plug is raised at the bottom, worked by a rod passing through rings of fire-clay in the liquid mass: the steel then runs out of the bottom of the ladle and fills the ingot mould, which is made of iron, and for the larger castings is generally lined with a mixture of fireclay and plumbago; the slag being lighter remains at the top and is not mixed up with the steel, unless indeed any difficulty occurs with the bottom plug, when the steel must be poured from the top of the ladle, as water is from a jug; in this case, small quantities of slag may become entangled in the steel, and the ingot will probably be spoilt.

Sometimes the metal is run direct into the ingot mould without the use of a ladle, but this is not generally considered a convenient plan. The whole operation from charging to casting, lasts from 7 to 11 hours, or longer, according to the size of the furnace and the weight of charge; the largest furnace yet made will melt over 30 tons at a time; the furnace bottom needs repair after each charge. The advantage of this process is that the composition of the steel can be very carefully controlled, but it requires very great attention and skill on the part of one or two responsible persons. It is very largely employed in making steel for ordnance, carriages, and other military constructions where uniformity and high qualities are more desirable than economy.

3. By far the largest proportion of mild steel (nearly 80 per cent., *vide* Fig. 3, p. 37) is manufactured by the **Bessemer process,** but it is not applied to military purposes to anything like that degree; the rapidity and economy of this plan are considerable, and the whole operation is startling and impressive. A large egg-shaped iron vessel called a "converter" (Figs. 7, 8, and 9, p. 45), can revolve on trunnions, one of which is provided with teeth which gear with rack-work, by means of which it can be turned up or down. The other trunnion is hollow, and through it comes a pipe communicating with a great number of small holes or "tuyères" fitted in the fire-bricks situated in the bottom of the vessel; a blast of air can thus be sent from an engine through the tuyères. The top of the converter is provided with a short chimney, and the interior is lined with a very refractory material called ganister. The operation is as follows: The converter being heated, it is turned down (Fig. 7), and melted cast-iron is poured in by a gutter, either from a reheating furnace, or in some cases, as first practised at Terre Noire, direct from the blast furnace, where it is reduced from the ore; when a sufficient charge has been poured in—only about $\frac{1}{8}$ of the total capacity, so that the iron may not rise to

the tuyères on pouring in—the blast is turned on, and when it is fully on, the converter is turned up (Fig. 8); the liquid iron cannot run down the small tuyère holes through which the blast comes, but on the contrary, the pressure is sufficient to cause a continuous stream of bubbles of air to rise up through the molten mass, thus oxidizing the carbon in it with the formation of carbonic oxide gas, which burns with a strong flame some 20 feet in length at the top of the chimney.

The temperature of the iron is considerably raised by the combustion of the carbon, and, although it becomes greatly decarbonized and consequently more infusible, it still remains liquid; after some 20 minutes the flame becomes much shorter and alters in character, signifying that the carbon is becoming very low, and that the operation is nearly completed; it is essential that the exact time of leaving off should be chosen, as if the "blow" is stopped too soon, sufficient carbon is not extracted; if, on the other hand, it is continued too long, the iron becomes oxidized, when not only is some of it wasted, but the quality of the rest is spoilt, for the iron itself burns, and excess of oxide is formed. The appearance of certain lines in the spectroscopic appearance of the flame is employed to find the proper time for leaving off, but it is often determined by eye alone. Before pouring out, however, comes the addition of manganese, which is absolutely necessary in this plan; as from the method of manufacture a good deal of ferrous oxide must necessarily be formed; the "blow" is continued for rather less than a minute, after which the steel is poured out into the ladle (Fig. 9), when it is sometimes stirred by means of an "agitator" to distribute the manganese and produce uniformity. The ingot casting is the same as in the "open hearth" system.

The rapidity of this plan enables an enormous output to be made with a moderate plant, as each "blow" lasts less than half-an-hour to produce 6 to 12 tons of steel, but this very rapidity prevents the careful control over the results which can be exercised in the last method: in a large firm this can be allowed for by sorting the ingots produced, and applying each to the purpose for which it is most suited; but it is most difficult to know what will be the exact proportions of carbon and the other qualities of any particular "blow."

It does not appear at present to be well suited for the manufacture of ordnance; and the following extract from the "Proceedings of the American Society of Civil Engineers" about the Monongehela Bridge, Pittsburg, represents the objection to the employment of this steel for work of the higher qualities:—"The difficulty seems to consist in controlling the uniformity of the steel within close limits for quality and strength with the Bessemer process. After a while, the attempt was given up, and the 'open hearth' was substituted—no trouble was then experienced in getting a uniform grade of steel of prescribed quality." Some 42,000 tons of "open hearth" steel are used in the construction of the Forth Bridge.

Bessemer steel is very much used for rails, and also for a great variety of other purposes, including compound armour.

Forging.

When steel is to be **forged**, it is cast into an ingot mould of a very simple form, efforts being made by the use of silicon, either in the pig or in a special mixture, or by fluid pressure as used by Whitworth, to get rid of blow-holes: the ingot is taken out, re-heated, carefully inspected, cracks cut out, and it is then either hammered, rolled, or pressed to the required shape, great care being taken about the temperature. In order that the blows may be well transmitted through the

mass, it appears that very heavy hammers are now generally preferred, probably because mild steel is not in as soft a condition as wrought iron when forged. As large masses of steel are now worked, it is probable that the numbers of already existing very heavy hammers may be increased, unless Whitworth's method of hydraulic pressure be employed instead. A few years ago larger ingots were produced in France and Germany than in England, but we are now abreast of other countries in this respect.

Tempering.

Increase of tenacity and elasticity can be obtained by the important operation of **tempering**, but this is at the expense of elongation before fracture. The temper obtained varies with the amount of carbon in the steel, the temperature, and the nature of the cooling material (rape oil is generally considered the best for large masses, as it has a less conducting power than water, and cools the metal more slowly) Krupp is said to temper his steel at a uniform heat; while our plan for ordnance is to temper a test piece at 1450° F., and if this does not give good results, the temperature is varied within certain limits, and then the whole mass is treated in the same way, as nearly as possible, as the test piece which gave the best results. As tempering alters the specific gravity slightly, warping and surface cracks are often produced by this operation, when large masses are acted upon. The effects of tempering have been modified by afterwards heating masses of steel for guns in boiling tallow for several days and allowing them to cool slowly; thus the steel is annealed, tallow is used on account of its high boiling point. The Schneider* steel plate at the Spezzia experiment in 1882 was tempered on the face to gain hardness, and it consequently warped so much that it had to be planed off at the corners to make it fit the frame it was placed in. Gun tubes often develop surface cracks after tempering, but sufficient thickness of metal is allowed so that they may afterwards be cut out.

Steel castings.

With **steel castings**, every effort is made to get rid of blow-holes; for all good work this generally involves a very considerable dead-head, which adds to the expense, and has to be cut off. Annealing for several days is resorted to by some, while others are content with slow cooling in hot ashes; tempering is sometimes used for special purposes, such as the head of an armour-piercing projectile, though it cannot be said that cast-steel projectiles are as yet very successful for armour-piercing purposes against steel or compound armour: but great progress has recently been made in steel castings; and they are now used for a great variety of purposes. In designing the shape of a casting, sharp corners and great differences of thickness must, if possible, be avoided on account of the difficulties of preventing unequal strains tending to produce rupture in contraction on cooling down.

A great variety of **tests** are applied to steel in different places, according to the purpose for which it is required; latterly there has been a growing tendency towards uniformity: the workman's rough tests of bending, breaking, and observing the fracture have been systematised in various pulling machines, actuated by a dead weight or by levers, or by hydraulic pressure, by all of which the limits of elasticity and tenacity are indicated; bending (*see* Figs. N and O, Plate I, p. 56), and torsional tests are also often applied, and in other

* *Vide* Lecture by Captain Orde Browne, "Pro. R.A.I.," 1883.

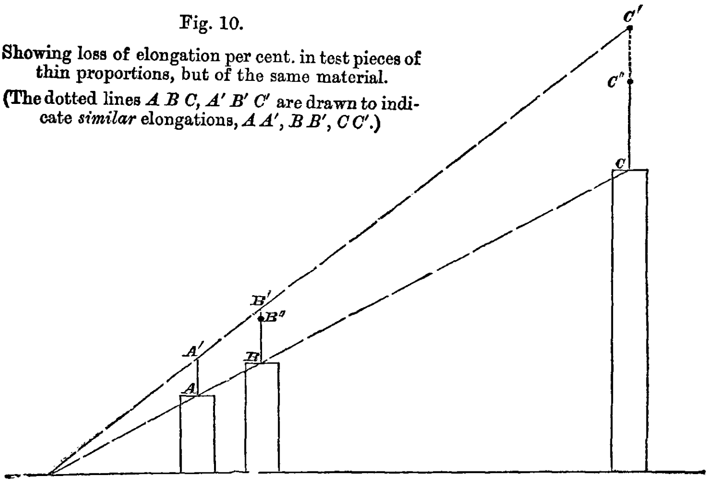
cases dynamical trials are made, as by dropping heavy weights on to rails, or by exploding gun-cotton, as is sometimes done with boiler plates, which are made of very mild steel, having but little carbon.

For ordnance, the tension and bending tests are generally sufficient; the elasticity, tenacity, and elongation are carefully recorded, and the area of the fracture and its general appearance are also noticed after tempering. In our gun manufacture, the behaviour of the tempered test piece serves as a guide for the treatment of the mass from which it came; but it appears now to be generally acknowledged, that the same heat will cause a different temper in a small test piece and in the large mass, as the latter will cool so much more slowly, and will not, therefore, be so much affected as the smaller piece. The French cut a piece off the tempered mass and then test it, in order to find out the properties really attained. The elongation on fracture has of late attracted attention, as it is a well recognised fact that a short test piece will elongate much more per cent. of its length than a longer one of the same material; this is explained by the circumstance that as the elongation is only considerable just immediately at the point where rupture takes place, the *total* elongation of the long piece is not actually much more than that of the short piece, and is consequently less in proportion to its length—or the elongation per cent. of its length is less—(*vide* Fig. 10, where a test piece of Whitworth's dimensions is sup-

Fig. 10.

Showing loss of elongation per cent. in test pieces of thin proportions, but of the same material.

(The dotted lines *A B C, A′ B′ C′* are drawn to indicate *similar* elongations, *A A′*, *B B′*, *C C′*.)



posed to elongate an amount AA′; if the same material is made according to Royal Arsenal pattern, it will only elongate BB″, instead of BB′, which would be in the same proportion as the last, while if the test pieces are still longer, the elongation CC″ is even less per cent.);* thus if a test is taken according to the Woolwich plan, it will not give such a favourable result as by the Whitworth method, unless the differences in the proportions of the test pieces are taken into account.

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* *Vide* paper by W. Hackney, "Min. of Proc. Inst. C.E.," vol. lxxvi.

It is usual to say that the elongation is so much in a test piece of so many inches, but this is not really correct, as a thick piece will elongate more than a thin one: it is now proposed that all test pieces shall be of the same proportion of length to thickness.

It is important that the tests imposed should be reasonable and possible, but this apparently has not always been the case, as, for instance, when iron was first employed in the construction of ships, the test employed by Lloyds' was the single one of good tenacity, irrespective of elongation, before fracture; the consequence was that much inferior brittle iron was built into many a vessel, accounting, doubtless, for numbers of casualties. On the other hand, to show the advantage of good elongation before fracture, many instances have occurred of steel vessels grounding on rocky places, when the bottom plates have been bent and crumpled, but not fractured, as they doubtless would have been if they had been brittle, although with a high tenacity. Going to the other extreme, the demands for tenacity *and* elongation, within certain limits of temperature for tempering, have been so high in some specifications that it has not been found possible to come up to the standard when large masses have been supplied; the steel has in some cases been rejected wholesale, whilst at other times it has been passed, when the tests being set aside became useless.

Not only are mechanical tests employed, but the **chemical composition** of steel is found by quantitative analysis in a most systematic manner: and each large factory has a regular laboratory, with one or more analysts—a class of men created by the requirements of the modern steel trade—in which sound scientific training in those concerned is a *sine quâ non* for successful manufacture.

The tests imposed by the Ordnance Committee on large masses of steel have been very stringent, and some manufacturers have found a difficulty in complying with the conditions, but the making of steel in the Royal Arsenal has given a decided impetus to gun steel, which differs somewhat from that required for most other purposes. The French Government, intent upon progress, have insisted on rigorous tests for steel which their own makers at first declined, but afterwards agreed to comply with; in the meantime, however, a large contract was giving to a foreign firm. It would appear to be the wisest course to insist on obtaining the very best material for gun steel.

Steel for Naval and Military Purposes.

The amount of steel used for naval and military purposes bears only a small proportion to that which is used for ordinary industries, and this ratio varies greatly in different countries, being probably highest in Russia and lowest in the United States. A great part of the steel for warlike uses can only be produced by special plant on a colossal scale—thus we hear of 100-ton hammers, 8000-ton forging presses, immense rolling mills for armour, 160-ton cranes, a tank containing 100 tons of oil for tempering, railway trucks for taking immense weights, and tools for machining enormous masses of metal. The number of factories in the world where the heaviest guns and armour can be produced is thus necessarily limited.

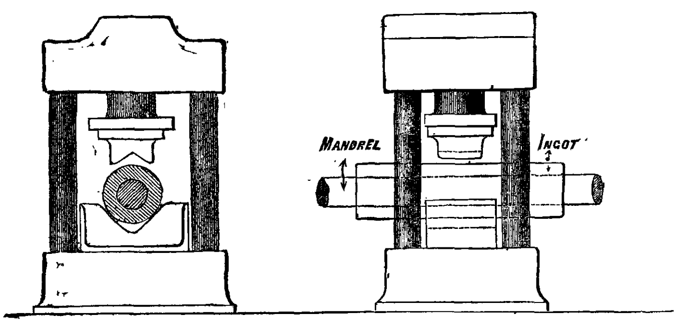
Ordnance. About five years ago a most important decision was made that our **ordnance**, of all calibres, was in future to be constructed entirely of steel, and wrought-iron coils were abandoned, as there was no longer a doubt that steel is much superior to wrought iron for this purpose,

from its greater strength. This has been especially noticed with guns firing the modern large charges of slow-burning powder. The steel for ordnance should be of such quality as to possess a considerable elastic limit, so that permanent deformation or enlargement of the bore may not take place to any appreciable extent; in our Service, two pieces for the tensile test are taken from each of the various parts of which the gun is to be composed; they are then heated to between 1350° and 1550° F., and plunged in a bath of oil of an initial temperature of 65°; when cold they are turned down to the testing size, the operative part being 2 inches long with a uniform diameter of 0·533 inch, the other parts are thicker and must fit the testing machine. The yielding point or elastic limit must not be less than 22 or more than 33 tons on the square inch. The breaking tension or tenacity must not be less than 35 or more than 45 tons on the square inch, and the elongation must not be less than 17 per cent. Two rectangular test pieces of tempered steel are subjected to a bending test. These tests will probably be made more severe as the manufacture of steel improves. Reliability and uniformity are now being attained, though only by most unceasing and intelligent care in all processes of manufacture; especially is this recognized to be the case when very large masses are forged, as the difficulties in obtaining uniformity in the mass become greatly increased. The percentage of carbon is about 0·4 to 0·45 with crucible steel when the manganese is low; but with the "open hearth" steel the percentage is a good deal less (0·28 to 0·31) when more manganese is present.

A modern heavy steel gun is constructed as follows:—The tube is made from the ingot by repeatedly heating and drawing it out under the hammer (see Plate I, fig. P, p. 56); a core is then cut out or trepanned, and thus most of the material cut to pieces in the ordinary process of boring is available for other purposes; the process is also quicker. Whitworth's firm, however, proceeds in a different manner, employing the plan adopted with such success in forging hollow propeller shafts, which are made thus. An ingot is bored, and the shavings remelted as they do not consider the interior of sufficiently good quality for further use without remelting. The hollowed ingot or cylinder is then heated, a hollow steel mandrel of smaller diameter than

Fig. 11.—Whitworth's Hydraulic Forging Press.

Drawing out a Tube.

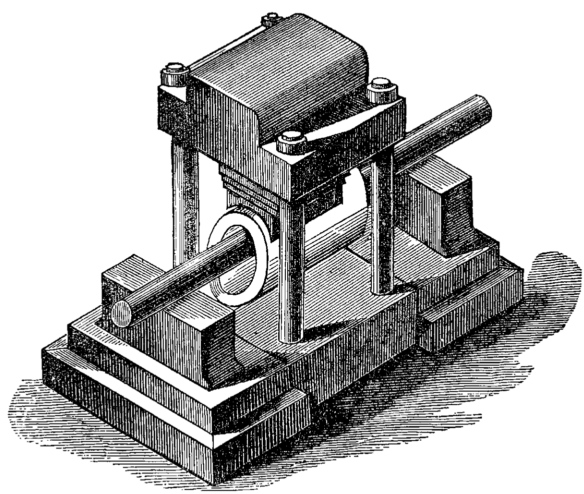


* Tests for steel revised by Ordnance Committee, 18th August, approved 2nd November, 1886.

the interior is passed through it, and both are placed under the powerful hydraulic pressing machine, which presses the metal of the cylinder against the mandrel by repeated pressings, while the cylinder and mandrel are turned round into fresh positions (Fig. 11); the consequence is the hollow cylinder becomes gradually less in diameter, but increases in length. This process is repeated with thinner mandrels and repeated heats until the cylinder assumes the desired proportions of the propeller shaft; the mandrels are kept cool by water circulating through them. Many Service gun tubes, for the larger natures, have been made in this manner.

The hoops for guns are also made from the ingot, which is trepanned, the ends cut off and remelted, and the rest is cut into thick rings, each of which is afterwards heated and made thinner by hammering on a mandrel placed through it and supported at each end (the hoop is thus suspended on the mandrel). This operation naturally increases the diameter, and the manufacture resembles that of the weldless tires of railway wheels. Whitworth's firm make some hoops in this way, substituting the press for the hammer (*see* Fig. 12), but they have proposed to construct others in the same way as the

Fig. 12.—Whitworth's Hydraulic Forging Press.
Enlarging a Hoop.



tube, in the form of long thin cylinders, and thus the heaviest guns could be made of fewer parts than those at present designed. For instance, the 110-ton gun is made of 43 parts, but one of Whitworth design would consist of only 11 pieces. Whitworth's establishment has a plant for making the heaviest ingots and forgings, and all the tools necessary for completing the heaviest guns, and Elswick will probably be able to cast and forge the heaviest ingots; the country will then possess two complete establishments to supplement the Royal Gun Factories, and there are of course several other firms which can produce excellent steel in large masses, and who are accustomed to the working of heavy steel forgings.

The barrels for small arms are made of mild steel from the bar, in an ingenious machine with a series of vertical and horizontal rolls, which in one heat draws out the barrel to the required length (*see* Plate I, Fig. A, p. 56). This has been in use some eight years at Enfield, and copies have since been supplied by an English firm to several foreign Governments. The bayonet is made of a hard steel welded by the aid of borax to a socket of softer steel: great care being taken not to overheat the end of the bar which will form the blade, while the socket being milder is made much hotter; the tests to which bayonets are subjected before being passed into the Service have lately been made a good deal more severe than before the late war in the Eastern Soudan.

Gun carriages.

The increased strains which the large charges of modern guns impose on **gun-carriages** have necessitated the employment of steel in their construction, in order to obtain the necessary strength; and steel plates and axles are now freely used, as in the 13-pr., but wrought iron is still preferred for some parts, as, for instance, the trail eye of a field gun-carriage, which is subject to constant jars. Steel castings are much used for heavy gun-carriages; some of 55 cwt. each have been made in considerable numbers for the sides of 45-ton gun-carriages; and steel castings on a grand scale have been made for the proof carriage of the 110-ton gun.

Steel **racers** have long been introduced for use with some of the heavier garrison guns, as they well resist the blow caused by the jump of the platform on firing, which indented the older and softer wrought-iron racers, and rendered subsequent traversing a difficult operation; the path and 28 rollers of cast iron on which the turrets of the "Inflexible" revolve have not been found hard enough; in future they will be of steel. Tubular steel linings are employed for the cylinders of Vavasseur mountings: and corrugated steel plate termed "Atlas metal," has been adopted for field artillery ammunition boxes.

Projectiles.

Steel has been applied to **shells** with great success, but at present the expense is considerable. Shrapnel are made for the 7-pr. and 12-pr. of Delmard's tubes, with thin walls and bursting charge in the head; the proportion of useful weight in the former being 45·4 per cent., while in other natures it is 25 per cent., and often much less. Steel has been tried for common shells, as they do not break up so often on striking earth as cast-iron shells. An example of the advantage of their use was furnished by an experiment, when a parapet was more readily breached by a 12-pr. with steel shells than by a 5-in. gun with 50 lb. cast-iron ones, the velocities being about the same in each case; but the walls of steel shells being made thinner than those of cast iron a larger bursting charge can be inserted. It is doubtful which is the best way to apply steel for these purposes, whether to cast it at once to the required shape or to forge it; the first is the simplest and cheapest way; but at present difficulties are apt to appear with blowholes which may lead to fracture in the bore when the walls of the shell are thin; however, with recent improvements in casting steel, this objection may very possibly be overcome. Another plan is to cut off a length of tubular steel, heat it, bend in one end for the head, and turn in and close up the other end for the base, or else weld in a base disc; another device has been ingeniously employed of cutting off a piece of thick bar steel, stamping it into a hollow, and drawing up the sides gradually when heated to form a shell.

With regard to armour-piercing projectiles, Service Palliser cast-iron chilled do very great damage to the targets, but they break up in doing so. Krupp has produced a good projectile, and the French Navy has for some years past been provided with steel armour-piercing shells for 32- and 19-cm. guns, all supplied by contract under severe tests for reception (*see* Plate I, Fig. F, p. 56.) The following conditions were imposed in September, 1884, when the French Government invited their steel makers to supply a large number (2,100) of armour-piercing projectiles for 32- and 27-cm. guns: showing that a high standard of excellence was desired. The total number was divided into four for each calibre, and the behaviour of two shells tested out of each lot was to determine the acceptance or rejection of the others. The heavier projectiles to be fired almost at right angles against a 30-cm. Creusôt steel plate with wood backing with striking velocity of 435 to 445 metrès per second, while the lighter ones were to strike a 25-cm. steel plate with a velocity of 445 to 465 metres per second. If the first projectile perforated the target unbroken and uncracked, the lot to which it belonged to be at once accepted; but if it broke up in perforating, the second projectile was to be fired, and only if it got through uninjured was the lot be accepted. If the first round did not perforate the target, the lot to be at once rejected. Facilities were given to allow the makers to fire trial shots against steel plates before submitting their finished shells.

Nordenfelt bullets for penetrating the sides of torpedo-boats are forged from steel bars by drawing down a part to form two heads; pieces are then cut off, stamped to true shape in a die, and oil tempered (*see* Plate I, Fig. C, p. 56). They are also made by Simond's round forging machine.

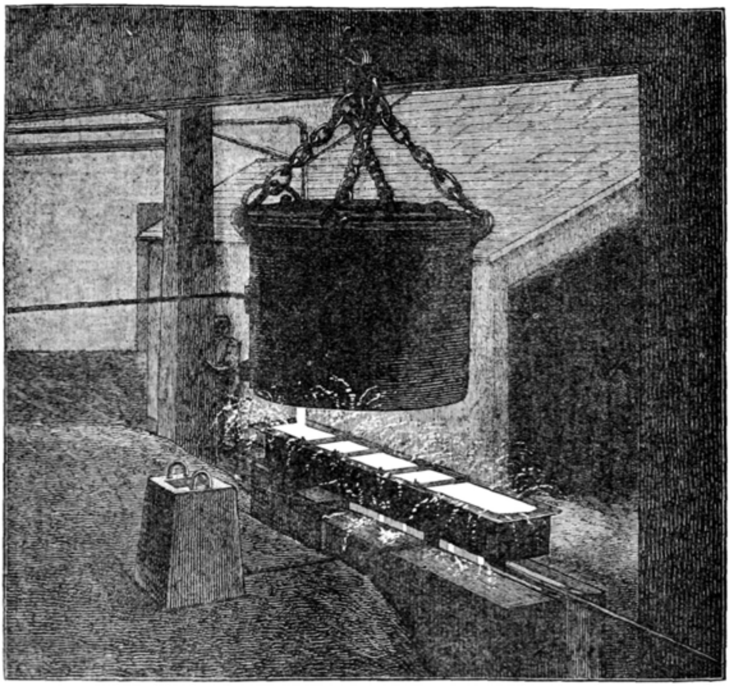
Our **compound armour** is constructed on two plans by the only two firms which at present manufacture it in England. **Armour.**

The Atlas Works (Sir J. Brown and Co.) make it on Ellis's patent (Fig. 13); a considerable thickness of wrought iron and a thin steel face plate are kept at a few inches distance from each other, with wedge plates round three sides, and small steel studs at several points keep them from coming too close to each other in the furnace; the whole mass is then strongly heated with the plates horizontal; when hot enough it is taken out and lifted by a crane, swung vertical, placed in a pit, and melted steel is poured from a large ladle into a trough which distributes little streams into the cavity between the two plates and thus joins them together; in a short time the whole plate is taken up and put back again into the furnace; when re-heated, it is taken out, and passed through the large rolling mill.

In Wilson's plan, adopted by Cammell and Co., a large wrought-iron plate built up of many thicknesses is passed through the rolls, and is then pushed horizontally into a huge iron chamber which can revolve on trunnions; when the plate is secured, the whole is turned up and becomes vertical, and liquid steel is poured in from a ladle and trough, between one side of the wrought iron and the side of the box, precautions being taken to prevent it from flowing elsewhere. In Wilson's plan the steel was formerly poured on to a wrought-iron plate provided with a rim and placed horizontally; but this was given up, as the scum, &c., tended to remain on the face of the plate. The whole is afterwards rolled.

The steel constitutes about one-third of the weight in both systems, its object being to break up the projectile on impact; it consequently has a considerable amount of carbon in order to give it the necessary

Fig. 13.—MANUFACTURE OF COMPOUND ARMOUR.
(Sir J. Brown and Co.—Ellis' patent. From a Photograph.)



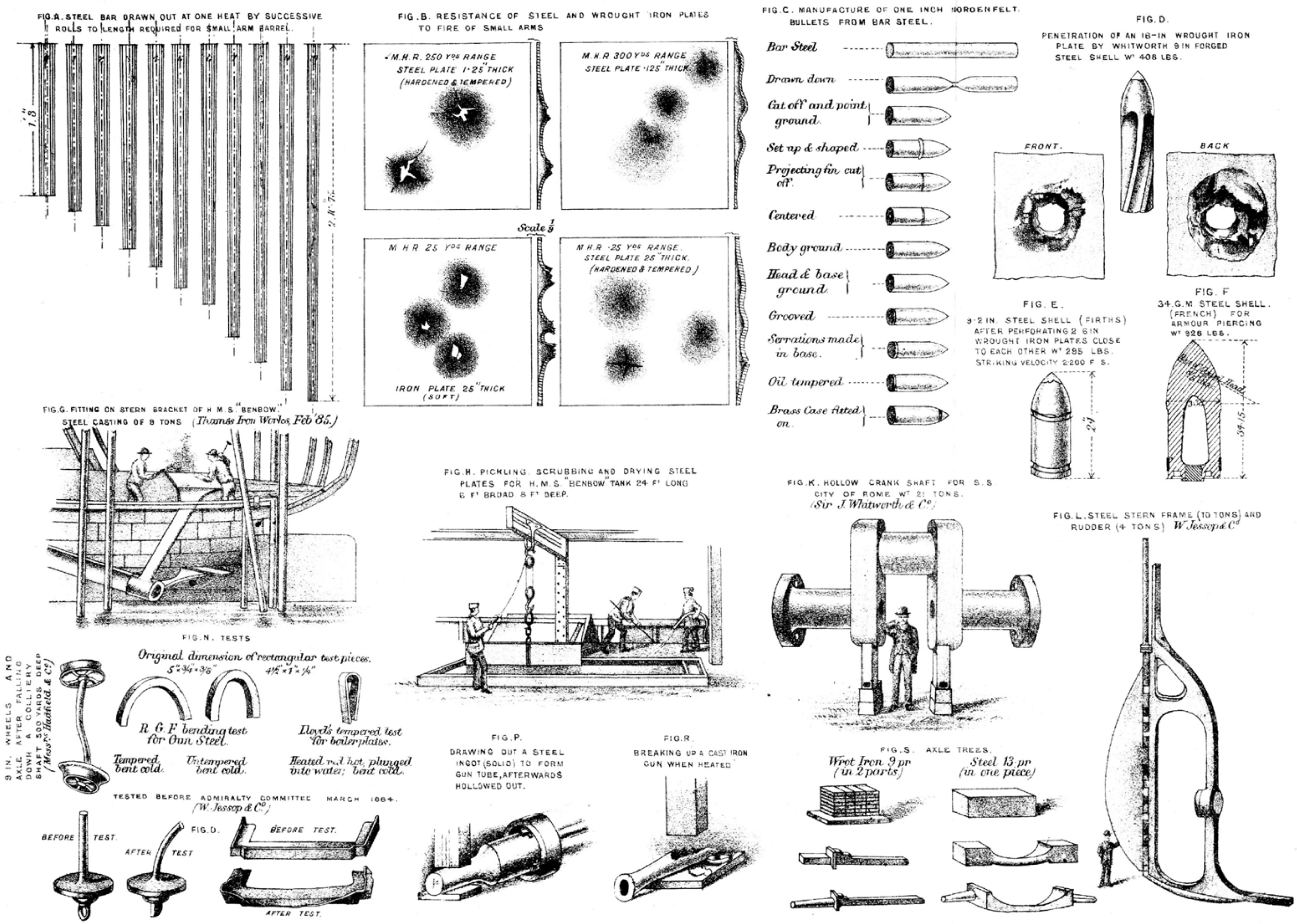
POURING IN MELTED STEEL BETWEEN THE WROUGHT-IRON AND THE STEEL-FACE PLATE.

hardness, while the wrought iron at the back of the plate is intended to hold the plate together, and to prevent the formation of cracks and splits as far as possible. This class of armour has achieved good results; the experiments at St. Petersburgh and at Spezzia during the last two or three years, as well as some of our own at Shoeburyness, having been favourable. Ellis's plan has the advantage of a very good front surface, but the results attained by each are generally considered to be about the same as far as present experience has shown.

Compound armour is now made in large quantities in Russia, Germany, and France, on the Wilson system; in the latter country there are three factories busily engaged, and in Germany the Dillengen works have been in operation for three years, producing compound armour for the three ships "Oldenburg," "Bremse," and "Bremmer." The Russian works at Kolpino, 16 miles from St. Petersburgh, are completed, and compound armour-plates are made there. The works will probably be well employed, as the Russians are rapidly developing their Navy.

Excellent **steel armour** has been made at Le Creusôt, of a milder quality than the face plates of compound armour, and it has given good results when tested at Spezzia, in competition with compound armour; under certain circumstances steel appears to have shown

MILD STEEL.



To face p. 56.

DANGERFIELD, LITH. 22, BEDFORD ST COVENT GARDEN. 12/85. 10943.

itself superior to wrought-iron plates; as, for instance, when the armour is thick and when the backing is not very rigid.

Thin shields are used for the protection from musketry fire of men working machine-guns (*see* Plate I, Fig. B); they are $\frac{1}{4}$ in. thick, and oil tempered; the resisting power is considerably better than that of wrought iron.

The Royal Navy and the merchant service (including many vessels available for war service in case of emergency) take vast quantities of steel plates, angles, and rivets, as well as tubes for boilers. On the Clyde scarcely an iron plate is now used for a marine boiler, so completely has steel taken its place. Ships.

Steel plates for Her Majesty's ships which are below the water-line are previously pickled in dilute hydrochloric acid, brushed with steel wire brushes and washed (*see* Plate I, Fig. H, p. 56); this removes the black oxide from the surface, and enables the composition to be properly applied, thus preventing corrosion; this plan also gives the advantage that minute cracks (if they exist) can easily be detected on the clean surface, and an untrustworthy plate can be rejected.

Rivet-holes are punched, and then rimed out to remove most of the metal weakened by the operation of punching; butt-straps (which connect plates together) are annealed only after punching, and plates which have to be much bent are also annealed.

Castings are used for stems, stern-posts (*see* Plate I, Fig. L, p. 56), rudders, and stems having torpedo tubes and rams combined. These have been supplied by our makers not only to our own Government, but also to those of Italy, Japan, and Denmark. These castings stand a breaking strain of 28 to 34 tons per square inch, with a minimum elongation of 10 per cent. in a length of 8 inches, while a second test piece will stand bending to a right angle. The "Benbow" (10,000 tons), lately built on the Thames, and which is to be armed with 110-ton guns, is altogether of steel, and contains some fine castings, the stern brackets (*see* Plate I, Fig. G, p. 56) of 9 tons each being, it is believed, the largest yet made.

Crank and propeller shafts (*see* Plate I, Fig. K, p. 56) are now very generally made of mild steel, and the weights dealt with are equal to the largest forgings required for the heaviest guns. Whitworth's forging for the "Jumna's" crank shaft weighed 45 tons, and when finished was 23 tons. Consequently any factory having a plant for this nature of work needs comparatively little adaptation to fit it for gun making, as the heaviest part of the 110-ton gun (the tube) when finished only weighs some 25 tons.

CHAPTER V.—PRINCIPLES OF GUN CONSTRUCTION.

(*See* also Part II, Chaper II).

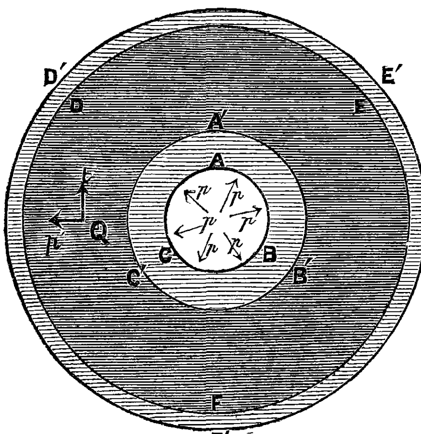
A GUN is essentially a strongly constructed tube, and, in common with every other construction, the strength of each part should be proportional to the stresses to which it is likely to be subjected.

When the powder charge is exploded, we may regard the chamber, where the maximum pressure is felt, as a closed cylinder containing fluid under great pressure, one end being the bottom of the bore, and the other the base of the projectile.

As the pressure of a fluid acts equally in all directions and at right angles to surfaces with which it is in contact, the powder pressures produced by the explosion of the charge act radially to the curved surface of the bore, as shown in Fig. 1, and they also act at right angles to the end of the bore and the base of the projectile (*see* Fig. 16, p. 72); neglecting for the present the pressures in this last direction, which produce a longitudinal traction on the gun, tending to ring fracture, we will consider the stresses induced by the explosion of the powder charge in a plane at right angles to the axis of the gun.

Suppose that ABCDEF, in Fig. 1, represents a section of a homogeneous gun in its unstrained state, and that A′B′C′D′E′F′ represents the same slightly expanded under internal pressure p, which acts normally to the surface of the bore: it can be seen, by assuming the sectional area constant, that the inner circumference expands in a greater proportion than the exterior; this is also proved

Fig. 1.

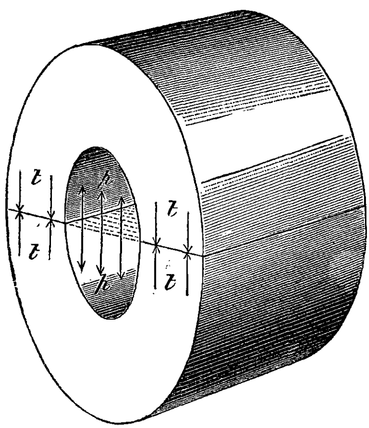


by more correct assumptions; consequently, if considerable internal pressure were developed, the inner surface would fracture and crack before the exterior had exerted all its strength.

We note that the powder pressures which act *radially* on the surface of the bore, produce a variable circumferential or **hoop tension** on the gun, which is greatest at the surface of the bore: a **radial pressure** also acts through the metal, being greatest at the surface of the bore, and decreasing to 0 at the outside (neglecting the pressure of the atmosphere). Generally at any point Q in the metal of the gun, a hoop tension t exists, and also a radial pressure p, each acting in the direction shown in Fig. 1.

If Fig. 2 represents part of a gun, a plane which contains the axis will divide the piece into two halves lengthways; the tendency to produce rupture along this plane is $2pr_0l$, in which $2r_0$ is the internal diameter in inches, l is the longitudinal length considered, and p is the powder pressure in tons per square inch, acting at right angles to

Fig. 2.



the imaginary surface and equal at all points, because it is a fluid pressure; this is resisted by the hoop tensions of the metal acting over the thickness of the two sides, or $2t(r_1 - r_0)l$, r_1 being the exterior radius in inches, and t being the *average* of the variable hoop tensions in tons per square inch. Consequently for equilibrium we have—

$$2pr_0l = 2t(r_1 - r_0)l,$$
$$\therefore\ pr_0 = t(r_1 - r_0) \ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(i)}$$

To develope the full powers of modern guns with slow burning powder, it is found best to employ enlarged chambers; by formula (i) it is seen that increase of internal diameter of the inner tube leads to a lessening of the maximum safe pressure. In like manner it is practically found that a large bore pipe of a certain thickness of metal will not contain water under a pressure which a smaller bore pipe of the same thickness will safely bear.

The hoop tension may be represented graphically. Let *c*OC (Fig. 3) be a diameter 17·4 inches of a cross section of a homogeneous gun, whose internal diameter *a*A is 8 inches (the walls are thus rather

Scales for Figs. 3 *to* 6 *and* 9 *to* 11.

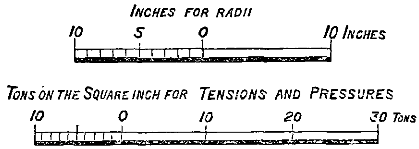


Fig. 3.

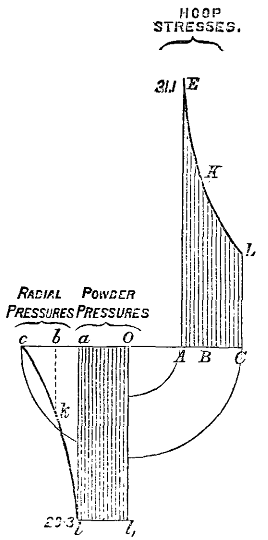
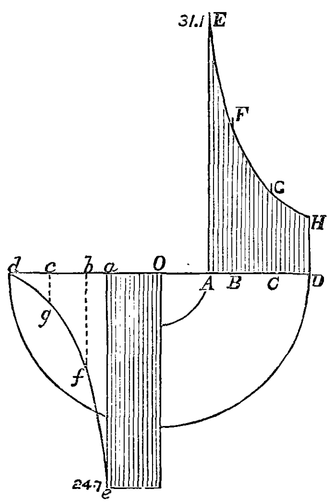


Fig. 4.



NOTE.—Though Figs. 3 and 4 represent homogeneous guns, the radial distance O*b*, OB, and O*c*, OC, are marked off to correspond with the radii of contact surfaces in Figs. 5, 6, 9, 10, and 11.

more than half a calibre in thickness), and suppose the particles of the metal under no stresses before firing, the intensity of stress circumferentially (or hoop tension) at various distances from the centre on the explosion of the powder charge may be represented by ordinates *above* the line ABC on the right side of the diagram, and thus the curve line EL is obtained (*see* Part II, Chapter II), the ordinate BK at any point B, being proportional to the hoop tension at B. AE at the surface of the bore is the greatest hoop tension to which any part of the metal is subjected; the area AL thus represents the sum of the hoop tensions of *one* side of the gun, or the strength of that side. As pressures at all points in a fluid are the same, the sum of the pressures *al* of the exploded powder charge over half the diameter may be represented by a rectangular area O*l*, and we place it below the diameter *c*OC, as pressures are negative tensions.

For equilibrium, according to equation (i), the sum of the tensions of the metal on one side of the gun equals the sum of the powder pressures over the radius, or the area AL = area O*l*.

Suppose in Fig. 4 that the thickness of the metal is increased by

3·1 inches, the walls are now nearly one calibre in thickness; a short calculation tells us that the weight of the piece is increased 106 per cent., but the increase of strength is only proportional to the difference between the areas AH in Fig. 4, and AL in Fig. 3, supposing the maximum hoop tension at the surface of the bore to be the same in both cases. According to the laws we have glanced at this only gives an increase of about 22 per cent.; if the weight were again doubled by making the homogeneous gun thicker still, the strength would be augmented by an amount less than even 22 per cent. The strength and weight of a gun made on this plan are thus seen to increase in very unequal proportions.

A series of verticals to the curve EL, and to the horizontal $ll_{,}$, not only represent the *magnitudes* of the hoop tensions and powder pressures respectively over the halves of a diameter, but they also represent their *directions*.

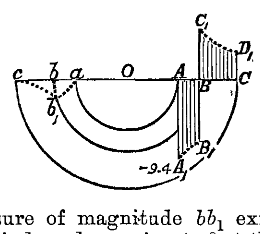
The radial pressures in the metal may also be represented by vertical ordinates to the curves *cl* in Fig. 3, and *de* in Fig. 4, below the diameters, and on the left side of each diagram; these will serve to indicate the variations in the magnitude, but they are *not* in the direction in which the radial pressures act. The hoop stresses are the most important to consider.

Initial Stresses.

A glance at Figs. 1, 3, 4 shows that while the interior of a homogeneous gun may be subjected to the greatest stress which it is safe to apply, the outer portions of the gun are of little assistance, and a great part of their strength is not made use of in a homogeneous gun, but the principle of **initial stresses** places the inner parts of the gun which require the most strength in an initial state of hoop pressure by the grip of outer cylinders; this does not create strength, it is simply an arrangement that the inner part which is most strained is supported by the outer parts, which are weakened initially to strengthen the interior; but as the exterior has strength to spare, this is an advantageous plan.

When this is done, the initial state of a gun composed of a steel tube and one hoop may be represented graphically by Fig. 5, in

Fig. 5.



which a radial pressure of magnitude bb_1 exists at the surface of contact of the two cylinders, decreasing to 0 at the interior of the tube, and also at the exterior of the hoop, as shown by the curves b_1a, b_1c.

As a consequence of this normal pressure at the surface of contact, the exterior cylinder is subjected to hoop tension in exactly the same manner as if an *interior* powder pressure of magnitude bb_1 were exerted over its interior surface (*i.e.*, the area contained by Ob, and bb_1 must equal the area BD_1); the inner tube, on the other hand, experiencing a normal pressure on its *exterior*, will consequently be *compressed*, and the hoop stresses will be *pressures*, as shown by the

area *below* AB. As the particles of the gun are in equilibrium, the area of pressures AB_1 must equal the area of tensions BD_1.

Let us suppose such a gun (Fig. 6), and also a homogeneous one of the same thickness (Fig. 3 repeated) fired with equal charges, and

Fig. 3 repeated.

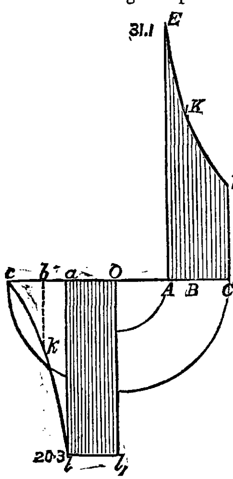
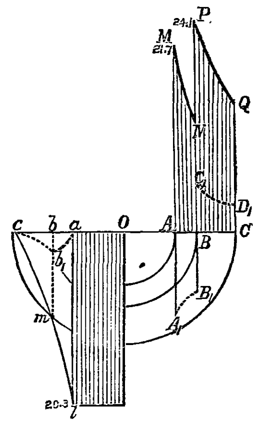


Fig. 6.



that the maximum hoop tension at the interior of the bore is again 31·1 tons on the square inch; in Fig. 3 it rises to that amount, but in Fig. 6 it will be less by the quantity AA_1, since before a hoop *tension* can really act, the initial hoop *pressure* at the bore must be overcome. In Fig. 6 the interior of the tube and hoop will respectively be under firing tensions of about 21·7 and 24·1 tons on the square inch; these tensions can be more readily borne than the 31·1 tons at the bore of the gun in Fig. 3. Fig. 6 is derived by superposing areas from Fig. 3 on Fig. 5. Thus make—

Hoop stresses..	AM of Fig. 6	$=$	AE of Fig. 3	$-$	AA_1 of Fig. 5.
	BN ,,	$=$	BK ,,	$-$	BB_1 ,,
	BP ,,	$=$	BK ,,	$+$	BC_1 ,,
	CQ ,,	$=$	CL ,,	$+$	CD_1 ,,
Radial pressures..	bm ,,	$=$	bk ,,	$+$	bb_1 ,,

and sketch in the curves, finding other points, if necessary, in the same manner.

We notice from Figs. 3 to 6, that the hoop stresses are sometimes tensions and sometimes pressures, but the radial stress* is always one of pressure, both in homogeneous guns and also in those made with initial stresses.

In this Chapter and in Chapter II, Part II, the hoop stress is represented by t, which is regarded as negative when a pressure is intended. The radial stress being always a pressure is represented by p.

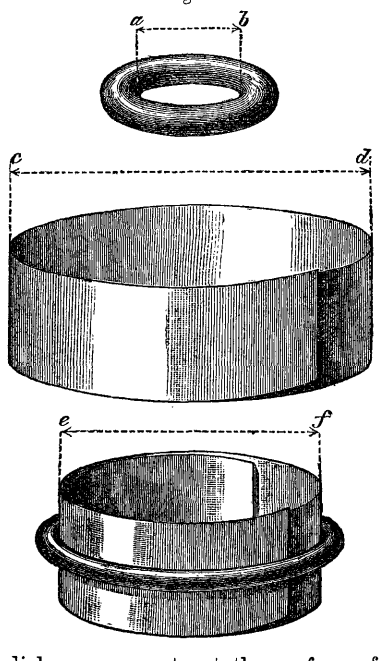
The **maximum hoop tension** imposed on any part of a gun should **always** be **less than the limit of elasticity of the metal,**

* Except in the case of cast guns, *see* p. 70.

because if this limit is exceeded, permanent deformation and enlargement of the bore will result.

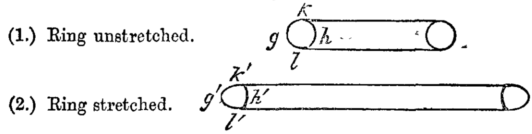
A simple illustration of initial stresses may be given by means of an india-rubber ring (Fig. 7) of interior diameter *ab*, and a loose cardboard roll of greater diameter, *cd*. If the former is stretched, put over the latter and left to itself, contraction of both will take place to some diameter *ef*, intermediate between *ab* and *cd*. A state of stress is thus produced between the ring and the roll, the one being larger, and the other smaller than at first, and each having its elastic tendency to return to its own original dimensions, resisted by the reaction of the other;

Fig. 7.



a normal or radial pressure acts at the surface of contact, which causes a *lengthening* of the india-rubber ring circumferentially, indicating *hoop tension;* and this same normal pressure makes the cardboard roll *smaller* in circumference, indicating *hoop pressure.* The cardboard roll is stronger than before, to resist an interior normal pressure, while the ring is weaker, but still it may be strong enough for the tension which will come upon it.

Fig. 8.



If Fig. 8 represents a section of the india-rubber ring (1) in its unstretched state; (2) when it is expanded over the cardboard roll;

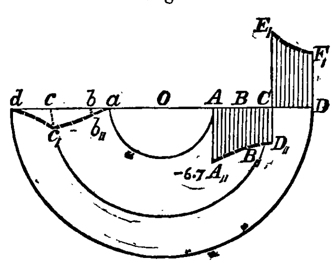
we note that although the ring becomes larger in diameter when it is stretched, and slightly changes in volume, owing to its elasticity, it becomes thinner in section, both in the direction of the radius and of the axis of the roll, *i.e.*, *gh* contracts to *g'h'*, and *kl* contracts to *k'l'*. In accordance with the same law of elasticity, the outer hoop of a gun also becomes thinner and shorter when it is shrunk on; and, conversely, the inner parts become thicker and longer under external pressure, and this has been verified by direct measurement, though the linear changes in the case of steel are minute.

When R.M.L. guns were manufactured, it was at one time the practice to screw the cascable securely into the breech-piece, so that the face of it bore against the end of the A tube before the jacket was shrunk on; when this outer part was afterwards put on, it was found that in some cases the exterior of the cascable could be turned round as much as 2 inches circumferentially, corresponding from the dimensions of the screw to an interval of about 0·022 of an inch between the cascable and A tube; this was doubtless due to the difference of the longitudinal extension of the interior of the breech-piece and of the exterior of the A tube, resulting from the exterior pressures caused by the shrinkage of the jacket.

When a piece of steel is broken by tension in the testing machine, it will be elongated to a certain extent, and it will be narrowed in diameter, especially near the part where fracture took place; the tension has thus induced lateral pressures. According to Wertheim's approximation if a tension acts in one direction, two pressures or negative tensions, each $-\frac{1}{3}t$, are produced in steel or iron, in directions at right angles to each other, and in a plane at right angles to the original tension. A more general statement of this law will be found in Part II, Chapter II.

Returning to the consideration of the hoop stresses in a gun, let us suppose another cylinder is placed over the two of Fig. 5 to compress both of them, and let us investigate how the initial stresses will be altered. If cc_1 (Fig. 9) represents the intensity of the radial pressure at the fresh surface of contact; let us at first suppose the two inner tubes to be one, subject only to the pressures produced by the third cylinder, and we have a diagram (Fig. 9) similar to Fig. 5 in

Fig. 9.



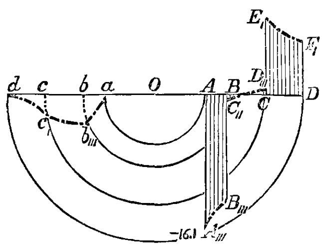
which a radial pressure cc_1 produces hoop *tensions* on the outer hoop, and hoop *pressures* on the inner cylinder, and for equilibrium the totals of these equal each other, or area $AD_{,,}$=area $CF_{,}$. The hoop tensions on the outer cylinder will remain the same whatever may be the condition of initial stress in the inner cylinder; but if, as in the case before us, the inner cylinder is a compound one already in a state of stress, the area of stresses from Fig. 9 must be superposed on the

pressures and tensions in Fig. 5, to show the new state now existing in Fig. 10, which is derived from Figs. 5 and 9 by making—

$$
\begin{array}{r l}
\text{Hoop stresses} & \left\{\begin{array}{llllll}
AA_{\prime\prime\prime} & \text{of Fig. 10} & = AA_{\prime} & \text{of Fig. 5} & + AA_{\prime\prime} & \text{of Fig. 9.} \\
BB_{\prime\prime\prime} & ,, & = BB_{\prime} & ,, & + BB_{\prime\prime} & ,, \\
BC_{\prime\prime} & ,, & = BC_{\prime} & ,, & - BB_{\prime\prime} & ,, \\
CD_{\prime\prime\prime} & ,, & = CD_{\prime} & ,, & - CD_{\prime\prime} & ,, \\
CE_{\prime} & ,, & = & & CE_{\prime} & ,, \\
DF_{\prime} & ,, & = & & DF_{\prime} & ,,
\end{array}\right. \\
\text{Radial pressures} & \left\{\begin{array}{llllll}
bb_{\prime\prime} & ,, & = bb_{\prime} & ,, & + bb_{\prime\prime} & ,, \\
cc_{\prime}' & ,, & = & & cc_{\prime} & ,,
\end{array}\right.
\end{array}
$$

Fig. 10 represents the initial stresses in the chamber of a 6-inch gun, Mark IV, to resist a maximum powder pressure of 24·7 tons on

Fig. 10.



the square inch, the maximum firing tensions on the interior of the A tube, breech-piece, and jacket not exceeding 15, 18 and 18 tons on the square inch respectively.

If other hoops are shrunk over these, the initial stresses will be again changed according to the same principles. We find that the inner tube is subject to a double compression, but the second cylinder, being subject to compression from the outer cylinder and to extension from the inner one, almost regains its orignal neutral state of hoop stress, but we notice it is subject to considerable radial pressure.

If now the gun represented under initial stresses in Fig. 10 is fired and the powder pressure rises to 24·7 tons on the square inch, we have Fig. 11 in which the conditions above stated for the 6-inch gun are fulfilled.

Fig. 11 is derived by superposing areas from Fig. 4 (repeated) upon Fig. 10, thus make—

$$
\begin{array}{r l}
\text{Hoop stresses} & \left\{\begin{array}{llllll}
AR & \text{in Fig. 11} & = AE & \text{in Fig. 4} & - AA_{\prime\prime\prime} & \text{in Fig. 10.} \\
BS & ,, & = BF & ,, & - BB_{\prime\prime\prime} & ,, \\
BT & ,, & = BF & ,, & - BC_{\prime\prime} & ,, \\
CU & ,, & = CG & ,, & + CD_{\prime\prime\prime} & ,, \\
CV & ,, & = CG & ,, & + CE_{\prime} & ,, \\
DW & ,, & = DH & ,, & + DF_{\prime} & ,,
\end{array}\right. \\
\text{Radial pressures} & \left\{\begin{array}{llllll}
bp & ,, & = bf & ,, & + bb_{\prime\prime\prime} & ,, \\
cn & ,, & = cg & ,, & + cc_{\prime} & ,,
\end{array}\right.
\end{array}
$$

Comparing Figs. 4 and 11 we see then in guns of the same dimensions and weight under fire, the limit of elasticity would be exceeded in the homogeneous gun at the surface of the bore when the tension is 31·1 tons on the square inch; but the gun under initial stresses is

nowhere subjected to a tension of more than 18 tons, which it is well able to bear. The questions now arise what pressure should be given

Fig. 4 repeated.

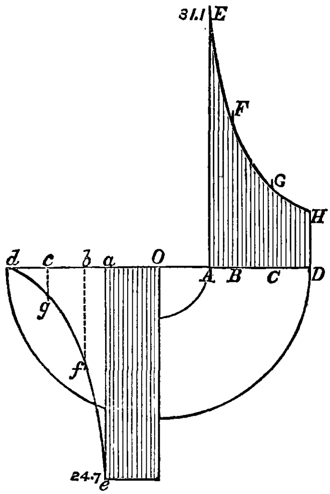
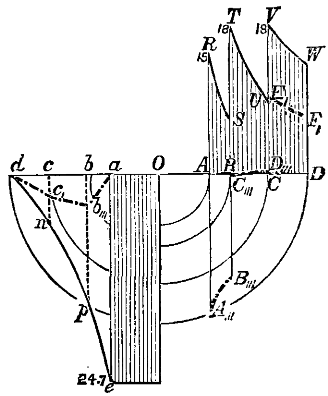


Fig. 11.



by the outer cylinders in manufacture in order to secure the desired result; and how is that pressure practically effected?

A perfectly exact answer to the first question is somewhat complicated, but the following formulæ (*see* Part II, Chapter II) give approximately correct results :—

$$p_2 = \frac{r^2_3 - r^2_2}{r^2_3 + r^2_2} t_2 \quad \text{.............................} \quad \text{(ii)}$$

$$p_1 = \frac{r^2_2 - r^2_1}{r^2_2 + r^2_1}(t_1 + p_2) + p_2 \quad \text{.................} \quad \text{(iii)}$$

$$p_0 = \frac{r^2_1 - r^2_0}{r^2_1 + r^2_0}(t_0 + p_1) + p_1 \quad \text{.................} \quad \text{(iv)}$$

in which p_0, p_1, p_2, are the radial pressures in tons per square inch at the surface of the bore, and at the surfaces of contact between the A tube and breech-piece, and between the breech-piece and jacket respectively, on firing.

t_0, t_1, t_2, are the maximum allowable hoop tensions in tons per square inch at the interiors of the A tube, breech-piece, and jacket respectively, on firing.

r_0, r_1, r_2, are the radii in inches of the interiors of the A tube, breech-piece, and jacket respectively: r_1 and r_2 are also the *exterior* radii of the tube and breech-piece, and r_3 is the exterior radius of the jacket.

If other cylinders are added the formulæ can be extended on the same notation.

The factor of safety of a gun to resist hoop stress is the ratio of the maximum powder pressure which can be sustained without exceeding the elastic limit, compared with the usual pressure. **Factor of safety.**

Example 1.

Find the maximum powder pressure and hence the factor of safety of the 6-inch gun, Mark IV, in which the usual powder pressure is 17 tons per square inch, the maximum hoop tensions on firing to be 15 tons per square inch on the A tube, and 18 tons on the two others, *i.e.*, $t_0 = 15$ tons, $t_1 = 18$ tons, $t_2 = 18$ tons.

The radial dimensions of the chamber are $r_0 = 4$ inches, $r_1 = 5{\cdot}6$ inches, $r_2 = 8{\cdot}7$ inches, $r_3 = 11{\cdot}8$ inches. Substituting their values in equations (ii) (iii) and (iv) we obtain successively,—

$p_2 = 5{\cdot}32$ tons on the square inch $=$ *cn*. *See* Fig. 11.
$p_1 = 14{\cdot}98$ „ „ „ $=$ *bp*. „
$p_0 = 24{\cdot}71$ „ „ „ $=$ *ae*. „

The maximum safe powder pressure is thus 24·71 tons on the square inch, and as the usual powder pressure is 17 tons, the factor of safety (f) is $\dfrac{24{\cdot}71}{17} = 1{\cdot}45$.

Shrinkage.

The necessary initial stress is generally obtained by shrinkage; a cylinder, whose interior diameter when cold is slightly less than the exterior diameter of the A tube, is heated and expands so that it will readily pass over the other; on cooling down, the outer cylinder again contracts and radial pressure is developed between the surfaces in contact: other cylinders are shrunk on in the same manner in succession. The difference between the interior diameter of an outer cylinder and the exterior diameter of the other before the operation takes place is called the shrinkage, and it is obtained from the following formulæ (*see* p. 251) :—

$$ {}_1S_2 = \frac{p_0 - p_1 + t_1 - t_0}{E} \times 2r_1 \ldots\ldots\ldots\ldots\ldots\ldots \quad \text{(v)} $$

$$ {}_2S_3 = \frac{t_2 + p_0 - (p_0 - p_2)\,\dfrac{r_2^2 + r_0^2}{r_2^2 - r_0^2}}{E} \times 2r_2 \ldots\ldots\ldots \quad \text{(vi)} $$

$$ {}_3S_4 = \frac{t_3 + p_0 + (p_0 - p_3)\left(\dfrac{r_3^2 + r_0^2}{r_3^2 - r_0^2}\right)}{E} \times 2r_3 \ldots\ldots \quad \text{(vii)} $$

and so on: equation (v) *can* be expressed in the same form as the others, but it alone can be reduced to the simpler form given above. ${}_1S_2$, ${}_2S_3$, &c., mean the shrinkage in inches between the first and second cylinder and between the second and third, &c., counting from the interior; E is the modulus of elasticity in tons per square inch, supposing it is the same throughout the gun; it may, however, be advantageous to make the A tube of a steel having a lower modulus than the rest of the piece; thus we will suppose a gun made of two parts, a tube and a jacket, the former having the lower modulus of elasticity, and as a simple case imagine that there is no initial stress;

the *extension* produced on firing by the hoop tension will be the *same* at the surfaces of contact, since they will remain in contact; but as the tube yields more readily than the jacket, it follows that the hoop tension on the outer part of the tube must be less than the hoop tension on the inner part of the jacket; by this arrangement, either alone or in conjunction with initial stress, the outer parts of a gun can be made to take a larger share of the firing stress than if the modulus of elasticity is the same throughout.

Example 2. Find the shrinkages between the A tube and breech-piece, and between the breech-piece and jacket, in the 6-inch gun, Mark IV, if the modulus of elasticity is 12,500 tons throughout; dimensions given in Example 1, previous page.

Substituting the various values of p, t, and r for the 6-inch gun in formulas (v) and (vi), we obtain—

$${}_1S_2 = 0{\cdot}011399 \text{ ins. and } {}_2S_3 = 0{\cdot}018001 \text{ ins.}$$

that is, the exterior of the A tube should be made 0·011399 of an inch larger in diameter than the interior of the breech-piece, and after this has been shrunk on, the diameter of the exterior of the breech-piece should be made 0·018011 of an inch larger than the interior of the jacket. The shrinkages are practically said to be 11 and 18 thousandths of an inch respectively.

Besides the favourable arrangement of initial stress, a gun composed of several thicknesses has the advantage of a greater surface capable of inspection in manufacture, and if unsound parts exist they can be more readily detected than in a gun made in only one thickness. Even if a flaw does exist it will only run through one of the pieces of which it is composed, and cannot extend through the whole thickness of the gun.

According to Gadolin's theory, the cylinders should increase in thickness from the interior in geometric progression, but considerable variation may be allowed without any practical loss of strength, and other considerations in the design generally prevent this rule from being carried out.

Wire Guns.

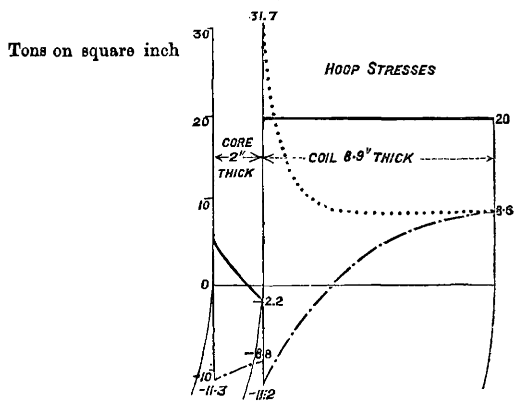
Wire guns. The method of giving initial stress by winding flattened steel wire under tension on a steel tube, has been adopted of late years for the newer designs of guns, and this plan gives good practical results; the tension of the wire can be well adjusted in winding on, and as it is thin and wound round many times, each part can be strained almost to its elastic limit on firing, and thus a fuller use can be made of the total available strength than in the case of thick cylinders; if flaws exist they can generally be detected, and steel, in the form of wire, can be made to have the greatest elastic limit of any known material. Practical difficulties, however, have arisen, as might be expected, the chief being the attainment of the desired longitudinal strength, as the wire gives no assistance whatever in this direction: in some systems the longitudinal strain has been taken almost entirely by a thick tube, but a *thin* tube in conjunction with coils of wire is the best to resist the transverse strain: hence rods and steel covers outside have been employed to help to take up the longitudinal stress, not always, however, with success, as Schultz in France designed a gun which unfortunately broke at the first round, the longitudinal strength not being sufficient.

Mr. Longridge* has investigated the subject mathematically, to find the stress at each coil of the wire under all circumstances; as with cylinders shrunk on, each succeeding layer alters the state of the preceding ones; for instance, if wire is wound on at first under tension it is extended circumferentially; but if the succeeding layers are also wound on under tension, the first layers may become compressed circumferentially. Hence three states are considered—

1st, the laying-on tension.
2nd, the state of stress of the various parts when the gun is completely manufactured and before it is fired; and,
3rd, the state at the moment of firing.

The annexed Fig. 12 shows graphically how he represents the hoop stresses in a 12-in. gun with a tube or core 2 inches and coils of wire 8·9 inches in thickness; the maximum powder pressure is assumed to be 30 tons on the square inch, and the wire coils are to be uniformly strained to 20 tons on the square inch when the gun is fired. The modulus of elasticity of the tube is only 4,500 tons, while that of the steel wire is as much as 22,000 tons, the normal forces, according to Wertheim's coefficients are not taken into account.

Fig. 12.



NOTE.—The laying on tension is denoted by the dotted line ....
The stress of finished gun at rest by the chain line — · — · —
The stress on firing is shown by the continuous line ————

Mr. Longridge advocates a thin inner tube of cast iron, as that material has a low modulus of elasticity; but this has not practically been followed by Elswick or by the Royal Gun Factories, both of whom have successfully constructed wire ordnance; as the metal of the bore must be of such a nature as best to resist the erosive action of fired gunpowder, and cast iron is quite unsuited for such a purpose; but it would appear that of steels, which equally well resist the erosive

Vide "A Treatise on the Application of Wire to the Construction of Ordnance." by James A. Longridge.

action of fired gunpowder, that one is the most suited for the inner tube which has a low modulus of elasticity. The method of joining the ends of the wire has presented some difficulty, and several designs have been adopted: but the practical difficulties have been overcome, and good results have already been attained, as for instance a 10-in. wire howitzer of 70 cwt. has given a projectile of 360 lbs. a maximum M.V. of 1030 f.s., and a 9·2 inch wire gun has attained a M.V. of 2400 f.s.

Although many miles length of wire are used in the manufacture of a heavy gun, the wire generally only constitutes about one-sixth to one-seventh of its total weight.

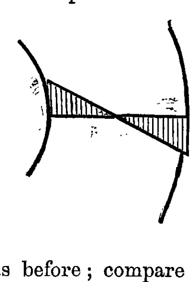
Cast Guns.

Cast guns.

A little consideration will show that simply **casting a gun cannot give great strength**, and the difficulties increase very rapidly with the calibre: in making a casting, the liquid metal flows into a mould, and in due time begins to solidify from the exterior. A more or less rigid crust then forms outside, while the interior is still fluid. In course of time the interior cools and solidifies; but in doing so contracts considerably in volume, and in this case a radial *tension* is produced, tending to pull in the outer crust with it: this it does to a certain extent, but the rigidity of the latter prevents it from giving way beyond a very small amount, the metal in the interior tends to tear itself apart in obedience to the law of contraction on cooling; in all large castings actual fissures are found in the centre, except perhaps in those of Whitworth, which are fluid pressed; this plainly tells us that the particles of the cast gun are arranged in a way calculated to help the powder-gas in bursting the gun, since the inner portions are already tending to rend apart. (*See* Fig. 13, which represents the initial hoop stresses in a cast gun; the vertical ordi-

Fig. 13.

Initial Hoop Stress in a Cast Gun.



nates mean the same as before; compare it with the state of initial stresses in Fig. 5, and 10.) It is true that the outer part has been made stronger by being pulled in and compressed, but this is exactly the part which it is useless to fortify.

In making guns by casting, the difficulties increase with the size, and thus very heavy natures cannot be made on this system; although it does very well for small guns, specially when a good tough material, such as bronze, is employed.

The adoption of the built-up system, on the other hand, has opened out a wide field which has been fully entered into, as during the past 30 years the heaviest guns in our Service have increased from $5\frac{1}{2}$ to 110 tons.

The Americans bestowed very great care on the manufacture of cast-iron guns of large calibre, and to a certain extent they attained success. They cast some of them hollow and kept the core cold by causing water to circulate through it: thus cooling the gun from the interior, and causing the initial stress to be correct in principle. But this plan does not give such good results as the built-up plan previously mentioned; for even the best cast iron does not possess the tenacity and elasticity of steel, and the Americans have lately followed the lead of the European nations by adopting steel guns.

With reference to the American guns, cast hollow, we may notice that many modern plans in gun-making are almost the same as the very oldest—

(1.) The first cast guns were cast hollow, and it was not until 1739 when the process of boring was improved, that guns were cast solid, and bored out afterwards, as it was found difficult to keep the core in the middle of the casting.

(2.) The modern plan, until lately adopted, of building up guns with coils of wrought iron is almost the same as that employed at the very first, for then wrought iron was employed, cast iron being unknown; and coils were doubtless used on account of their easily giving the required cylindrical shape, and of the difficulty of obtaining and working a large solid forging.

(3.) The likeness of the newest guns to the very old ones in their great length, is a subject of ordinary comment. With the old pieces the gunpowder employed really was a powder, very weak and slow in its action, hence a long bore was necessary to give it time to burn and press on the projectile in the bore. But as the manufacture of gunpowder continued, it was "incorporated," "pressed," and broken up into grains or "corned," when it burned with very much greater rapidity and violence. Hence shorter bores were used, not always, however, with advantage, as was shown in the first Afghan war, in which the hill men with their long matchlocks and inferior powder sometimes hit our men at ranges at which our Brown Bess could not reply. Now that elongated projectiles, are used, it is found necessary purposely to make the powder slow burning, and again the bores are lengthened in order to obtain the high velocities required.

(4.) When the Enfield rifle was converted into a breech-loader on the Snider system, an old musket was found in the collection of arms in the Tower almost exactly corresponding to the breech action then introduced.

(5.) When rifled guns were introduced, the swell at the muzzle, common to all S.B. guns was abandoned, but in the more modern pieces with well-sustained powder pressures and high velocities, they have again been adopted, since a slight tendency to split at the muzzle has been noticed (due doubtless to the metal not being supported in front, as is the case at all other parts of the bore).

Longitudinal Strength.

Locking the parts together.

In some steel guns longitudinal strength has been given only by a thick A tube, as the hoops which were shrunk over were not attached to each other; but now it is considered necessary that the parts of a modern steel gun should be firmly locked together, to avoid any loosening from the effects of firing and to give longitudinal strength: at one time this was done by means of a ring with a groove cut inside it, the ring being cut into two or three segments, the groove in it was fitted over projections which were on the ends of both the parts to be attached together: a small hoop shrunk over prevented this "split ring" (as it is called) from falling off. More recently, however, the attachment has been by means of an outer cylindrical portion, whether jacket, breech-piece, or hoop, which fits over the other part to be secured to it by bayonet joints—a series of projections exist inside the outer portion, and also a series of projections on the outside of the inner portion; the outer portion is dropped on by sliding its projections between those of the others; when down in its position the projections on the one have passed to a place where there are none

on the other portion, the outer one can then be turned round slightly; this is done till all the projections are in a line; the empty parts between them are filled with steel wedges driven in, and a secure fastening is thus made. This plan has the advantage that transverse strength is also given by the outer portion, as it is shrunk on.

In the construction of guns now adopted the A tube gives little or no assistance in resisting the longitudinal traction, because the breech-screw is not now cut in it, as was formerly the case (Fig. 14), when it was found that a tendency existed to shear the threads of the screw, because the chamber part of the A tube expanded on firing, while the rear part did not do so, and thus the threads had an unequal bearing: this difficulty has been overcome (Fig. 15) in recent constructions by cutting the threads of the screw in the jacket or breech-piece instead of in the A tube.

Fig. 14.

Breech part of 4-inch B.L. Gun Old Pattern with Interrupted Screw cut in the A Tube.

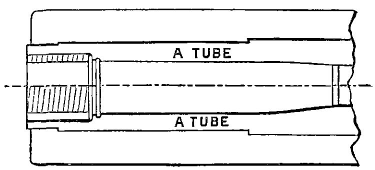


Fig. 15.

Present Pattern with Interrupted Screw cut in the Jacket.

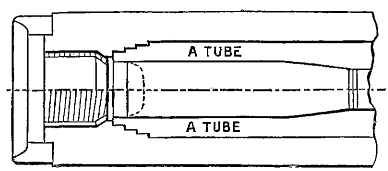
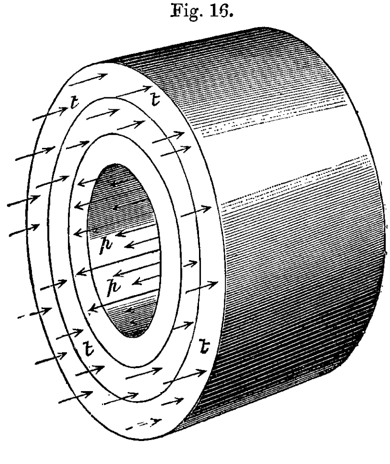


Fig. 16.



With regard to an estimation of the longitudinal strength of a gun, or its ability to resist ring fracture (*see* Fig. 16, where for simplicity only one of the halves into which the gun tends to split is shown); the stress tending to produce rupture in this direction—the fluid pressure per square inch multiplied by the number of square inches in the sectional area of the bore, is resisted by the tenacity of a part of the cross section of the metal of the gun (according to the design).

$$\pi r_0^2 p f_1 = \pi(r_3^2 - r_1^2)t$$

$$\text{or } r^2_0 p f_1 = (r_3^2 - r_1^2)t \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(viii)}$$

in which p = usual powder pressure in the chamber in tons per square inch.

t = elastic limit of the steel in tons per square inch.

r_3 = external radius in inches of the part bearing longitudinal stress.

r_1 = interior radius of the same.

r_0 = radius of the chamber.

f_1 = factor of safety in this direction.

This formula is only an approximation, as it takes no account of several circumstances: for instance, the initial stress caused by shrinkage, which produced elongation of the bore and contraction of the exterior cylinder, is not regarded, nor is account taken of a corresponding tendency to shorten the bore, due to the radial pressure on firing, but the formula is practically useful. In most constructions the factor of effect in this direction is much better than in the transverse direction; for instance, if we substitute the values for the 6-inch gun taken previously, we find—

$$f_1 = 7{\cdot}1.$$

Guns will, however, give way if the longitudinal strength is too small, and this was shown by the 100-ton gun on board the Italian ship "Duilio," which burst with a ring fracture at the line of least resistance, where the inner tube tapered from the chamber to the bore, and where the coils joined;—a strange instance of longitudinal pressure was afforded by the gun, which burst or at least broke away on board the Chilian ship "Angamos," when firing at the "Union" in dock at Callao in December, 1880. The piece, an 8-inch B.L. 11½ tons, new type, with high velocity, had fired many rounds, when at last it shot backwards out of its wrought-iron trunnion ring through the side of the vessel into the water. The explanation given is, that the wrought iron had become slightly enlarged by continued firing, while the steel, from its higher elasticity, had returned to its original shape; and at the last round, on the expansion caused by firing, the steel contracted more quickly than the wrought iron, and, so becoming loose, the gun was projected backwards by recoil, as no shoulder had been made on the tube, and the attachment was simply frictional.

The tendency to blow out the end of the bore is easily prevented in practice by the thickness of the metal, and guns have seldom given way in this direction.

The pressures tending to split the gun longitudinally and to produce ring fracture are in about the same proportion to each other all the way down the bore, but each becomes less towards the muzzle, as the projectile in moving along acts as a kind of safety-valve, and so the pressures are lowered.

All calculations of the strength of guns can only be regarded as approximations, as they are founded on the assumption of a steady fluid pressure, as in the cylinders of a hydraulic machine; but the pressure of fired gunpowder is sudden, and must set up vibrations in the metal, which cannot be determined. Calculations are useful as guides to the designer, but a gun cannot be regarded as strong enough for the pressures to which it is intended to be subjected, until after the proof rounds have been fired, and it has been determined by a comparison of careful measurements to the thousandths of an inch, before and after proof, that the interior of the bore has not permanently expanded by the firing, and that consequently the elastic limit of the steel has not been exceeded.

CHAPTER VI.—DETAILS OF GUN CONSTRUCTION.

In the last chapter we considered the general nature of the stresses and strains to which a gun is subjected on firing, and the means taken to meet them. We will now look at some of the details of construction.

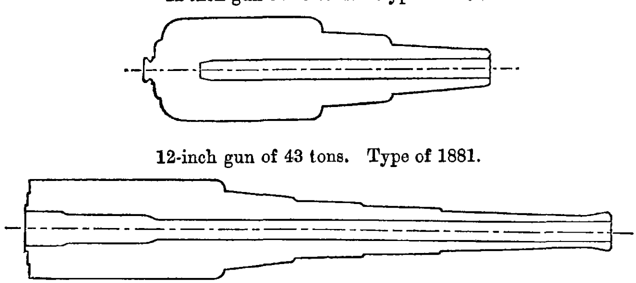
Bearing in mind that a gun is a machine to obtain energy in a projectile from the gunpowder, it follows that if changes are made in the latter, corresponding changes must be made in the former in order to utilise the altered qualities of the charge.

With the old powder the explosion was quick, and the pressure in the gun rapidly developed and reached a high maximum before the projectile had time to move far down the bore; the pressure, however, rapidly fell off towards the muzzle, as the space occupied by the gas increased, owing to the forward movement of the projectile. Hence we find guns constructed for the old powder short in the bore, thick at the breech, and tapering rapidly towards the muzzle (*vide* diagram of a 12-in. gun of 35 tons, Fig. 1, which has a bore 13·5 calibres long, and a thickness of metal at the chamber of 1·9 calibres).

Fig. 1.

12-inch gun of 35 tons. Type of 1871.

12-inch gun of 43 tons. Type of 1881.



Attempts were made to reduce the high maximum pressure on the gun, and it was accomplished by air spacing and by introducing P powder, then P^2, prism black, and more recently prism brown, which burn more slowly than that which was formerly used; these arrangements, as before mentioned, allow the projectile to move some distance down the bore before all the powder is converted into gas; the maximum pressure is thus reduced, but, the charges being larger, the sum of all the pressures is increased, if the gun is made of sufficient length to give a considerable number of expansions to the powder charge. Since the maximum pressure at the breech is less than formerly, metal can be spared from thence to add

to the length of bore at the muzzle; but this does not, however, give quite enough metal, and the new guns are rather heavier than the old ones of the same calibres. **Modern guns** are very **long** and they taper but slightly towards the muzzle.

These considerations led to the proportions illustrated in Fig. 1 by the 12-in. gun of 43 tons which has a bore 25·5 calibres long, and a thickness of metal over the chamber of 1·2 times its diameter; the more modern 9·2 inch wire gun is no less than 40 calibres in length.

More recently the employment of prism brown powder has led to **thickening the metal near the muzzle**, as the powder pressures are now well sustained over the whole length of the bore.

In obtaining the same velocity in similar projectiles with an old and with a new type gun, the one imparts the energy to the projectile rapidly, the other more slowly, and thus a very much greater maximum strain tending to cause fracture is produced in the first case than in the second. With the new type gun, on the contrary, a higher velocity may be produced in the projectile without exceeding the safe limits of maximum stress on the gun. This may be simply illustrated thus:—A truck standing on a line of railway cannot be made to take up a velocity of 60 miles an hour very rapidly by dashing an engine moving at a very high speed against it, as a general break up would ensue: but it *can* be made to move at the rate of 60 miles an hour, or even faster, by attaching to it a locomotive at rest, which then starts off and gradually increases in speed without breaking anything.

Breech and muzzle loaders.

The great length of modern guns appears to have settled the long-vexed question of the relative advantages of **breech** and **muzzle loaders** in favour of the former, at least for heavy guns, as Mr. Trevelyan, Under-Secretary of State for War, stated in his speech on the Navy Estimates in the House of Commons in March, 1881:—" High velocity is now required for the projectile, and this can only be obtained by length of gun, and a gun beyond the usual length cannot be loaded at the muzzle under ordinary circumstances."

Whatever may have been the relative advantages of breech and muzzle-loading a few years ago, it seems now to be universally admitted that the present conditions of very long guns with large diameter chambers, slow-burning powder, and the need for preventing the ready movement of the projectile, by holding bands, all compel the employment of breech-loaders.

Grooves.

An important matter in dealing with metal structures subjected to heavy strain is an unbroken smooth surface, small cracks and even tool marks serve as starting points for fracture in a very marked degree; and this is especially the case in the bore of a gun, where the erosive action of the powder gas serves to enlarge any break in the smoothness of the surface.

In this respect the bore of a rifled gun compares unfavourably with that of a smooth-bore, since each **groove** in the former is an element of weakness, as was shown by the experimental 80-ton gun, in which after severe experimental trials a crack began to develope in one of the grooves of the steel barrel.

However, since grooves are indispensable in rifled guns, the problem becomes (as far as the gun is concerned) to make them as little hurtful as possible. Numerous shallow and rounded ones are preferable to those which are few in number, deep and with sharp re-entering angles. If they are numerous, the pressures of the studs, gas-check,

or rotating ring on the sides of the grooves are more distributed, and if shallow and rounded there is less fear of the development of cracks than under opposite conditions.

The old Armstrong polygrooved system had sharp corners, and the Service M.L. guns for studded projectiles have only a few grooves; but the modern shallow and rounded polygroove now employed possesses as few disadvantages as possible.

With regard to the **twist of rifling**, the *increasing* gave rotation gradually, with nearly uniform pressure on the grooves and studs all the way down the bore when the quick burning powder which caused great maximum pressure at the breech was employed; while the *uniform* twist imparted angular velocity to the projectile very rapidly at first, but added little to it during the latter part of its passage down the bore; hence in this latter case the pressures varied greatly. **Twist.**

This difference is well shown by Table A (calculated by Captain A. Noble, F.R.S.), which shows the pressures between the studs and grooves in a 10-in. Service gun at different distances down the bore. (For formulæ for calculating the pressures on studs, see the article on "Gun Making" in the Encyclopædia Britannica, by Colonel Maitland, R.A.)

(1.) With an increasing twist.
(2.) With a uniform twist (the same as the other at the muzzle).

TABLE A.

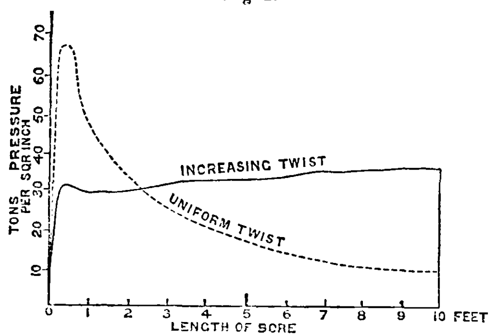
Travel of projectile in feet.	Pressure on the studs in tons per sq. inch.	
	Increasing twist.	Uniform twist.
0·000	0 0	0·0
0·833	31·2	68·5
0·945	28·7	47·7
1·834	29·0	34·6
2·723	30·2	27·5
3·612	31·4	22·6
4·500	32·3	18·7
5·389	33·0	15·8
6·278	33·8	13·5
7·167	34·5	11·8
8·055	35·2	10·6
8·944	35·8	9·7
9·833	36·3	9·1

From this it appeared that the maximum pressure on the grooves was nearly double as much in the one case as in the other, see Fig. 2, which is simply Table A plotted out to scale; the horizontal line represents the length of the bore, and the curved lines are obtained by vertical ordinates which represent the pressures at consecutive points along the bore; it has practically been found that the greatest wear takes place in guns rifled with the uniform twist at that part of the groove which is nearest to the chamber.

With a studded M.L. projectile the maximum pressure on the studs was about the same in both cases: because with the increasing twist the rear studs practically give all the rotation, while with the uniform twist both front and rear are equally in bearing on the grooves all the time the projectile is passing down the bore.

With modern conditions of well-sustained powder pressures all the way down the bore of a gun, and with great initial maximum pressure at starting, the necessity for the increasing twist is not so great as

Fig. 2.



formerly: and the twist is now made uniform for some distance at the muzzle to give steadiness of rotation. The part of each groove towards the breech is, however, made with the increasing twist.

Vent. With regard to the opening which must exist in the chamber of every gun, viz., the **vent**, it may be either *radial*, entering the bore at right angles, or nearly so, to the axis of the gun, or *axial* when it is in prolongation of the axis of the bore; the former may be either *forward*, striking the cartridge at 0·4 of its length from the end, or *rear*, entering the chamber near the junction of the cylindrical part with the flat end of the bore. It is in all cases a source of weakness, as it gradually enlarges, and cracks may develope from it after long continued firing. This means the ultimate condemnation of a cast gun when, after a great number of rounds, the cracks or fissures have reached certain specified dimensions; generally speaking, the defects which condemn S.B. guns will be found at or near the vent. The Service R.M.L. guns (except 12·5 inch of 38 tons which have axial vents) for studded projectiles resemble S.B. ordnance in having copper radial vents, but the steel bore suffers less than the cast iron; defects, however, occur in them from the same cause. Removable steel vents for field guns prevent injury to the bore, and they can soon be replaced when worn out.

In the first breech-loaders in our Service the vent was a bent channel, entering the chamber at the axis of the gun, and it was placed in the removable vent-piece; this gave the obvious advantage that any damage at or near the vent only caused the condemnation of the vent-piece instead of the whole gun. But with this arrangement it was found that the charge gave a good deal less velocity to the projectile, and quite as much strain to the gun; therefore for R.M.L. guns forward vents were employed in all except—

(1.) Field guns constructed before silk cloth was generally introduced for cartridges (as there was danger of smouldering fragments remaining in the bore, and prematurely igniting the next charge, when firing rapidly with blank ammunition), and

(2.) Howitzers, since they often use very small charges, which might fail to be ignited with a forward vent.

These both have radial (rear) vents.

When the large grained slow-burning powder was introduced, it was found to make no practical difference in the velocities and pressures, whether the cartridge was ignited near the middle of its length or at the end; the reason probably is that with the large interstices between the grains, each one of them is ignited at about the same time; while with the old smaller grained powder, as the interstices are smaller, the grains nearest to the vent are ignited first, and therefore a charge ignited near the middle of its length burns up more quickly than one lighted at the end, and so it gives a higher velocity to the projectile.

The 38-ton gun, Mark II, 80-, and 100-ton guns, which are muzzle-loaders, are axial vented, as the following advantages are obtained:—

(1.) Wear round the vent bush and fissures in the bottom of the bore are of comparatively small importance, as they can be readily bored out without injuring the walls of the gun.

(2.) The vent can be easily got at, and so a fresh bush can readily be put in on service when required.

A means of stopping the rush of gas and flame through an axial vent must be adopted for the sake of safety; this arrangement also lessens the erosive action of the powder gas at that place; there is no difficulty in effecting this. In the new breech-loaders axial vents are placed in the breech-screws, and automatic safety arrangements are made which prevent the tube from being fired until the breech is properly secured.

The **chambers** of cast-iron guns and of the first B.L. ordnance are cylindrical, the former of the same, the latter of very slightly larger diameter than that of the bore; but howitzers, mortars, shell guns, and carronades, as well as the Service R.M.L. guns, have conical chambers (speaking generally) whose mean diameter is less than that of the bore. With the new high-velocity guns it is, however, found that the best results are furnished by cylindrical chambers considerably larger than the bore; these enable the large charges to be made up into tolerably compact forms (of lengths $3\frac{1}{2}$ to 4 times their diameters), and this plan helps to prevent the abnormal very high local pressures which are induced by *long* cartridges. As before mentioned, the increased diameter of the chamber is a source of weakness; but this does not so very much matter, as the maximum pressures may be kept tolerably low with the slow-burning powders now employed. Chamber.

In all M.L. ordnance care is taken to round off the junction of the curved part of the chamber with the flat end, in order to prevent the formation of cracks. The front part of the enlarged chamber of a new B.L. gun tapers towards the bore, and is sometimes lined with copper to resist erosive action: the incline is a rapid one, as it causes the gas to converge, and it prevents it from striking the bore at the seat of the projectile: thus the first part of the rifling is less eroded than it would be if the incline were more gentle. This incline stops the projectile at the same place each time the gun is loaded.

The **grip** in the early R.B.L. guns had the advantage of always stopping the projectile at the same place, hence the space occupied by the cartridge was always nearly the same, the gravimetric density was fairly constant for each round, and consequently nearly equal muzzle velocities were produced (other things being equal): this accounted for the great accuracy of the first R.B.L. ordnance. The R.M.L. guns which replaced them were less accurate, chiefly because Grip, choke, and stops.

some gunners ram home with more force than others, and, as the cartridge is slightly compressible, different charges occupied different spaces, according to the strength of the man who happened to be ramming home; the velocities of the projectiles, and, consequently, the ranges varied, and the accuracy was impaired. To prevent this, marks were made on the rammers to tell when the projectile was in the right place, but gunners cannot be expected to look very carefully to a little mark on the stave in the heat of action, and so, consequently, a **choke** or slightly narrowed part just in front of the chamber is provided in the 13-pr. and 2·5-in. R.M.L. guns, and with R.M.L. howitzers **stops** are provided at the ends of the grooves; these devices prevent the projectile from being rammed too far.

Bor

With regard to the **bore,** since the strains become less and less towards the muzzle, defects in front of the trunnions are of rather less importance than those behind them. Rules are laid down for the guidance of inspectors, who are directed, when in doubt about any defect, whether in the chamber or in the bore, to note if it increases after firing a certain number of rounds, and if it does not materially do so, the gun is considered serviceable.

The bores of guns may be injured by shells accidentally bursting in them, and in some of these cases the steel tube has split at the muzzle. The premature bursting of a shell in a gun is very injurious; but a break up in the bore and the consequent ignition of the bursting charge is comparatively harmless. A longitudinal crack is more dangerous than one in another direction.

Liners.

Notwithstanding the lessening of erosion from the sealing of windage by the use of gas checks and rotating rings, continued firing will obliterate the grooves at the breech end of a gun with the heavy charges now used: the new heavy guns are consequently made with thin **liners** of steel in which the grooves are cut. This lining is in two parts: that at the breech end extends to a little in front of the trunnions, and it can be renewed when the grooves are worn out: the erosive action is not of importance towards the muzzle.

Means are taken to keep the bores clean and free from rust by oiling: and heavy guns which are not often used are lacquered internally; the lacquer can be removed very rapidly if necessary by firing small blank scaling charges. Depressing the gun prevents water from lodging in and rusting the chamber, and tampeons and vent stoppers keep out damp air. All breech-gear should be kept clean and free from rust in order to keep it in proper working order.

Gas escape.

Coming now towards the exterior, we find that all R.M.L. built up and converted guns are provided with small channels called **gas escapes,** through which smoke issues, if the inner tube is cracked through, thus giving warning that it is time to cease firing. With cast guns there is no self-registering means of recording if the bore has been seriously damaged after any round, and so the firing may not be stopped until a serious accident occurs.

Breech fermeture.

With B.L. ordnance the most successful plans adopted for **closing the breech** are the side wedge of Krupp and the interrupted screw: the latter is used by us, as well as by the French, and has the advantage of being well protected, as it is behind the gun.

Obturation.

The escape of gas at the junction of the breech block and the end of the bore must be prevented; this is called **obturation**; the system employed in the R.B.L. (screw) guns was the mechanical fit of two conical surfaces, one at the end of the bore and the other on the face

of the vent-piece: with smaller natures these were made of copper, and with the larger pieces a detached flat cup was employed to prevent escape of gas; with the former, continued firing eroded the surfaces and escape of smoke took place, causing fouling of the breech screw and consequent difficulty in turning it and securing the breech; provision was made for rebushing with copper, but this took time and skilled labour; with the larger natures of guns the tin cups often jammed at the end of the chamber from their want of elasticity, and caused delay in reloading. In the first of the modern B.L. (interrupted screw) ordnance a carefully made flat steel cup (E.O.C. cup) was attached to the breech screw; when the gun was fired the expanding gases pressed the sides of the steel cup against the bore and over the junction with the block, all small orifices were thus closed; immediately after firing the elasticity of the steel caused the cup to regain its former shape, and the breech could be opened again.

The French have obtained good results from the lateral expansion of a wad or pad of asbestos and mutton fat encased in canvas acted on by a circular steel block (de Bange plan) at the end of the bore, and this system is now adopted in our service. The durability of the apparently frail pad is remarkable: at the Okehampton practice in 1883 several spare wads per gun were taken, but no extra ones were found to be necessary. It is found to act much better than the steel cup, which rapidly scores from the action of the powder gas if grit is present.

With quick firing guns, machine guns, and B.L. small arms, the obturation is effected by a metallic cartridge case.

Trunnions.

The use of the **trunnions** is to give a means of support to the gun, to communicate the shock of discharge to the carriage, and to allow of easy alteration in the elevation of the piece. They are generally placed a very little way in front of the centre of gravity to allow of elevating with ease; this causes a statical pressure on the elevating gear called preponderance, which is necessary for steadiness. Of late years the amount of preponderance has been reduced to facilitate rapid laying.

With S.B. guns the axis of the trunnions was below that of the gun, to allow the use of quarter sights in some natures, and to allow sufficient elevation to be given with others. With all rifled pieces the axes of the gun and trunnions intersect. Mortars have their trunnions at the end of the breech for convenience in giving high angles of elevation, and carronades had a loop underneath instead of trunnions.

The Italian 100-ton B.L. gun is not provided with any trunnions,* but it is rigidly attached to its carriage. The necessary alteration in elevation is given by raising or lowering the rear end of the slide, the front of which pivots on a large horizontal bolt. This arrangement is intended to adjust the strains caused by firing the gun advantageously, and it also allows the use of a small port.

In our Service certain heavy guns intended for hydraulic mountings also have no trunnions.

Ordnance in parts.

Under certain circumstances, when only a certain weight can be carried, **ordnance** have been made **in parts** which can be screwed together on service, thus making up a heavier gun than can be otherwise readily transported. The 2·5-in. jointed gun is made up of two

* *Vide* "Engineer," 26th January, 1883.

parts, each of about 200 lbs., which is about the limit to be carried by a pack animal in mountain artillery: the two parts are connected together by a junction nut, which contains the trunnions, on the same plan as the attachment is made between a fire hose and a standpipe; a gas ring is employed at the join to prevent escape of gas: a light howitzer in three parts has also been made. The Russians employed heavy howitzers on this principle in their late war in Turkey, and a jointed wire siege howitzer has been fired at Lydd.

CHAPTER VII.—LAYING.

The intention in laying a gun is to point its axis in such a direction that the projectile is likely to hit the target on firing. It is done in several ways. Most commonly on the outside of the gun a visual ray (called the line of sight), whose inclination to the axis is known, is pointed directly on to the target; this is called **direct laying.** When a gun is fired through the small port of a turret or cupola, if the line of sight were on the gun itself, the wall of the structure would frequently hide the target from view, preventing the piece from being laid by this means; as turrets and cupolas turn with their guns in traversing, the line of sight is placed outside the whole structure. This visual ray passes through the target, the top of a point or fore-sight secured on the fore part of the gun (or turret), and a notch on a tangent scale which is capable of moving in a slot (at right angles, or nearly so, to the axes of the gun and trunnions) in the breech of the gun, or breech side of the turret or cupola. If this scale is pushed in or out of the slot, and the gun is moved at the same time so that the visual ray is kept passing through the three points, the inclination of the axis of the piece to the line of sight (called the angle of elevation) is altered, and the scale records the magnitude of the angle, or the range due to it with Service projectiles.

When the scale is lowered to the bottom, the line of sight is parallel to the axis of the gun.

The axis of the piece must be pointed to some spot P above* the target (Fig. 1), to allow for the effect of gravity which acts on the projectile in flight; and with service guns rifled with a right-handed twist the axis must point to some spot P′ (Fig. 2) to the left, to allow for the lateral motion of drift to which their projectiles are subject.

Fig. 1.

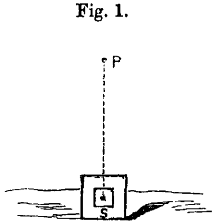
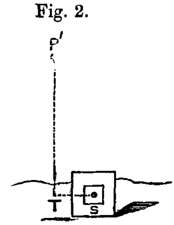


Fig. 2.



If through S, the centre of the target, in Fig. 2, ST is drawn horizontally, and from P′ a perpendicular P′T is dropped on to it; then P′T subtends the angle of elevation of the gun in a vertical plane, and TS subtends the angle of deflection in a horizontal plane, necessary to be given to the axis of the piece.

In the German cupola experimented with at Bucharest in April, 1886, the gun could be laid at *short* ranges by looking through the bore before loading; the No. 1 was not then so much exposed as when laying with the sights outside the cupola.

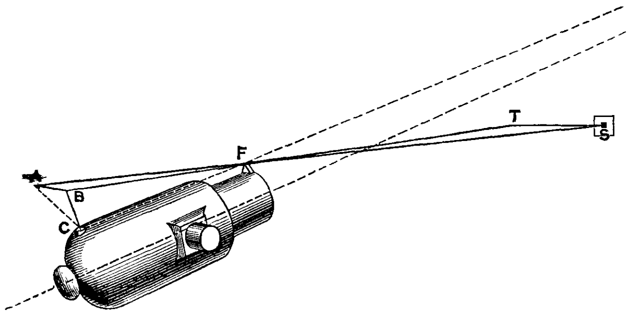
* Except at very short ranges when the jump (*see* p. 188) of the gun on firing jerks up the muzzle sufficiently.

As an example, suppose the rifled gun in Fig. 3 correctly laid, so that with a given charge it will hit the target S.

The tangent scale works in a slot which is perpendicular to the axis of the gun and nearly so to that of the trunnions; in the head of the tangent scale is a leaf with slight notch capable of sliding on a graduated deflection bar, parallel to the trunnions, which should be level.

Let TS be the horizontal distance of S from a vertical plane passing through the axis of the gun; this is the "drift" or distance to the left of the target it is necessary to point the axis of the piece.

Fig. 3.



NOTE.—As angles in 3 planes at right angles to each other have to be considered, this figure is drawn in perspective; the angles ABC, ABF, BCF, FTS are all right angles.

Let F be the foresight,

BC = amount on the tangent scale (let us suppose at first that this is at right angles to both the axes of the gun and trunnions),

BA = amount on deflection scale;

then AFS is the line of sight, and BFT is an imaginary line through the top of the foresight in the vertical plane which contains the axis of the gun.

Join AC,

Suppose angle $ACB = \theta$,

And angle BFC, angle of elevation $= \epsilon$.

Let TS, the drift $= D$,

And FT, the range $= R$;

then in the similar triangles, ABF, FTS,

$$\frac{AB}{BF} = \frac{ST}{TF} = \frac{D}{R},$$

Now $AB = BC \tan\theta$, for ABC is a right angle;

And $BF = BC \operatorname{cosec}\epsilon$, since BCF is a right angle;

$$\therefore \frac{BC \tan\theta}{BC \operatorname{cosec}\epsilon} = \frac{D}{R},$$

$$\text{whence } \tan\theta = \frac{D}{R} \operatorname{cosec}\epsilon \quad \ldots\ldots\ldots\ldots \text{(i)}$$

It is found practically, that for ordinary service angles of elevation, the values of the mean ranges and lateral deviations vary in such a way that when the angle ϵ is changed, the value of the three variables in the expression $\frac{D}{R}$ cosec ϵ remains nearly constant. In other words θ is nearly a constant angle for different service angles of elevation; this is not however strictly true, but it is a fair approximatton. θ is always small, never exceeding about 3°.

Advantage is taken of this circumstance (with guns) by inclining the slot for the tangent scale laterally in the direction of AC at an angle θ to the perpendicular to the axis of the trunnions. This angle varies for different ordnance, and is practically found by firing a series of rounds at different ordinary elevations to find the corresponding mean ranges and drifts; the value of tan θ, from the above formula, for each elevation is then calculated, and the mean of all the values of θ thus found is the angle of the slot with the perpendicular to the trunnions, and it is called the **permanent angle of deflection.** By this means one adjustment of the scale is sufficient instead of two, and the deflection leaf is only used for occasional fine adjustment and for accidental circumstances, such as cross wind or difference of level of wheels.

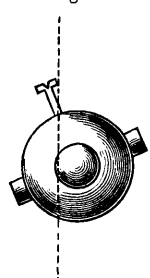
Permanent angle of deflection.

With howitzers, however, the varying charges prevent the use of this plan, and no permanent angle of deflection can be found for them, which will do under all circumstances; their tangent scales are therefore perpendicular to the axes of the piece and of the trunnions, and they are provided with long deflection bars.

If, from inequalities or slope of the surface of the ground or platform, **one trunnion is higher than the other** (suppose the right higher than the left), the notch of the tangent scale, when raised, moves more to the left (Fig. 4) than the foresight does, and so the axis of the gun is pointed more to the left, and the projectile will fall to the left of the target (the side of the lower wheel), if allowance is not made for this circumstance.

Difference of level of wheels.

Fig. 4.



Looking from breech.

NOTE.—The dotted line is a vertical through the tip of the foresight. When the wheels are level a vertical line will cover both sights, or the deviation of the tangent scale from it will only be due to the permanent angle of deflection.

The angle of elevation will also be a little less, but for ordinary small differences of level of wheels this is practically insensible, and may be neglected.

The practical rule given in the Manual of Field Artillery Exercises to allow for the difference in level of wheels is—

"Multiply the difference in level of the wheels in inches (or the inclination of the trunnions in degrees) by the number of degrees of elevation for the range, for the number of minutes deflection to be given on the side of the highest wheel."

This may be explained by a consideration of the following facts:—

$$\text{Since } \tan 1^\circ \text{ is nearly } \frac{1}{60}$$

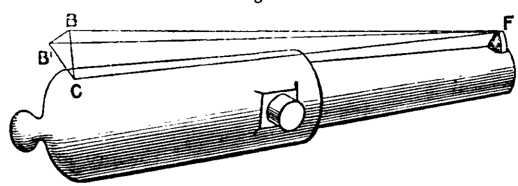
$$\tan 2^\circ \quad \text{,,} \quad \frac{2}{60}$$

&c., &c.,

(only however approximately true for *small* angles,)

and as the track (5′ 2″) of the wheels of field and siege gun carriages is, roughly speaking, 60 inches; if one wheel is 1 inch higher than the other, the axis of the trunnions must be inclined at an angle of 1° approximately, and generally (for small angles) if one wheel is n inches higher than the other, the axis of the trunnions is inclined at n° to the horizontal, and at the same time the tangent scale has been moved through n° laterally from its position when the trunnions are level.

Fig. 5.



Suppose (Fig. 5) F the foresight, BC the tangent sight which records m degrees of elevation; and that the latter is turned sideways through an angle of n degrees by an alteration in the level of the wheels, so that B is moved round to B′, and the angle BCB′ = n°. Join B′F, BB′.

From what has just been stated above, it follows that

$$BB' = \frac{n}{60} BC \text{ approximately.}$$

Now BC subtends m degrees at F;

$$\therefore BB' \text{ subtends } \frac{n}{60} \cdot m \text{ degrees at } F,$$

or mn minutes; and the angle $BFB' = mn$ minutes.

Hence the above rule.

With mountain guns the track is 3 feet for the 2·5-in. gun, and 2 ft. 3 in. for the 7-pr. gun of 200 lbs.: "In the former case the number of degrees of elevation must be multiplied by 0·6 of the number of inches difference of level, and in the latter case by 0·45 of the number of inches difference of level, to ascertain the number of minutes deflection which should be given on the side of the higher wheel. To effect this, the prickers of the guns are marked with divisions of 0·6 in., and of 0·45 in. respectively.

With the jointed 2·5-in. gun of 400 lbs. it has sometimes happened that the junction nut, which also constitutes the trunnion ring, has been overscrewed, the consequence being an effect on the sights similar to that of having one trunnion higher than the other, and an increased difficulty in accurate laying.

The inclination of the trunnions to the horizontal is found with howitzers by means of a clinometer or quadrant-level placed on the plane cut across the piece; with field guns it is found by the pendulum deflector attached to the carriage, or a sponge stave with one end resting on the top of the higher wheel may be held across the other and levelled with a spirit level or clinometer; the distance in inches of the top of the lower wheel from the stave is then measured. Method of finding the inclination of the trunnions.

The necessary calculations for the difference of level of wheels are most tiresome to make at a time of hurry and excitement, and they constitute a serious difficulty in the attainment of the two great desiderata in artillery fire, viz., rapidity and accuracy.

It is ordered that (when firing at a stationary object) the sights must always point directly on to the target for the sake of good shooting, deflection on the scale being given for cross wind, difference of level of wheels, or other cause of deviation; allowance must *not* be made by laying so many yards right or left. Correction of shooting.

One minute of deflection subtends about 1 inch at a hundred yards, and so generally n inches at n hundred yards and m minutes of deflection subtend nm inches at n hundred yards.

Thus, if the mean of several rounds at 1800 yards falls 3 feet 6 inches to the right of the target, the deflection which should be given is found thus—

Deviation in inches = range in hundreds of yards multiplied by deflection in minutes;

$$\therefore 42 = 18n,$$
$$\text{whence } n = 2\tfrac{1}{3} \text{ minutes deflection left.}$$

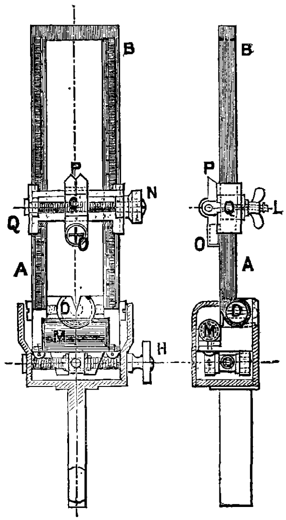
If a group of rounds falls short or over, *correction* must be made by giving more or less elevation; but this varies considerably for different projectiles and ranges, and no general rule is applicable, but the particular correction for each case must be known from the results of previous experiments: this information is recorded in the range tables of modern guns. (*See* 8th column, Table XIV, p. 309.)

Scott's Revolving Sight, the invention of Major L. K. Scott, R.E., automatically obviates the need for making the calculations for deflection on account of difference of level of wheels, or other causes. The steel tangent scale frame, A B (see Fig. 6) can turn 10° sideways to the right and to the left on a pivot D, of which the centre and the tip of the foresight are at equal distances from the axis of the gun. A line of sight through the notch on the pivot allows the gun to be laid with sights at zero. Scott's sights.

A thumb screw at the side H causes the sight frame to revolve slowly laterally, and an attached cross spirit level M indicates when verticality is attained, or the frame may be set at the permanent angle of deflection for drift by the spirit level, instead of vertically if desired. The sight can thus always be set up at the same angle, and the error which would otherwise arise from the difference of level of wheels is eliminated, and no calculations have to be made.

A slide, NQ, can be moved up or down the frame according to the

Fig. 6.



elevation required for any range; it is clamped by a screw, L. Deflection is given by means of a screw, N, which moves a traveller, PO, containing the rough laying notch, P, and the eyehole, O; the latter is used in conjunction with cross wires below the foresight for accurate laying.

Ranges, degrees (and if necessary deflection for drift), are marked on the vertical sides of the frame. The whole is connected with a stem which fits into the ordinary sight slot of a gun, and the frame can fold down out of the way, like the backsight of a rifle, when the gun is travelling.

The foresight is protected by being hooded, thus a fine point can be used and glare is prevented.

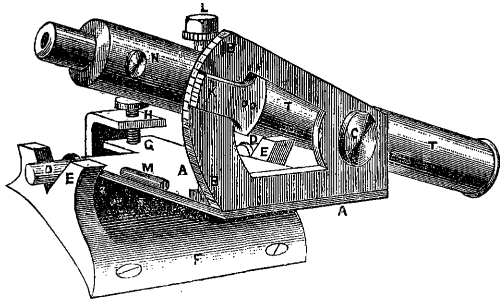
When opening fire, all the guns of a battery are laid on the same object, with the same elevation and deflection; the first gun only is fired, and the point of impact noted from the sights of the gun alongside; if the projectile falls wide of the mark, the correct deflection at this next gun is obtained by turning the screw N till the line of sight is laid in the direction of this point of impact; the gun is then moved till this corrected line of sight is again on the target, and the proper amount of deflection will then have been obtained without calculation.

Major Scott's overbalance **telescopic sight** (*see* Fig. 7) consists of three principal parts—

(1.) The lower limb A
(2.) The vertical arc B
(3.) The telescope T

The telescope T is pivoted at C, and can slide over the vertical arc B; the latter is rigidly screwed to the lower limb A which forms the framework, holding the various parts of the sight together; on this lower limb are trunnions DD whose axis is parallel to that of the telescope, when elevation and deflection are both zero; the

Fig. 7.

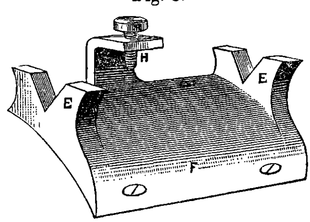


trunnions of the sight must always be parallel to the axis of the gun when they rest in **V** bearings EE provided for them in a bracket F (*see* also Fig. 8), which is rigidly attached to any convenient part of the gun. The term overbalance is employed because the trunnions DD of the lower limb not being vertically below, but to one side of the telescope T, the latter overbalances to one side, and causes the projection G from the trunnions to press up against a screw H which works in the bracket F. This screw levels the sight; the leverage obtained by this means causes the trunnions to be invariably forced to the bottom of the **V** bearings.

The telescope T is revolved in a vertical plane up or down the vertical arc B on its pivot C by means of the tangent screw L according to the elevation required. The vertical arc B is graduated to half-degrees, and the vernier K reads to two minutes; the micrometer head L is also graduated to two minutes. The levelling screw H causes the trunnions of the sight to revolve in the **V**s of the bracket until the bubble in the cross level M, which is rigidly fixed at right angles to the axis of the trunnions DD, indicates that the plane in which the telescope revolves is vertical.

Fig. 8 shows the bracket F which is attached to the gun, and on it

Fig. 8.

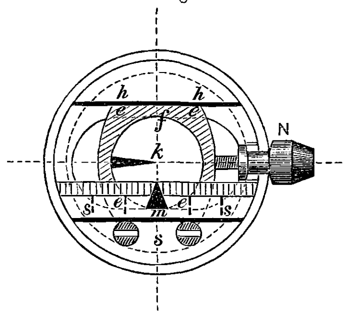


are the **V**s in which the trunnions of the sight rest, and it also has the levelling screw H.

The sight must be taken off the bracket before firing; it is put in again by turning the telescope so that it is nearly vertically above the projection G; when it will be found that the trunnions of the sight can be fitted into their **V** bearings; the telescope is then allowed to fall gently over to the right side, when the projection G will press up against the levelling screw H, and the instrument will have a steady bearing.

In the field of the telescope (*see* Fig. 9) is a deflection scale (*ee*) graduated to three minutes, and also two pointers (k and m). In

Fig. 9.



NOTE.—The contents of inner circle m, e, f, represents all that can be *seen* when looking through the eye-piece of the telescope: the head of the thumbscrew N is outside.

laying the gun, the point k is aimed on the target; deflection is given by turning the screw N (*see* Figs. 7 and 9), when the two pointers will be moved together in the field of the telescope; the vertical one indicating the amount of deflection given.

In opening fire, as with the other revolving sight, the guns are all laid alike, and if the first round (common shell, percussion fuze) falls wide of the mark, the No. 1 of a gun alongside, who is watching the effect of this shot through his telescopic sight, will at once measure the error by turning the deflection screw N of his sight till the end of the pointer k (*see* Fig. 9) is in the direction of the point of impact; he then moves his gun till the pointer k aims at the target; the correct deflection will thus be obtained without calculation.

The deflection scale may also be employed for approximately ascertaining the height of burst of a shell above plane if the range is known.

An objection to the use of such a sight as this is its liability to derangement if accidentally left on its bearings when the gun is fired, or from shaking about whilst travelling; it has no less than four screws for giving movement (*see* Fig. 7), viz., one for levelling H, one for elevating L, one for deflection N, and a fourth, which is not shown, for focussing; but the improvement in accuracy of shooting is so considerable that its employment under certain circumstances seems essential. Many other telescopic sights have also been tried.

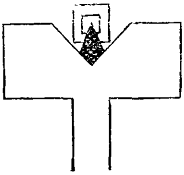
Full and Fine Sights.

In order to aid in obtaining uniform results in laying by different men, it is ordered that all pieces are to be laid with a **full sight**, *i.e.*, the top of the leaf of the foresight is to be level with the shoulder of the notch of the tangent scale; if this matter were not settled, some men would lay with a finer sight than others, thus giving less elevation to the axis of the piece, and the range would be less than expected. A half sight is employed in the Royal Navy.

Fig. 10 represents the appearance of the sights on a gun when a full sight is taken on the centre of a distant target, according to the service plan.

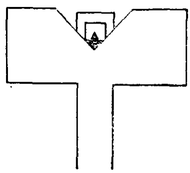
Fig. 11 shows a finer sight taken on the same target; this gives less elevation than is intended.

Fig. 10.



Full sight.

Fig. 11.



Fine sight.

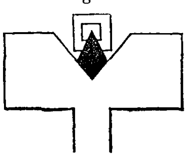
The shaded part represents the portion of the foresight seen, when looking through the notch on the tangent scale in both cases.

A *shallow* notch* is used with siege guns in which accurate shooting is required; with small arms the notch on the backsight is now being given up, and a horizontal bar with a vertical rim on it is used instead, as it has been found that the sides of the notch cause indistinctness at the longer ranges; with the 6-inch B.L. naval gun, a straight horizontal wire is provided at the tangent sight, and there is no notch, the direction being obtained by aligning a projection on the tangent sight just below the wire with a vertical on the foresight.

Distance of the eye from tangent scale.

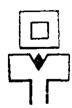
In laying a gun, it is ordered that the *eye should be a little more than a foot in rear of the tangent scale*, and this distance should not be varied from round to round. If the eye is too close, the three objects to be aligned cannot all be focussed by the eye at the same time, and indistinctness leading to inaccuracy is the result. If the eye is much farther to the rear, a different appearance is observed: thus the sights and target with the eye a foot in rear of the tangent scale present the appearance shown in Fig. 12; but from a position a yard or so in rear of the trail, the gun appears to be laid low, as shown in Fig. 13, although nothing has been moved. Hence it is necessary for the eye to be always in about the same position in laying the gun.

Fig. 12.



Eye about a foot in rear.

Fig. 13.



Eye about 4 or 5 yards in rear.

The following are **practical rules**, and are important.

Practical rules.

Get the gun roughly into the correct direction before looking over the sights; support the head in an easy upright position, by holding on to the gun; press down on the cascable of a field gun to ensure the breech resting firmly on the elevating gear; lay quickly so as not to fatigue the eye, and choose a well-defined part of the target on which to align the sights.

It will be noticed, that with the service sights, a good part of the target is hidden, and that in laying the gun some time and attention must be devoted to getting the top of the foresight level with the shoulder of the tangent sight.

Hence various plans have been tried with cross wires; in the 13-pr., and other modern pieces, below the notch on the tangent scale

* The converted 71-cwt. and 80-pr. guns should be laid with a half sight for agreement with the range table made out for the Royal Navy.

is a **small round hole**, and at an equal distance below the top of the sight leaf of the foresight is an aperture containing **cross wires** in a short tube or tunnel to prevent glare. The gun is first laid with the upper arrangement of notch and sight leaf in the ordinary way; but if there is time for laying with greater care this operation only serves the same purpose as the "finder" attached to a large telescope, and on looking through the small hole, the intersection of the wires is seen somewhere near the centre of the target; keeping the wires crossing the centre of the round hole, the gun is moved so that the intersection of the wires just covers the centre of the target; rather greater accuracy is thus obtained than by the former means, and the possibility of variations arising from some taking fine and others full sights is prevented. The small hole also serves to bring the foresight and the distant object to the same focus, on the same principle as a photographer uses a small stop when taking both near and distant objects at the same time. The orthoptic sight, often used for match-shooting with rifles, is on the same principle: it simply consists of an ebonite disc with several small holes in it to suit different lights.

Fig. 14 shows the sights laid on the target by the ordinary plan.

Fig. 15 shows a probable appearance on then looking through the round hole.

And Fig. 16 shows the appearance when the gun is properly laid by the more accurate plan.

Fig. 14.

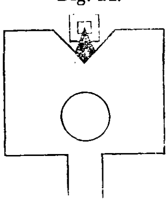


Fig. 15.

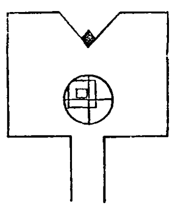
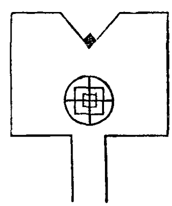


Fig. 16.



NOTE.—The notches and round holes in all these rough sketches are exaggerated for the sake of showing the target and foresight distinctly, the holes are really about the size of a pin's head.

Moving Target.

Laying and **firing at a moving object** is difficult, and requires great skill and judgment. The range is different for each round, and as it continues to alter while the projectile is in flight, the target itself cannot be aimed at unless estimated alteration be made in the laying, but the sights must point to some spot in front of its path, where it is expected it will be when the projectile has traversed the range; but as this spot will generally be ill defined, or perhaps not defined at all, a better way is to give a little more or a little less elevation, according as the target is retiring or advancing; the gun can then be fired when the object covers the sights.

With regard to the changes in the range: suppose, for the sake of simplicity, that the object moves in a straight line at uniform speed, and that you can fire at a uniform rate. The simplest case is that in which the object either approaches or recedes directly from the gun, for here the variations in range (under the above conditions) are uniform, and scarcely any more deflection is necessary than for a fixed target. The sights may be adjusted to a certain range, and laid on some spot (which is judged to be at that range) in front of the target's

path. You must then wait to fire the gun until the object has reached a position at such a distance from this spot, that both it and the projectile will probably get there at the same instant. This distance is due to the time of flight of the projectile and the speed of the target.

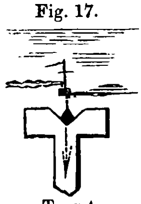
Thus, suppose at 2000 yards range a ship is steaming directly towards the gun at 10 knots (a knot = 2027·55 yards) an hour, and that it is to be fired at by a 64-pr. of 64 cwt. once in $1\frac{1}{2}$ minutes; from the range table we find that the time of flight of the projectile for that range is 5·84 seconds, and in that time the ship will advance 33 yards. The gun must be fired when it is judged that the part of the ship aimed at is 33 yards from the spot the sights point to. A better plan is to lay with 33 yards less elevation, and fire the gun when the part of the ship aimed at crosses the sights.

If the correct range is obtained at first, the next round must be fired with a tangent scale set to 500 yards less range, since in the $1\frac{1}{2}$ minutes taken to load, lay, and fire the gun, the ship will have moved 500 yards. If an error is made in judging the first round it must be added to or subtracted from the next, but this is only a rough guide, as calculations based upon the result of a single round cannot be depended on to give exactness.

Thus, suppose the first shot is 200 yards short, the next should be laid with tangent scale set to 500 − 200 = 300 yards less range than the last.

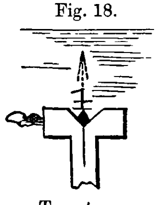
The appearance of the sights and ship just before the word "fire" is shown when the latter is advancing towards the gun in Figs. 17 and 19, and when retiring from it in Figs. 18 and 20, or the gun is fired when the target comes on to the line of the sights. Thus Figs. 17 to 20 represent the appearance of the sights and target when the gun is laid, and while waiting to fire. It will be noticed that in the case of a retiring target a good deal is hidden by the sight. When the ship moves, as is generally the case, across the range (*see* Figs. 19 and 20), the difficulties increase, for here not only the

Fig. 17.



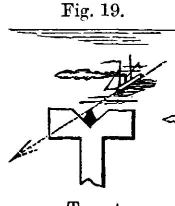
Target advancing direct.

Fig. 18.



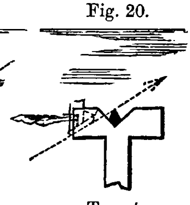
Target retiring direct.

Fig. 19.



Target advancing across.

Fig. 20.

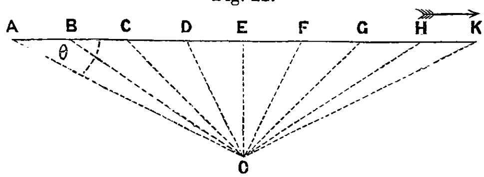


Target retiring across.

elevation changes, but the deflection does so also, and the changes in range are not constant, as is seen by Table A referring to Fig. 21, which represents a ship steaming at the same speed as before across the front of a gun in such a way that when nearest to it the range is 1000 yards. This would be passing directly in front; one passing diagonally is subject to the same rules, except that some of the rounds towards one end of AK (Fig. 21), cannot be fired.

Thus, suppose the ship moves along the line AK from A to K at a perpendicular distance OE = 1000 yards from O, the 64-pr. gun, which fires at the same rate as before, viz., 1 round in $1\frac{1}{2}$ minutes. On AK mark off equal distances AB, BC, CD, &c., each equal to 500 yards—the distance the ship moves between each round.

Fig. 21.



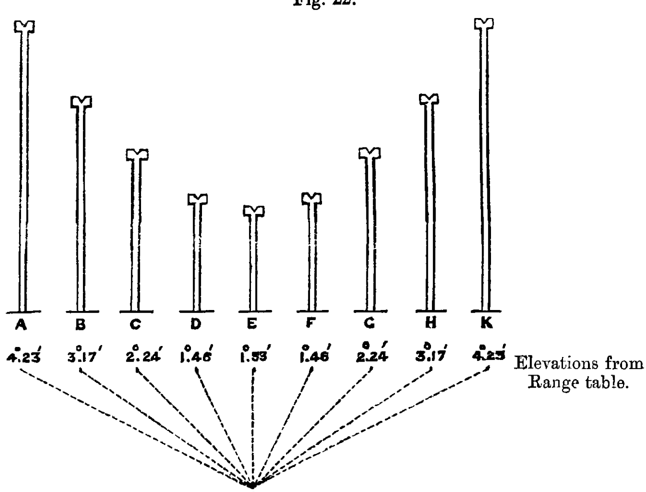
Suppose the first shot is fired when the vessel is at a range of 2236 yards; when the gun is next fired after $1\frac{1}{2}$ minutes the range OB will be 1802 yards, and all the ranges will be as follows:—

TABLE A.

Ranges.	Yards.	Irregular differences in yards. Less.	Irregular differences in yards. More.
OA	2236		
OB	1802	434	
OC	1414	388	
OD	1118	296	
OE	1000	118	
OF	1118		118
OG	1414		296
OH	1802		388
OK	2236		434

The lengths of tangent scale used for each round being as in Fig. 22.

Fig. 22.



Note that the difference between the length of tangent scale for A and B is much greater than between D and E. Irrespective of the fact that A and E are at different distances, it is practically easier to hit the target at about E, where the range varies slowly but the deflection rapidly, than near A, where the range varies rapidly but the deflection slowly; because it is much easier to correct for deflection than for range.

Frequently other guns or works on the flanks will mask many of the rounds, but those which can be fired will follow this plan of decrease and increase of elevation.

The distance in front to aim, or the deflection required in these cases will be as follows:—

TABLE B.

Range in yards.	Times of flight from range table in seconds.	Distance the ship will move in that time in yards.	Lateral movement of ship as seen from the gun in the same time in yards.	Deflection right in minutes.
OA 2236	6·59	37	16·5	26
OB 1802	5·15	29	16·0	32
OC 1414	3·93	22	15·6	41
OD 1118	3·04	17	15·2	49
OE 1000	2·72	15	15·0	54
OF 1118	3·04	17	15·2	49
OG 1414	3·93	22	15·6	41
OH 1802	5·15	29	16·0	32
OK 2236	6·59	37	16·5	26

A system of sighting has been introduced for naval service, in which additional lateral movement can be given to each sight to compensate for the movement of the enemy and for that of the ship; the fore-sight is moved along a scale graduated for the estimated speed of the enemy, and the other is adjusted for the speed of the ship which carries the gun.

When an enemy passes at right angles to the line of fire, the same deflection will approximately suffice for all ranges; at a long range the same deflection covers a wider lateral distance than at a shorter one, but this is approximately counterbalanced by the longer time of flight of the projectile; these differences would exactly balance each other if the velocity of the projectile were uniform, but a fair approximation can be made by taking the mean velocity over some ordinary range (say 1,500 yards) for all cases, and supposing this to be the uniform velocity. When the enemy's vessel moves in a direction oblique to the line of fire this plan must be modified, as we see from Table B that much less deflection must then be given: if we call the angle $OAB = \theta$ in Fig. 21, the proportion between the deflection to be given when the enemy is at A and at E is $\sin \theta : 1$. It will not be possible, however, to determine the angle θ, and less deflection must be given by estimation when the enemy's direction becomes oblique to the line of fire; when $\theta = 0$ no deflection should be given. Similar remarks apply to the movement of the ship which carries the gun.

It is evident that very great accuracy cannot be obtained when firing at a moving object. Practically, however, as ships and masses of men are large objects, it is not very difficult to hit them somewhere if the range is fairly well estimated, though it is of course best to strike them at the spot desired; for instance, one shell well placed may do more harm to a ship than several which strike on unimportant parts. It will often be hardly possible to make any calculations under the circumstances of actual warfare; but a rapid estimation must be made at each range, having due regard to the effect of the last round. These estimates are likely to be near the truth, if the principles involved

are understood. Considerable practice is needed to make good shooting at a moving target.

Very great assistance is obtained from having the ranges of different prominent points on the probable path of the moving target carefully measured beforehand: this can generally be done on land, when acting on the defensive; and coast batteries should know the distance of the buoys, placed near where ships are likely to pass. In firing over the sea from even a slight elevation, a depression range-finding instrument very readily indicates the ranges of a moving object, and the assistance thus given may be very great.

Indirect System of Laying.

Various circumstances will arise when the target cannot be seen directly from the gun, either because the position of the target is masked, or because it is desirable to protect the No. 1 of the gun from the enemy's fire. At the experimental German cupola at Bucharest in April, 1886, partly direct and partly indirect laying, was employed. The cupola was provided with a hole near the top at the *opposite* side to the muzzle of the piece. A sight was then taken through this hole whilst the muzzle of the gun was turned *away* from the enemy in a protected position: the cupola was then revolved through 180 degrees, the elevation was given by quadrant, and the gun was fired.

Reflecting sights.

Guns are laid with **reflecting sights**—

When the size of the port in a casemate is not large enough to use the ordinary sights; and also when

It is desirable to protect the man employed in aligning the sights from the enemy's fire.

Chase sights are employed when a gun is fired through a small port; they are both placed on the thinner part of the gun in front of the trunnions. And as there is not room for a man's head to be placed behind the tangent scale, on account of the coils of the gun, a small mirror on a movable socket is placed just behind the tangent sight notch, and the man who lays the gun stands at some convenient position at the side of the muzzle, and aligns the sights in the mirror exactly as he would do in laying direct; he is well protected from the enemy's fire. The 6·6-inch siege gun has sights on this principle, but they are placed in the usual position, and are only intended to enable the No. 1 to be under cover whilst laying. The 100-ton gun is also provided with a reflecting sight.

With Moncrieff's protected barbette system, in addition to a set of direct sights, in Mark I a mirror at the breech, set at an angle, with a cross engraved on it, is used in conjunction with a notch, which slides in a graduated vertical groove at the trunnions. The trunnion notch is first adjusted to the required elevation, and a man below then looks up at the mirror, and the gun is moved until by reflection the distant object and the trunnion-sight notch coincide with the intersection of the lines engraved on the glass.

Another way, in Mark II is to have one mirror set at an angle attached to a trunnion so that the distant object is reflected on to another mirror, which slides under cover in a graduated slot in the side of the elevator. The gun is laid by sliding the lower mirror to a certain graduation (required by the range) on the slot, and then moving the gun till the distant object is seen reflected in the lower

mirror covering the intersection of cross lines, which are engraved on both the mirrors.

Reflecting sights have the disadvantage of a limited field of view, which may cause delay in finding the object in the mirror, even when the gun is approximately brought into the correct line before using these sights; and reflection, especially when double, causes indistinctness, particularly at long ranges; but in this last case guns are best laid direct, as the No. 1 then runs but little risk, and his aim is better.

Hidden targets.

When firing howitzers or mortars, and sometimes guns, at considerable angles of elevation, the object fired at is often **hidden** by the parapet, or by some other object in front, while a view of it can be obtained by standing on some place close in rear.

When this is the case, the tangent scale and deflection leaf are adjusted as usual, and the No. 1 stands behind on a higher position, directly in rear, from which he can see the target.

He holds a plummet in his hand, and orders the piece to be moved till the vertical line covers the distant object and both the sights (or a chalked line on the exterior of a mortar) parallel to its axis.

It frequently happens, however, that the target cannot be seen from any conveniently accessible point close in rear.

In this case the line between the piece and the target must be found in one of the ways directed in the drill book, or the line may be roughly obtained from a good map; in fact, the direction must be found by a process of surveying. When found the line should be marked out to the rear by a bannerol 300 yards or more behind the piece, clear of smoke; this is called the aiming back point. The distance of 300 yards is necessary in order that errors arising from very small lateral displacements of the piece may be too small to be of any practical effect. A second bannerol should be placed in the same line in case the first is accidentally disturbed.

Cross bar sights.

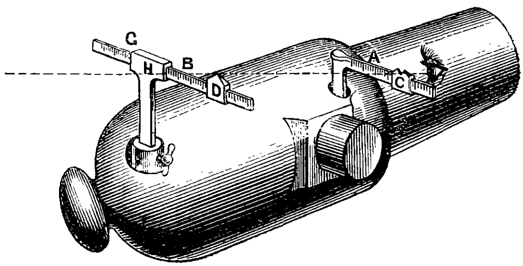
The gun or howitzer is then laid by means of **cross bar sights,** the invention of Major French, C.M.G., R.A. (Fig. 23), with which all the siege ordnance are now provided. These sights have cross bars divided into scales of equal parts parallel to the axis of the trunnions; the whole of the rear bar can slide laterally through the head of the rear sight, and there is a scale of degrees and minutes on the cross bar, which registers the amount of sliding or deflection at G. The other bar is a fixture. Each bar carries a sliding sight leaf C, D, which is reversible, so that either a notch or apex point may be used in each as required. In making use of these sights the No. 1 moves in front of the trunnions, and looks over the sights (on which suitable deflection has been given) towards the aiming back point, and lays the piece in the proper *direction; elevation* must generally be given by quadrant. As the practice goes on correction in deflection and elevation is given according to the effect of fire, which is reported to the battery from some spot where it can be seen. By night a lantern is placed at the foot of the bannerol, and means must be taken to ensure laying on the proper mark, as there will probably be many lights towards the rear.

It may sometimes happen that the target may be visible by day, and it may be required to continue the firing by night or through smoke or fog: this is a simpler case than the former—the piece would be laid direct, and then before it is fired the bannerol which is to form the aiming back point would be placed in position by a man looking backwards over the sights (*i.e.*, from the foresight towards the rear),

directing others to place the bannerol in line with the sights. For succeeding rounds the piece could be laid on the bannerol or the lantern at its foot if the target is hidden.

Fig. 23 shows the general arrangement of French's sights (for details *see* List of Changes in War Material, 1st April, 1882).

Fig. 23.



A and *B* are graduated bars; the latter slides through the head of the rear vertical scale.

C and *D* are sliding sight leaves, which can be clamped in any position, and can be reversed for laying to the front.

At *G*, deflection is marked on bar *B*.

And at *H* is a screw to clamp the cross bar *B*.

Wheel guides fitted on double-decked platforms and the riband on Clerk's platform bring the piece to the same position for each round on firing, and thus the above-mentioned plan can be employed without appreciable error. It is best that one wheel should always be in contact with the same riband on firing, but even if this is not done the resulting error, with a fairly distant back aiming point, is almost inappreciable.*

Auxiliary mark.

It may frequently happen that at a small angular distance from the line of fire to the rear, there may be some conspicuous object, such as a spire or factory chimney, which offers a good mark: after laying the piece as before described, the continuation of the firing may be facilitated by simply sliding the sights on the arms till they are in a line with the **auxiliary mark**; since deflection can be given 8° right and left of the line of sight, it is probable that some such mark can often be selected; a note is taken of the reading of each of the sliding leaves on their cross-bars and they are similarly adjusted for each round when the piece is laid on the easily seen auxiliary mark. Elevation is given by quadrant, but it can be more quickly done if a good horizontal mark is available, in which case, however, it is not easy to give correction in elevation should it be required from settlement of the platform or other

* Before wheel guides were used, hanging scales, and afterwards French's sights, were employed to lay the piece each time in a position parallel to itself at only a few inches distance; when French's sights were used for this purpose two plummets or one plummet and a board with a vertical line on it were placed at a few yards distance in rear of the piece in line with the target, the front sliding leaf was slid along till it came into the same line, and the rear slide was adjusted to the same reading, and then the trail was moved until it also came into the same line; this plan is not so convenient as that which is now employed, and the plummets, &c., being just in rear of the piece, were liable to be struck by the enemy's fire, or to be otherwise disturbed.

causes. An auxiliary mark *in front* may often prove useful by day; but unless it has a light it will be of no use after dark.

Method of Laying Garrison Guns.

With garrison artillery on traversing platforms the guns can always easily be brought to the same place for each round, consequently with them another plan is employed for firing at an unseen target.

The elevating arcs or index plates of the heavier R.M.L. guns are graduated to degrees and minutes, and these, in connection with readers on the carriages, always record the quadrant elevation of each gun. A pointer hangs from the platform close to the ground just over a graduated brass arc concentric with the racers, and this tells the lateral angle of training from the fixed line of direction, which is due north and south.

With coast batteries.

An observer posted on some well-concealed station outside the fort, which must be 60 feet or more above the level of the sea, points a **depression range** and **position finder** on the vessel to be fired at, and keeps the intersection of the cross wires in the field of the telescope, covering the water line by means of suitable following screws attached to the instrument.

At the battery and in electric communication with the range and position finder is an indicating instrument, which automatically records the range and the direction of the ship from each group of guns.

When firing at a moving object, its probable position after a certain time can be predicted: the guns are then laid on that spot, and they can be fired electrically just before the ship arrives there.

The officers and men actually manning the guns may perhaps not see the enemy's ships after the first round on account of the smoke, and they may be ignorant of their commander's plan of fighting, but this system allows the commanding officer to make use of the concentrated fire of his guns on the important parts of the enemy's attack, and he can also observe the results of his fire, as he will be clear of the smoke of his own batteries.

This system confers great advantage on the defence, as ranges and positions are readily found and communicated to the commanders of guns. In the Naval Service, on the other hand, a good range and position finder is still a desideratum. Similar instruments have been most usefully applied to submarine mining, a most important addition to the defence of maritime positions. It is important that the range and position finders should be well cared for and cleaned by those who really understand them, or they may get out of adjustment.

The whole of this ingenious plan is due to Major Watkin, R.A. Arrangements have been made with a similar object on Continenta coast forts.

CHAPTER VIII.—STRESSES ON GUN-CARRIAGES, AND RECOIL.

Stresses in three planes.

To investigate the **pressures** acting on a gun-carriage on firing the piece, we will consider them **in three planes** at right angles to each other. The most convenient to take are—

(1.) A vertical one through the axis of the gun;
(2.) A vertical one at right angles to the first; and
(3.) A horizontal plane.

The stresses in the first of these planes are the most important.

(1.) *Pressures in a Vertical Plane containing the Axis of the Gun.*

Pressures in a vertical plane containing the axis of the gun.

Consider a gun and carriage (either field, siege, or garrison) as one system. On firing, a resultant pressure acts along the axis of the piece and tends to produce two motions.

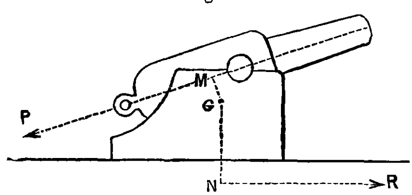
(i.) Rotation round the centre of gravity of the system—a couple.
(ii.) Motion of translation of the centre of gravity of the system.

We will consider these tendencies to motion separately.

Couple.

(i.) In all our land-service carriages, the direction of the force of the powder, acting along the axis of the piece, is above the centre of

Fig. 1.



G is centre of gravity of gun and carriage.
P is powder pressure.
R is resistance of buffers, &c.

gravity of the system: hence a moment $P.GM$ is produced (Fig. 1); and this is further increased by the friction of the bottom of the carriage against the ground, or by the compressor or buffer, which exerts its resistance in an opposite direction to P on the *lower* side of the centre of gravity, and so adds* a couple (R.GN) tending to

---

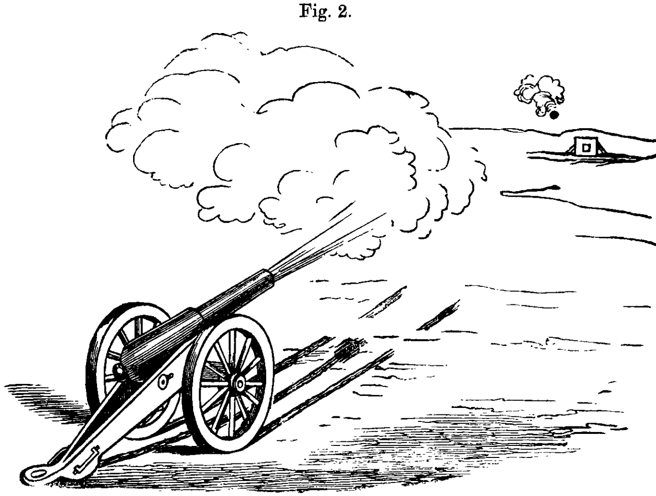
* A simple illustration may make this clear. If a man receives a blow in the face, he may stagger back and nearly fall; but if at the same time his feet are tripped up he is more likely to come down. The blow in the face corresponds to the force of the powder acting along the axis of the gun, and the tripping up of the man's feet is similar to the action of the compressor below the carriage; hence it follows that the tendency to rotation is greater when the resistance of the buffer is increased. When the new 12-pr. B.L. gun is fired from a carriage with wheels skidded, the jumping up of the fore part is greatly increased.

produce rotation in the same direction as the other. In Vavasseur's mountings for naval service the buffers which check recoil are just behind the trunnions, and consequently the couple is almost done away with.

Practically, the tendency to rotation produced by these couples is soon checked; but the fore part of a carriage jumps, and the rear is pressed violently downwards. In field and siege carriages, the trail strikes the ground with great violence, causing a destructive cross-breaking strain. With an overbank carriage the moment is greatly increased, because the line of action of the powder gas is further from the centre of gravity of the system than with an ordinary carriage; the result being that the trail delivers a severe blow on the platform when the piece is fired, and trail plates are used to protect it. This stress on the trail is met by constructing it deep in section and placing the bulk of the metal of which it is made at its upper and lower surface, distant from its neutral line, in the girder form. A short trail is advantageous, as it digs into the ground, and so helps to stop the recoil, which is now becoming very considerable with the modern high-velocity field pieces; but it is not so convenient in limbering up as the longer one, and it is not quite so easy to traverse.

The jumping of the front of a field carriage is easily noted if the ground is tolerably soft. After the first round, the ruts made by the wheels on recoil are seen to be not continuous (Fig. 2); but there

Fig. 2.



are intervals of about 12 inches or more between the places where the wheels rested on firing, and where they touch the ground again on recoil. The wheels of the 13-pr. rise some two or three inches above the ground. A long trail and a heavy weight lessen the jump.

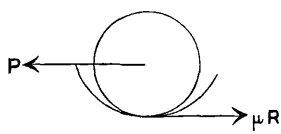
The same result would occur with garrison gun-carriages; but with them the front must not be allowed to rise, or the piston-rods of the hydraulic buffers would be bent: hence, clips are attached to the front and rear of the carriages; these slide under the flanges of the platforms, and prevent the tendency to rise; but clips only transfer the movement and cause the front of the platform to jump. Some platforms have been known to rise on firing, and come down again with such

violence, that their racers were greatly indented, and subsequent free movement in traversing was prevented. This difficulty is now met, either by causing great weight to rest on the front racer—the two fore trucks being at a long distance from the two rear ones, and the carriage placed just over the front racer on firing; or by the means of a vertical yoke, as used in the 45-ton gun carriage, where the tendency to rise is resisted by two rollers gearing in a circular groove in the roof of the casemate, and it is also diminished by the buffers pulling directly on the yoke. The adoption of wide steel racers and broad trucks has also lessened the chances of injury from this cause. Clips holding the platform down to the racer have been tried experimentally, but they were not then successful: the foundations were gradually torn up as the firing went on, but clip racers are used for heavy guns on board ship, care being taken to strengthen the structure to which they are attached.

Play in the trunnion holes.

Since the trunnions work in the trunnion holes, there must be a little play, which for the sake of clearness is exaggerated in Fig. 3. Although in the service ordnance the axes of the piece and trunnions intersect, and the force of the powder acts through the centre of gravity, a couple is nevertheless produced by the friction μR of the trunnion against its bearing (*see* Fig. 3); and this causes the breech to give a blow to the elevating gear.

Fig. 3



With S.B. pieces the axis of the trunnions was below that of the gun; it will readily be seen that the position of the trunnions below the axis of the gun must have increased the couple; on the other hand the couple might possibly be reduced to less than its present dimensions, by placing the axis of the gun a little below that of the trunnions.

Jump.

These couples causing the blow on the elevating gear give rise to the **jump** of the gun, and thus more elevation is given by many minutes than is intended. This would not much matter if the amount were always the same; but when field-guns are fired on soft ground, the trails plough deeply, and the jump is not the same as when a gun is fired resting on a hard surface, such as asphalte; consequently variations in range result from this cause.

The breech of the gun tending to revolve in common with the rest of the system, and finding its motion violently checked by the carriage or trail, which is stopped by striking the platform, inflicts a *severe blow on the elevating arrangement*, which is the less able to bear it, as it must of necessity be a movable gear with a certain amount of play for freedom of working; it therefore receives a severe shaking.

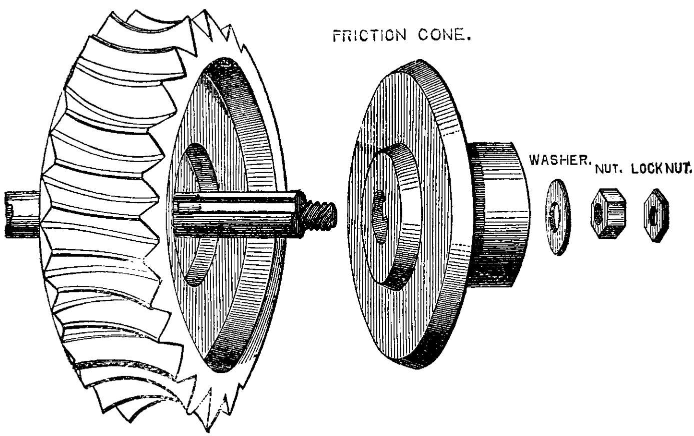
Blow on elevating gear on firing.

This violent blow tends to break the teeth of the elevating gear, which is now very generally made on the worm and worm wheel plan. This has the advantage of being very compact, and not requiring to be clamped, as the worm cannot be moved by the tendency of the gun to move the worm wheel (from reasons connected with the angle of friction of the surfaces of the teeth in contact). This, however, leads

to the disadvantage that something is likely to break when the blow is considerable.

The friction cone (Fig. 4) remedies this defect by enabling the gun to twist the cone round inside the worm wheel, when it gives a severe blow on firing, as the two are only caused to revolve together by friction, which is, however, quite sufficient to keep the breech of the gun from running down in laying, and the giving way of the gear on firing is not found to impair the accuracy of shooting: the amount of friction is regulated by a nut. This plan cannot be applied in the very heaviest guns as so much power is lost by friction; spur gearing is used for them in spite of the disadvantage of clamping.

Fig. 4.



NOTE.—The worm which works the worm wheel; also the pinion fixed on the shaft, and the elevating arc on the gun worked by the pinion, are not shown for the sake of simplicity. The elevating arc, pinion, spindle, and friction cone move when the breech runs down on firing. The worm wheel cannot be moved by this means, as the worm prevents it.

The capstan-head plan has not only the disadvantage of requiring the use of a separate frictional clamp, when the required elevation has been given, to prevent the breech from running down, but it is also not continuous, as the lever which works it must be shifted for a fresh purchase.

(ii.) The tendency to *motion of the centre of gravity of the system*, produced by the act of firing can be resolved,—

Motion of translation of centre of gravity of the system.

(*a*) vertically, and
(*b*) in direction of the ground or platform.

Vertical component.

The first of these tendencies to motion causes a stress on the axletree and eventually a downward thrust on the wheels of a field-carriage. Since, however, the fore part of the gun-carriage jumps up, there cannot be a downward thrust at the moment of firing on the axletree. Soon afterwards, however, it falls down again with considerable violence, and sufficient strength must be given to meet

the stress thus occasioned. A downward blow is also produced on the sides of a garrison carriage and on the platform, where the girder form is used for strength, and the "fish-bellied" in the larger natures, where the trucks are far apart to lessen the jump.

Great improvements have been made of late years in the strength of field axletrees, rendered necessary by the increased stresses. The wooden bed having been given up, the box-girder took its place, and afterwards the "bow string" of steel for the 13-pr.; in the former the bed is the top of a strong girder of deep section, the bottom of which is formed by the axletree: the latter is only pierced with holes for attachment to the bed at its thick ends. This arrangement prevents the thinner parts of the metal from being nipped or pierced when strained,—an important matter, especially in dealing with steel. It is also easily taken off the bed when required. The axletree for the new 12-pr. is a solid bar of steel without a bed.

Component parallel to the ground.

The component parallel to the ground is always considerable and causes the motion of **recoil**, which is checked by various means, nearly all of which, as before stated, increase the moment tending to produce rotation.

This pressure produces a *horizontal strain* on the axletree of a field carriage, as the weight of the wheels causes a resistance to motion which increases if they are skidded, or if they have sunk deeply into shingle, sand, or mud. The tensile stays transfer the pull from near the middle (where the axletree is weaker) to parts nearer to its points of support, where it is stronger.

It appears now to be a necessity to construct field carriages so that the horizontal strain is brought to bear *gradually* on the axletree, consequently in the new 12-pr. carriage the axletree has about 0·2 inch play in the recess in the brackets of the carriage in which it fits. The axletree is connected with the trail by stays, each formed in two parts connected by strong spiral springs: when the gun is fired the springs in the stays are compressed, and thus a gradual pull is communicated to the axletree.

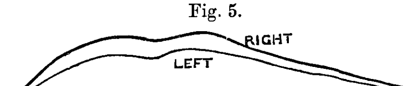
The elastic Russian Englehardt carriage is on the same principle.

(2.) *Pressures in a Vertical Plane at Right Angles to the Axis of the Gun.*

Pressures in a plane at right angles to the axis of the gun.

With S.B. guns no pressures act in a plane at right angles to the axis of the piece except accidental ones resulting from any want of symmetry; but with rifled guns a marked difference has been noted in the strains on the capsquares; the right, showing evident signs of sustaining far more injury than the left, after continued firing. This results from a tendency causing the gun to revolve in an opposite direction to the projectile; for if we imagine the latter to be prevented from revolving as it passes down the bore, the gun must be turned in the *opposite* direction if it is free to revolve. The superior weight of the gun and carriage and the position of the wheels prevent much rotation in this plane; but an upward blow on the right capsquare is inflicted, and this always lifts the right wheel of a field gun-carriage a little higher than the left on recoil; this is well shown by the annexed diagram (Fig. 5), which is reduced from tracings made by

Fig. 5.



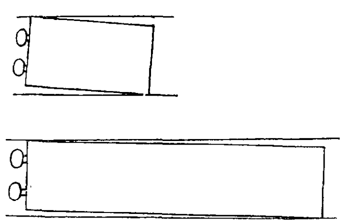
The two tracings superposed for comparison.

needles attached to the axletree arms of a 13-pr. on vertical surfaces of grease. The mean of many observations always gives the same result, that the right wheel rises higher than the left by about half an inch.

(3.) *Pressures in a Horizontal Plane.*

Let us now consider what pressures act in a plane, which is horizontal, or nearly so; such as the surface of the ground or platform. If everything is perfectly regular and symmetrical, there will be no forces acting in this plane, but practically this is not the case; for instance, a field-gun, after it has recoiled, seldom or never points in the same direction as when laid, because a certain lateral twist has taken place depending on one wheel meeting with greater inequalities than the other in recoil. If one wheel is skidded and the other is accidentally left loose, the muzzle slews violently round on recoil to the side of the unskidded wheel; but this does not affect the accuracy, as this side movement does not begin till the projectile is out of the bore. With a central buffer on a garrison carriage, this twist could hardly happen; but when two buffers are used, it is usual to connect them together, so that the fluid pressures in each may be the same; if they were not, a lateral twist would ensue, which might cause a jam or scoring of the side of the platform, or a seizure of the metal. This is the more likely to take place with short carriages, in which the guides have a certain play between the sides of the platform, than in longer ones having the same play, for the same reason that a short drawer in a table is far more likely to stick fast than a long one (Fig. 6) from any accidental twist, occasioned by pulling one handle with more violence than the other. **Pressures in a horizontal plane.**

Fig. 6.



This difficulty has been practically experienced with some short carriages on iron platforms, as they jammed in running up by accidentally turning laterally between the sides of the platform. The bearing surface of the flange of the girder of the platform and the guide of the carriage being small, seizure has occasionally taken place.

Recoil.

The majority of the strains imposed on a carriage on firing are met by strength of material, and the tendency to motion is generally stopped: some of the elevating gears, however, are constructed in such a way that the breech will run down when a very heavy strain is imposed. But the chief motion is **recoil**, and this nearly always takes place; but generally much controlled. Krupp has, however, made two classes of carriages which have no recoil, one muzzle pivot- Recoil.

ing with a ball and socket joint in an armour plate, and the other a swivel gun; in both of them the destructive blow is lessened by increasing the weight of the piece, which is made very long. Machine-guns have *no* recoil, and quick firing guns hardly any, and heavier ones on Vavasseur mountings only a very little; in each of them rapidity of fire is a first essential. The Maxim gun utilizes recoil in a most ingenious manner to work the mechanism; consequently, when once started, this machine-gun will continue firing automatically as long as desired, or until the supply of cartridges is exhausted.

No very satisfactory relation between the velocity of the projectile and that of the gun and carriage on recoil has been determined; but since action and reaction are equal, *at the instant* the projectile leaves the bore the momentum of the gun and carriage and half the charge must equal that of the projectile and the other half of the powder charge, or

$$(W + \tfrac{1}{2}w_1)U = (w + \tfrac{1}{2}w_1)V \quad \text{(i)}$$

but the formula generally employed is

$$WU = (w + Cw_1)V, \quad \text{(ii)}$$

in both of which

W = weight of gun and carriage in pounds,
w = weight of projectile in pounds,
w_1 = weight of powder charge,
U = velocity of gun and carriage in f.s.,
V = velocity of projectile in f.s.,
C = a constant determined by experiment.

With slower burning powder-charges and even only the same velocities of projectile, the recoil is so considerably increased, owing to pressure of the issuing powder-gas after the projectile has left the bore, that buffers have had to be readjusted to meet the increased pressure.

Formula (ii) gives fairly good practical results; but it must only be taken for what it is worth. No account is taken of several facts, or rather they are intended to be accounted for by the use of the constant C.

1. The velocity of the powder-gas is not the same as that of the projectile.
2. Leakage by windage.

It is found practically, that the gun and carriage have moved only a few inches when the projectile has just left the muzzle; and the maximum velocity of recoil is not attained till a short time afterwards.

At a French experiment it was found that a 24-cm. gun had only recoiled 1·181 inches during the time it had taken for the projectile (61·7 lbs.) to travel all the way down the bore: the velocity of recoil was then 12·86 f.s., and it attained its maximum of 17 f.s. at a period 0·048 second later.*

Knowing the muzzle velocity of the projectile and the various weights referred to, we can approximately find the velocity of recoil, and hence the energy of recoil, by aid of the above formula.

The *energy* of recoil, $\frac{WU^2}{2g}$, of the gun and carriage is always *less* than $\frac{wV^2}{2g}$, the energy of the projectile; while the *momentum* WU of

* *Vide* Trans. U.S. Ord. Notes, No. 313.

the gun and carriage is, by both formulæ (i) and (ii) taken as *more* than that of the projectile wV.

If two pieces of ordnance of different weights fire equal projectiles with equal charges and velocities, the *momentum* of each gun and carriage will be equal, or

$$WU = \text{a constant},$$

$$\therefore\ U \propto \frac{1}{W},$$

$$\text{and } \therefore \text{ energy of recoil, } \frac{WU^2}{2g} \propto \frac{W}{W^2},$$

$$\propto \frac{1}{W};$$

or the energy of recoil diminishes as the weight of the gun and carriage is increased; and this is found to be the case in practice, since a heavy gun exerts a less destructive effect than a light one, when firing equal projectiles with equal velocities.

Find the velocity of recoil of the new 12-pr. B.L. and the old (screw) 12-pr. R.B.L. from the following data:— Example 1.

Ordnance.	Weight of gun and carriage (tons).	Weight of projectile (lbs.).	Weight of charge (lbs.).	M.V. (f.s.).
New 12-pr. B.L.	0·9	12·5	4·0	1717
Old (screw) 12-pr. R.B.L. .	1·046	11·25	1·5	1239

Assume the coefficient C as unity.

For new 12-*pr. B.L.*	*For old (screw)* 12-*pr. R.B.L.*
$0{\cdot}9 \times 2240V = (12{\cdot}5 + 4) \times 1717$	$1{\cdot}046 \times 2240V = (11{\cdot}25 + 1{\cdot}5) \times 1239$
whence $V = 14{\cdot}05$ f.s.	whence $V = 6{\cdot}742$ f.s.

Compare the work done on the carriages of the same pieces. Example 2.

For new 12-*pr. B.L.*	*For old (screw)* 12-*pr. R.B.L.*
$\text{Work} = \frac{0{\cdot}9(14{\cdot}05)^2}{2g}$	$\text{Work} = \frac{1{\cdot}046 \times (6{\cdot}74)^2}{2g}$
$= 2{\cdot}76$ ft.-tons	$= 0{\cdot}74$ ft.-tons

Or the work done on the newer carriage is $\frac{2{\cdot}76}{0{\cdot}74}$ times $=$ 3·7 times as much as on the older one.

If the new carriages were simply so much stronger than the others by sheer strength of material, the weight would be too much increased: hence the need of special arrangements to prevent fracture, and the adoption of the carriage with the elastic stays already mentioned.

Small arms. The amount of recoil of a small-arm is practically limited by the amount of blow on the shoulder which an ordinary man can sustain

repeatedly without lessening his accuracy of aim; unless any arrangement be devised to cause the blow to come with less suddenness on the man's shoulder, on the principle of the elastic field carriage, but it is hardly likely that anything of the kind will be necessary. As this blow depends on the momentum of the bullet and powder-charge, wV, it follows that if very high velocity is required, the weight of the bullet must be reduced; since wV being constant, w must diminish if V is to be increased; but if the weight of the bullet is lessened its diameter must be made less, or otherwise the retardation of the air upon it will be increased. The new rifle for the Service carries out this principle, as it fires a lighter bullet with a higher velocity than the Martini; but as the bore is small, the sectional density is good (*see* p. 160), and thus the resistance of the air is not able to exercise a great retardation, the velocity is well maintained, giving rise to a flat trajectory which aids greatly in accuracy, as mistakes in judging distances become of less importance. The practical limit preventing a still further reduction of bore seems to be that difficulties are apt to arise with lubrication, and fouling occurs more readily than with a larger bore.

Methods of Controlling Recoil.

The work done on recoil can be decreased by diminishing either the weight of the projectile, or its velocity, or both: but it is essential that these should (generally) be as great as possible, and also it is an advantage to have a light piece of ordnance. Hence the force of recoil is considerable, and recourse must be had to mechanical means to control it.

This may be done by—

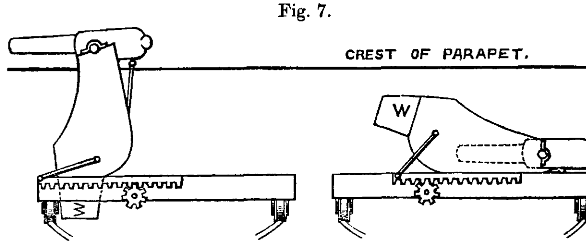
1. Raising a weight.
2. The friction of solids.
3. The resistance of liquids.
4. The resistance of air.

More than one of these means may be employed at once; in fact most of the serviee garrison carriages each employ all the first three plans. The gun and carriage rise by recoiling up the slope of the platform; resistance is offered by the oil in the hydraulic buffer, and there is friction between the bottom of the carriage and the top of the platform: this last plan will, however, probably not be adopted in future land service carriages, as it is so variable; consequently the newer carriages are on live rollers.

Raising a weight.

(1.) All garrison and siege carriages recoil up platforms sloping to the rear; this materially serves to check the recoil, and it also places

Fig. 7.



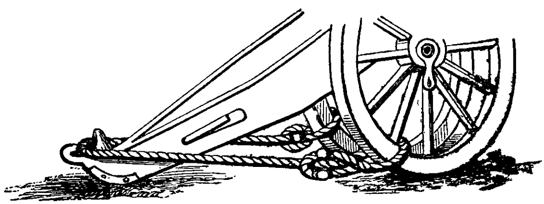
Firing position. Counter-weight down. Loading position. Counter-weight raised.

the gun and carriage so that they can easily be run out again into the firing position.

In Moncrieff garrison carriages (Fig. 7), a counter-weight is raised. This not only possesses the advantage of storing up the force of recoil, but the gun disappears from the enemy's view after firing.

(2.) If no special means are taken to check the recoil of a field-gun the friction of the ground will bring it at last to rest; but the distance of recoil is apt to be very great indeed with the new high-velocity guns, and various means have been tried to increase the frictional resistance by skidding the wheels, either with an ordinary brake, a nave brake, check ropes (Fig. 8), or drag shoes. The labour of running up after continued firing has always been considerable, as Major Mercer's battery found at the Battle of Waterloo, when the exhausted gunners not having strength to continue to run up their guns, fired them at last from positions dangerously near the limbers. The friction of solids.

Fig. 8.

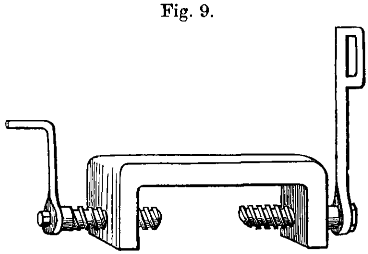


Check ropes for 2·5-in. Mountain Gun Carriages.

Since the energies of the projectiles of the field-guns which are now being introduced into the Service are much greater than those of the old smooth-bores, it is evident that a good means of controlling the recoil of field-guns is a great desideratum; the band-nave brake fairly answers this purpose, but seizure of the metal is apt to take place.

The frictional plan of controlling recoil is developed in the Elswick and other compressors, which consist of iron or steel plates on the carriage pressing against and between iron bars on the platform (Fig. 9).

Fig. 9.



Compressor for Howitzers on beds. Plates and bars not shown.

This method is much more used in the naval than in the land service, as it serves very conveniently to hold a carriage when the ship is rolling or pitching. But it possesses the serious disadvantage of being uncertain in its action after long disuse, when the plates may

become coated with rust, and several accidents have happened, when, after firing once successfully, the gun has run violently against the stops at the second round, while the compressing levers were adjusted as before; the reason being that the rust was rubbed off at the first round, and the plates and bars consequently became too loose, and the amount of friction was not sufficient. This has led to the following order:—

"After *each* round, for the first three or four rounds, and until any rust, scale, or dirt on the bars becomes worn off, the adjustment of the compressor will be tested in the manner specified." (Proceedings D. of A., 30th September, 1880.)

The following extract from an account by Lieut. Madan, R.N., of the late Chili and Peru war, in the Journal R.U.S.I., may be of interest on this subject. In November, 1880, the "Huascar" was bombarding Callao when "the compressor (ordinary Armstrong plates and bars) on one of the gun-carriages was not sufficiently set up, and the gun recoiled with great violence when fired. The woodwork and buffers in rear were completely wrecked, and about a fortnight elapsed before the gun could be fired again."

Wet compressor bars are a source of damage, as the friction is less than may be expected.

The resistance of liquids.

(3.) The resistance of liquids flowing through small apertures increases very rapidly with the velocity; and this principle is widely employed in various hydraulic buffers.

The ordinary hydraulic buffer possesses decided advantages for land service over the Elswick compressor, as the adjustment is so simple—to have the proper quantity of oil in it; but several improvements have been made upon those which were first constructed.

Tension or pull-out buffers are better than push in ones, as the effective length of the buffer is greater, and consequently the mean pressure is less than in the other plan. Most of the newer patterns of hydraulic buffers have an arrangement by which the size of the apertures can be adjusted to suit varying charges. This can be done by having projecting ribs inside the cylinder, along which slides a disc with deeper groves in it; this disc is attached to the piston-head. By turning the piston-rod round, the clearance between the rib and the groove is altered, and so the size of the aperture is adjusted.

Other buffers* have grooves inside the cylinder of varying depth, so that the aperture becomes smaller as the rod advances and the velocity falls off; hence the resistance is kept nearly uniform as the velocity decreases, and the recoil can be stopped in a very short distance.

Krupp's buffers, as well as some in our Naval service, are entirely filled, and the apertures are valves, kept shut by a spring; consequently they will hold a gun-carriage in a sea-way with the valves closed. On firing the gun, the pressures are sufficient to open the valves, and the recoil is then checked after acting for a short distance.

The resistance of air.

(4.) This plan is used in the Moncrieff hydro-pneumatic carriage (Fig. 10) for siege purposes. The force of recoil still further compresses air in a cylinder, which is always kept at a certain compression. The gun disappears on firing from the enemy's view, and the elastic force of the compressed air serves to raise the gun again into the firing position when required. A little water is used at the bottom of the cylinder, where the valves are situated, as it is much

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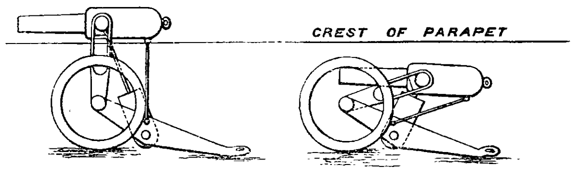
* For details of these and other buffers, *see* R. C. D. Treatise, and also Manual of Gunnery for H.M. Fleet, 1886.

easier to make a valve water-tight than air-tight. Powerful hold-fasts are required; one to the front and another vertical to hold down the forepart of the carriage. A similar plan, but with a shorter piston rod, has been employed for a few guns on garrison carriages.

Fig. 10.



Firing position.
Air in cylinder only slightly compressed.

Loading position.
Air in cylinder fully compressed.

CHAPTER IX.—ATMOSPHERIC EFFECTS ON FIRING GUNS.

Blast.

THE long **flash from the muzzle** of a piece of ordnance, and the rapid rush of the suddenly produced powder gas, cause a powerful blast, which acts destructively on objects close at hand lying in its path: boats, deck planking, and shutters have been shattered or torn up on board ship; but in forts little or no damage is likely to happen, since the exterior, which is exposed to the blast, is always of strong construction, as it is officially directed that care is to be taken that the masonry under the muzzles of the guns is very firmly secured. (*Vide* Permanent Fortification, by Captain Lewis, R.E.)

Effect on embrasures, &c.

In siege batteries the parapets become damaged from the effects of their own fire; but this, to a certain extent, is less the case now-a-days than formerly, as embrasures have only been retained for flank fire. It has been found difficult to preserve the shape of the shallow troughs through which siege guns are now fired, even when iron plates are used: these have been torn up from the baulks to which they were secured at Lydd, and curled up *towards* the gun, showing that the destructive effect was due to the indraft. It has also been noticed that doors in permanent works have burst open outwards, *i.e.*, *towards* the piece when fired. When the blast is confined in a narrow embrasure the sides soon crumble away after repeated firing, and thus constant repair is entailed.

Not only, however, is the embrasure gradually destroyed, but frequently the revetment of its sides (if it is not a permanent work) catches fire or smoulders, rendering great care necessary in taking out fresh cartridges in loading. Revetments covered with raw hide, which is uninflammable, have been tried to prevent this tendency to hold fire, and they answer fairly well; but in any case care is necessary from sparks which may remain in the neighbourhood after firing. Mantlets of interwoven rope are soaked in chloride of calcium to prevent ignition from the blast of the gun.

Partly burnt grains of powder.

One of the effects of the explosion of a very heavy gun is sometimes the projection of a considerable number of burning grains of powder, which are extinguished by the cooling influence of their movement through the air, or by the rapid expansion of the produced gas.

Fig. 1.

Sketches of grains picked up in front of 100-ton gun fired at Woolwich 26th October, 1881. Real size: the remains of large prisms.

They can occasionally be picked up after firing, about 100 yards or so in front of the gun, curiously pitted over with small cup-shaped indentations (Fig. 1). When a gun is chambered, fewer of these unburnt grains are observed, and the flash is noticed to be longer than with an unchambered piece; the quality of the powder also influences the existence of these grains. A somewhat similar effect has been noticed in meteorites; Dr. Fletcher (" Guide to Meteorites," S. Kensington Museum) writes:—" The surface of a meteorite is generally covered with pittings, which have been compared to thumb marks. It is remarkable that pittings bearing a close resemblance to those of meteorites have been observed on the large partially burned grains of gunpowder which have been picked up near the muzzle after the firing of the 35- and 80-ton guns at Woolwich. The pitting of the gunpowder grains is attributed to unequal combustion, but that of meteorites seems to be due not so much to inequality of combustibility as to that of conductivity and fusibility of the matter on the surface."

The existence of the blast has led to an order in the Royal Navy that Her Majesty's ships are not to fire salutes towards each other when closer than 150 yards: beyond that distance, it is thought there is no danger. Ships saluting near each other.

The screen with copper wires nearest the gun, used with the instrument for finding the velocities of projectiles in flight, must be put at a certain distance from the muzzle, so that the wire may not be damaged by the blast, but only cut by the actual passage of the shell. Velocity screen nearest the gun.

A mortar or short howitzer with a large charge is probably the most trying piece to fire with an ordinary lanyard, as the gunner is placed near the muzzle, and, consequently, close to the blast: the Germans allow the use of cotton wool in the ear under these circumstances; a simple precaution in any case,—whether exposed to the blast or to the ordinary sound vibrations, is to look directly towards the point of explosion, for the same reason that a boat better resists the force of a wave at sea if it is placed end on to it, than if it offers its broadside, as then the vibrations of the atmosphere flow past the drum of the ear, and do not impinge directly upon it, and there is less fear of deafness or other injury. Effects on the drum of the ear.

The explosion sets up a *vibration of the atmosphere*, giving rise to **sound.** The effects of this with very heavy guns are often destructive to glass in all directions, and windows should be opened to allow the vibrations to pass without breaking the glass: swing windows are best, as they open the widest. When large accidental explosions have taken place, this shattering effect has been observed at long distances . as, for instance, when the Erith powder magazine blew up in 1864 a large window was broken in Charlton House, at a distance of 5 miles. A curious fact was observed by Mr. Symons, F.R.S., after the explosion on a barge in the Regent's Park Canal a few years ago ;—windows close at hand were noticed with the glass broken by falling away from the explosion; at a little greater distance came a neutral zone, where little or no harm was done; and further off the windows were again broken, but there the glass fell in a contrary direction, *i.e.*, towards the explosion. When the 80-ton guns were first fired from the turret at the end of Dover pier, it was considered necessary to wait until the wind blew off the shore and the inhabitants were warned to open their windows. It was thought that some of the cliff might, perhaps, be shaken down; but when the guns were fired in a seaward Sound.

direction the only damage was the breaking of some glass on a lighthouse quite close to the turret.

As the vibrations of sound travel at some 1130 feet per second, and as a gun can be heard at many miles,* a good approximation to the distance of a gun fired a long way off can be obtained by noting the interval which elapses between seeing the flash and hearing the sound of firing, as the light practically travels instantaneously. If the distance is known, the muzzle-velocity of an enemy's gun may be calculated if the time of the arrival of the sound of firing, and of the projectile are carefully noted.

The employment of firing guns as signals is well known. The firing at the beginning of an action gives very evident information to all troops within many miles; and the Germans took advantage of this in their late war with the French by always marching to the sound of the guns, and thus they rapidly brought up large bodies of troops to important points.

Attentive listening will generally tell whether musketry, heavy or light guns are being fired, and this may, under certain circumstances, be valuable information, as it serves to show what troops are engaged. On the other hand, pickets are often forbidden to fire to avoid giving information to the enemy, or to prevent unnecessary alarms among their own troops.

It will be noticed that the velocity of sound is greater than that of mortars, rifled howitzers, and some of the early rifled guns, but less than the mean velocity of most modern rifled pieces at ordinary ranges.

Colonel Maitland, R.A., wrote as follows in his "Report on War Material of the Turkish Army, 1878:"—

"In firing at works either to harass the defenders or to impede repair, the Russians employ shrapnel with time fuzes, and common shell with percussion fuzes. I have been much struck with the smallness of the effect produced by either, and consider that it is due in a great measure to the low velocities of the projectiles.

"Put a common case. A breastwork has been thrown up hurriedly in the night, or during fog; it is desired to strengthen or extend it, or render it suitable for guns; the Turks swarm round, some with entrenching tools, some bringing up hand barrows of earth, some branches of trees. The sound of a gun is heard; every man goes down behind the parapet into a hole, or under a rock. Presently the gradually increasing whiz of the projectile is heard as it approaches. It bursts, passes over or buries itself; all rise and continue their work. Hardly anyone is hit in this way, and the hindrance to repair is slight.

"The velocities of the projectiles are so low, that the sound of the gun is heard a considerable time before the first sound of the shell, while the first sound of the shell is heard long enough before the shell itself arrives, to enable men close to a shelter to wait for the whiz before ducking under cover. This may appear at first of small importance, but it is not so. Both the physical and moral effect of artillery fire are much impaired by it; the latter so much that it just makes the difference to the working parties between being very much afraid, and accepting the shells as rather an amusement.

"It may be said that the effects of high velocity shells arriving before the gun reports, and outstripping their own whiz, can be guarded against by establishing a good look-out; but smoke from a gun concealed among trees, as is often the case, does not show itself at once; and again, time is lost in giving warning.

. . . .

"On entering one of the emplacements at Rustchuk, the Turkish Staff Officer who accompanied me remarked (without any leading to the subject) that the Russian gun opposite was a particularly dangerous one, as the shell and the sound came together. The gun was supposed to be a long 15 c.m., and the range

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* It was stated in the "Times" that the sound of the guns firing salutes at Cherbourg, when Her Majesty and the Prince Consort were present at the opening of the docks, &c., was heard in Dorsetshire, a distance of some 80 miles. A still atmosphere favours the transmission of sound.

was given me as 2500 yards. This would imply a muzzle velocity of about 1520 f.s.

"From what I have seen and heard, I entertain no doubt that it is of great importance to be able to give projectiles a striking velocity equal to that of sound, at a reasonable range, say 1000 yards, while the mean velocity should equal that of sound up to much greater distances."

When the velocity of a projectile is less than that of sound, warning is given to those fired at, as the sound comes first; in either case it may sometimes be possible to have a look-out man to give notice when he sees the flash or smoke from an enemy's gun. This applies to the slow firing at sieges. An instance occurred at the siege of Cadiz, 1812, when the French employed three mortars or howitzers (one of which is now a trophy in St. James's Park) to fire at a considerable elevation at a long range to bombard the town.

"A watchman was placed in the high spire of one of the churches, who with a telescope observed the battery where these mortars were. As soon as he saw the smoke rise, he struck the great bell of the church. . . . . . The effect produced by the sound of the church bell was most striking. I was one day in the Plaza del Correo, a square much resorted to. . . . . . Amidst the crowd I was bargaining with a man for the purchase of a set of pictures, when the heavy single toll of the bell sounded. With a rapidity that was hardly conceivable to me, I found myself almost literally alone. . . . . After waiting, what on such occasions appears a long time, there followed the loud hissing sound of such projectiles, growing louder and louder until it thundered in its fall at the corner of the street into which the picture man had made his escape." Life of Rev. A. Dallas.

With regard to seeing the flash of a gun at a siege, General C. Gordon, C.B., wrote in his diary towards the end of his memorable defence of Khartoum, 4th December, 1884:—"With a good mitrailleuse (or machine-gun) and a sharp operator, *with telescope sight*, no gun could be served with impunity at 2000 yards range, though it could be served *against artillery fire*, for at that range there is plenty of time to dodge under cover, after seeing tho flash, ere the shell arrives."

At the siege of Sebastopol, when the firing of a Russian mortar was heard, it was said to be generally possible to look up and see the shell coming in the air, when a guess could be made of its probable place of descent, and thus it could be avoided.

At shrapnel practice (suppose 1500 yards range) with the newer field-guns, the range party observes the flash from the piece when it is fired, and the smoke begins to rise: the next thing noticed, after an interval of some $3\frac{1}{2}$ seconds, is a small cloud of smoke in the air near the target as the shell bursts, and then a little dust rises at intervals on the ground, and perhaps a few splinters of wood are seen tearing away from the target; while at about the same instant, or very soon after, the shrill noise of the unburst shell cutting through the air is heard, rapidly followed by the sound of its explosion; then comes the whirr of the fragments and bullets in the air, and their thud on the ground or target, while about a quarter of a second later the sound of the explosion at the gun comes up, and the smoke still continues to rise.

A double, or even a more continuous sound is generally heard by those at a distance when a gun is fired; this is more marked with the heavier natures; the secondary sounds are probably echoes produced by the contact of the sound-waves with neighbouring objects.

Sound of projectile in cutting through the air.

The passage of a rifled projectile through the air causes a shrill, tearing sound, and an experienced ear at experimental practice can detect from this sound whether the projectile is properly centred, if not it is said to be noisy in flight.

Projectiles visible in flight.

A gun projectile is generally visible in flight to an observer standing behind the piece a little to windward clear of the smoke, and it appears to travel quite slowly, always falling. Only its motion in a vertical and lateral direction is thus seen, and this is often not considerable. Mortar shells become difficult to follow with the eye when they have attained a considerable velocity in a downward direction. Small-arm bullets have only been seen by those with exceptionally good sight with the naked eye when the state of the atmosphere is favourable, but with a telescope they are often visible.

Smoke.

The volumes of **smoke** produced often hide guns firing, and also obscure the object aimed at. A very possible difficulty in firing B.L. guns from casemates and turrets may be the accumulation of smoke in the interior of the works, and means have been suggested for its removal: but as the new prism brown powder produces only a little smoke, this difficulty is not now likely to occur to so great an extent as under former conditions. At sieges, plans are adopted as previously mentioned, for continuing the fire in the same direction, though the target is not visible, but this plan has its drawbacks, as firing may be continued after a work is completely disabled; or the enemy may begin firing with a few guns in one battery, and when obscured by smoke they may open with another whose range cannot then be easily determined. Observing stations should always be established clear of the smoke, and the results of the firing, as far as it is known, should be telegraphed to the battery to correct the laying. The flashes at each discharge are generally seen through dense smoke, but this forms rather an indifferent object to lay on; it was done at Sebastopol, and may be done again; at the bombardment of Alexandria salvos of four guns at a time were fired by means of electricity from one of our ships, as it was found that the guns could be laid oftener by this plan than when firing independently, because then the smoke obscured the view for a longer time.

With a side wind, when firing deliberately, it is best to begin from the leeward side, as the aim of the next gun is obscured for a less time than when firing from the opposite flank. The smoke from guns is sometimes useful to hide deployments, or preparations for an assault.

The smoke from a shell bursting on graze affords information as to whether the range has been correctly obtained or not: this is quite wanting with long range infantry fire and machine-gun fire, with which it is difficult to find out whether the fire is effective or not.

Fouling in the bore.

Smoke is a solid product of the explosion in a fine state of division; but at the instant of explosion this product is liquid and a part of it cakes together in the bore and causes **fouling**. Each succeeding round, however, expels a good deal of that which was previously deposited. Fouling is more likely to occur in a small arm than in a piece of ordnance, and it is best prevented by the use of good powder in the charge. It gradually impairs the accuracy, and sometimes makes the breech difficult to open, and increases the recoil, as the resistance to motion of the bullet in the bore becomes greater, and, consequently the charge exerts its force for a longer time.

Gun-cotton leaves hardly any residue or fouling, and gives no smoke, and many attempts have been made to introduce it as a substitute for gunpowder. But it burns so rapidly that it has only succeeded with small arms, and, even with them its action cannot be said to be altogether certain; it has not been adopted in the Service for this purpose. The products of the explosion of gun-cotton are acid,

and they would act more injuriously on the metal of the bore than the products of fired gunpowder, which are alkaline.

Each succeeding round raises the temperature of the gun. The heat imparted to the gun is, of course, so much lost in expanding the gases which give velocity to the projectile. Doubtless as the gun becomes hotter a very little less amount of heat is given to the gun in each succeeding round, and thus we should expect to find the first rounds falling short; but this must be a very small difference, and practically, it cannot be said that its effect is generally noticeable, except with howitzers firing small charges, as so many causes are at work to create slight variations in muzzle velocity.

Gun heating by continual firing.

If the firing is very rapid and continuous, guns become so hot that water must be poured over them to enable the vent to be served, as was found at Maiwand; this is now, more than ever the case, in consequence of the magnitude of modern charges.

Brass guns have heated so much that they have "dropped at the muzzle" from the metal softening, and bending under its own weight, thus causing the piece to become unserviceable.

Expansion caused by heating after rapid and continuous firing may cause difficulty in working the breech-mechanism of some pieces.

The Martini rifle becames so hot with rapid and continuous firing, that the barrel cannot be touched with the hand; under such circumstances a leather guard is occasionally employed to protect the hand, and then the rifle can be held steadily when firing.

CHAPTER X.—MUZZLE VELOCITY.

(*See* also Part II, Chapter I.)

Electric chronographs.

HAVING considered the development of energy in the bore of a gun and the various stresses to which ordnance and carriages are subjected on firing, our attention now turns to the projectile after it leaves the piece with a definite muzzle velocity. The following chapter is devoted to a description of the principal means by which this has been practically determined.

Electric Chronographs.

For many years astronomers have been accustomed to find the time taken by stars in passing from wire to wire in the field of a telescope by means of electricity. A slip of paper is made to move at a uniform speed, and a clock beating seconds breaks a current of electricity at each beat; this makes marks or *dots on the paper* at equal distances apart, the distances varying with the speed of the paper.

As the star crosses each wire the *observer* breaks the current, and immediately the electrical arrangement makes a dot on the paper somewhere in the scale of seconds recorded by the clock. Thus, if Fig. 1 represents the dots made by the clock alone at regular intervals.

Fig. 1.

Fig. 2 shows two additional dots made when the stars pass two wires at about 0·4 and 4·8 seconds (if we begin counting from the dot on the left).

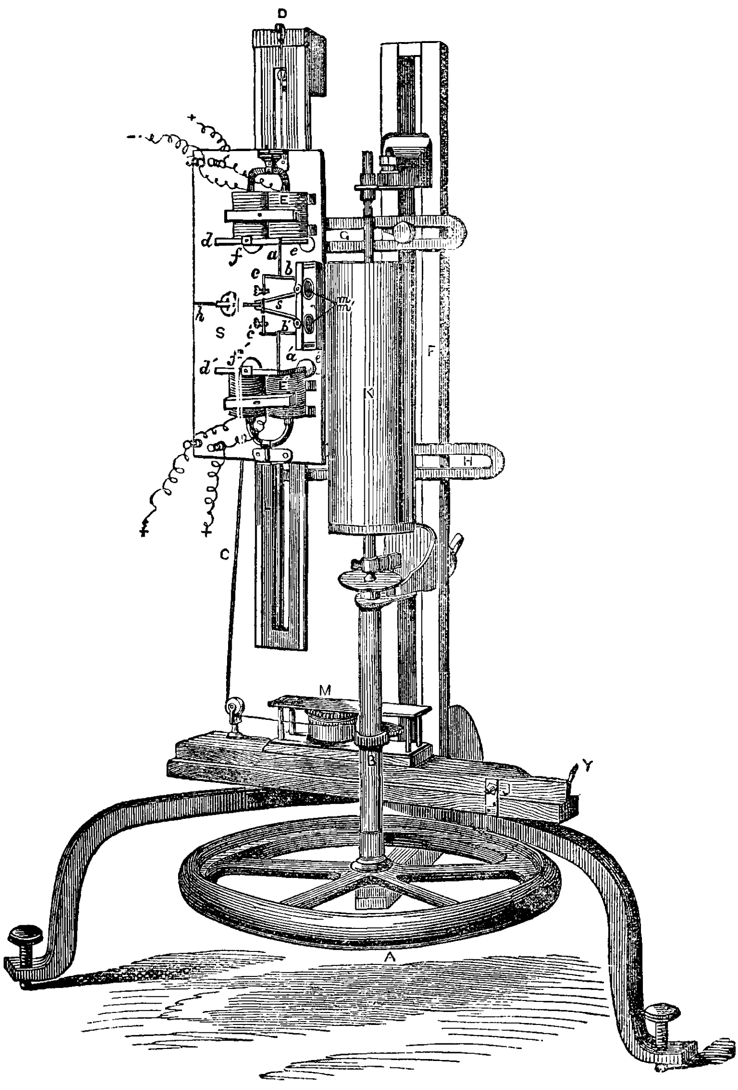
Fig. 2.

On this principle many instruments have been constructed for finding the time taken by projectiles to pass from one place to another; they generally consist of rapidly revolving cylinders covered with prepared paper, which is marked by a clock at regular intervals of time.

In some instruments brass discs or cylinders covered with lamp-black are dotted with bright spots by means of electric sparks, from the induced currents in Rhumkorff coils, when the circuits are broken. The projectile in its course, either in the bore or in the air, breaks the currents by cutting wires, placed at known intervals from each other, and thus the instruments are self recording. In **Mr. Bashforth's chronograph** used for finding the resistance of the air, several screens are used, but only one current, as a spring closes the circuit again at one screen before the projectile can reach the next.

Fig. 3 gives a general view of his chronograph: "*A* is a fly-wheel capable of revolving about a vertical axis spun round by hand, and carrying with it the cylinder *K*, which is covered with prepared paper for the reception of the clock and screen records. The length of the cylinder is 12 or 14 inches, and the diameter 4 inches. *B* is a toothed-wheel which gears with the wheelwork *M* so as to allow the string *CD* to be slowly unwrapped from its drum. The other end of

Fig. 3.



OD being attached to the platform *S* allows it to descend slowly along the slide *L*, about $\frac{1}{4}$ inch for each revolution of the cylinder. *E*, *E'* are electro-magnets; *d*, *d'* are frames supporting the keepers; and *f*, *f'* are the ends of the springs which act against the attraction of the electro-magnets. When the current is interrupted in one circuit, as *E*, the magnetism of the electro-magnet is destroyed, the spring *f* carries back the keeper, which by means of the arm *a* gives a blow to the lever *b*. Thus the marker *m* is made to depart from the uniform spiral it was describing. When the current is restored the keeper is attracted, and thus the marker *m* is brought back, which continues to trace its spiral as if nothing had happened. *E'* is connected with the clock, and its marker *m'* records the seconds.

Figs. 4 and 5 give the details of the screens. Fig. 4 represents a piece of board 1 inch thick and 6 or 7 inches wide, and rather longer than the width of the screen to be formed. Transverse grooves are cut at equal distances, something less than the diameter of the shot, as shewn in the diagram. Staples of hard brass spring-wire (No. 14

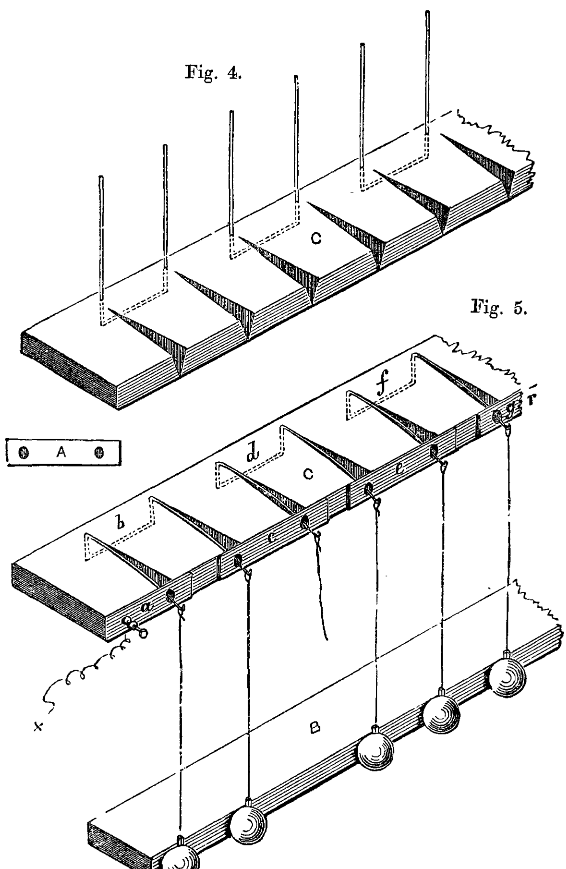


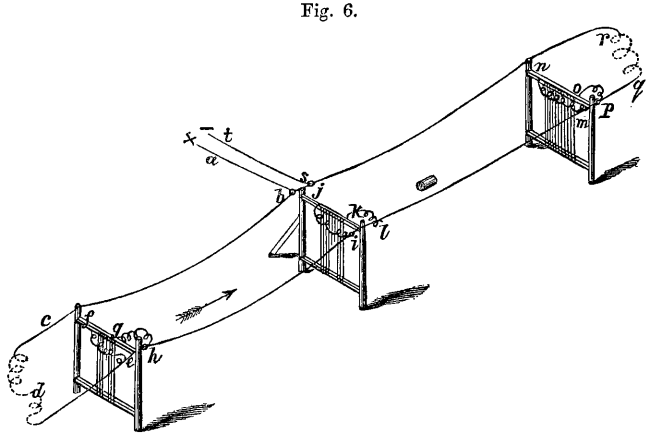
Fig. 4.

Fig. 5.

or 15), are fixed with their prongs in the continuation of the grooves. Pieces of sheet copper A are provided, having two elliptical holes, the distance of whose centres equals the distance of the grooves. The pieces of copper A are used to connect each wire staple, as C, with its neighbour on each side. Thus, Fig. 5, a, c, e, g, &c., represent these copper connections put in their places and holding down the wire spring, which, when free, are in contact with the tops of the holes; but, when properly weighted, they rest on the lower edge of the holes. Thus the copper c forms a connection between the staples b and d; the copper e joins d and f, and so on. An electric current will therefore take the following course, whether the springs be weighted or unweighted: copper a, brass b, copper c, brass d, copper e, brass f, copper g, &c. The current will only be interrupted when one or more threads have been cut and the corresponding spring is flying from the bottom to the top of its hole. About $\frac{1}{50}$th of a second is required for the complete registration of such an interruption, the spring traversing about half an inch. The shelf B is placed for the weights to rest against, partly to prevent them from being carried forward by the shot, but chiefly to prevent the untwisting of the threads which support the weights. The weights used were about 2 lbs. each, and the strength of the sewing cotton for supporting them was equal to a stress of about 3 lbs., which was sufficient to withstand a tolerably strong wind. As the weights were equal the threads were kept equally stretched.

The arrangement of the screens for an experiment is shown in Fig. 6. The wires for conveying the current are, like the common telegraph wire, carried on posts. abc is a continuous piece of wire; but there are interruptions between e and h, between i and l, between m and p, &c., in order to make the galvanic current circulate

Fig. 6.



through the screens. The course of the galvanic current is + a, b, c, d, e, f, g, h, i, j, k, l, m, n, o, p, q. r, s, t. The ends a, t, are connected with the instrument and battery. The shot, being fired through the screen, in passing cuts one or more threads at each screen, so that corresponding to the instant at which the shot passes each screen, there is an interruption of the current, and a simultaneous record on the paper." (*Vide* "Description of a Chronograph," by F. Bashforth, B.D., in which the instrument itself is also described.)

In Noble's chronoscope, where the time of passage from one plug to another in the bore of the gun is measured, the intervals are very minute, and a separate battery, current and induction coil, must be employed for each plug.

Want of space prevents a description of this and other instruments; the Navez-Leurs, which was one of the first, recorded the time by means of a pendulum swinging through a certain arc, which was then measured. In Watkin's first chronograph, time is found from the distance fallen by a weight, whilst in Watkin's recent instrument, and in Sébert's velocimeter (*see* Translation U.S. Ord. Notes, No. 313) a tuning fork the number of whose oscillations per second are known) traces out a wavy line on a rapidly moving surface, and thus forms an exact time scale, as each wave corresponds to a certain definite small fraction of a second.

If the motion of the recording cylinder of Bashforth's chronograph is not uniform, its rate of change of velocity must be found from the clock records, and allowance must be made, as explained in Part II, Chap. III. But for ordinary purposes, such as proving gunpowder, by finding what velocity a given charge will impart to a given projectile in the same gun, or for simply finding the muzzle velocity under any other circumstances, the **Boulengé chronograph** is largely employed, not only in England, but also on the Continent. This instrument (*see* Fig. 7), with the disjunctor close to it, is placed in a house, which may be at any convenient distance (say 100 yards or more) from the gun.

Boulengé chronograph.

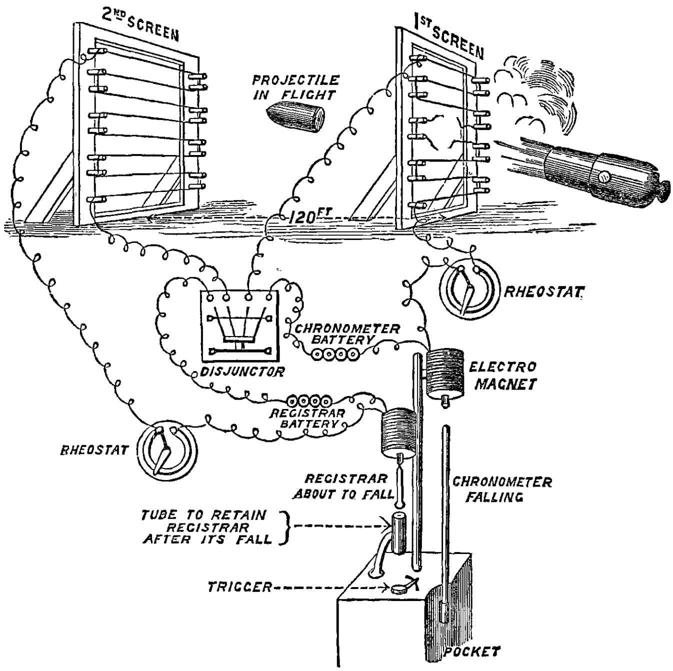
In the Boulengé chronograph, modified by Captain Bréger, the time is obtained by means of marks cut on a long weight falling under the influence of the uniform acceleration of gravity, and thus no clock is employed. (For further description *see* "Chronograph le Boulengé," modifié par M. Bréger, Capitaine d'Artillerie de la Marine, Commission de Gavre, Sep. 1880.

The instrument records the time taken by the projectile to pass from one screen to another 120 feet distant (*see* Fig. 7). Each screen consists of a large open vertical frame at right angles to the line of fire. At each side of the screens are placed a number of pins, covered with insulating material, and a fine copper wire is passed backwards and forwards from the pins on one side to those on the other side of the frame; the returns of the copper wire must be at such a distance apart, that the projectile cannot pass without breaking at least one of them; the ends of the wire are placed in the circuit of an electric current, which, coming from a battery, passes through an electro-magnet at the instrument itself. Each screen forms part of a distinct circuit, with a separate battery and a separate electro-magnet; both currents, however, pass through the same *disjunctor*, whose office is to break both of them, when required, at the *same* instant; and each circuit passes through a rheostat to modify the strength of the current sent to its electro-magnet; the rheostats shown in Fig. 7, and Plate II, Fig. *E*, p. 126, are those used in the R.G.F., and differ somewhat from the French ones, though their object is the same in each case.

The instrument.

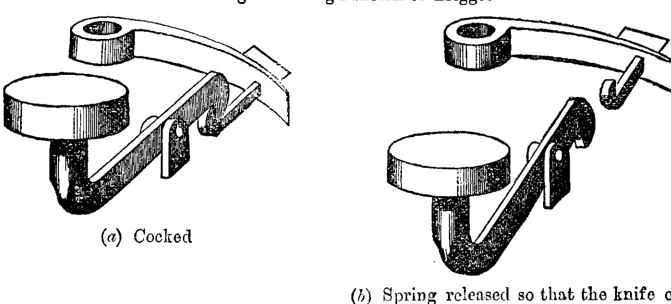
Two electro-magnets (*see* Fig. 7 and Plate II, Fig. (*A*), p. 126) are fixed on a vertical stem, and when magnetized by the current, they are each capable of sustaining a brass rod tipped with iron; the long one, called the "chronometer," *C* is covered with a removable copper tube, and it is placed so as to fall into a pocket below; the other rod is called the "registrar" *D* (Plate II, Fig. (*A*)). When it falls, it is retained by a tube *F*, and presses down a little disc immediately

Fig. 7.—Rough sketch of the general arrangement of Boulengé Chronograph.



NOTE.—Practically the wires cross each other, and the batteries are for convenience put out of the way. The screens, gun, and chronograph are are not really so near each other, but they are shown as above for the sake of distinctness.

Fig. 8.—Rough sketch of Trigger.



(a) Cocked

(b) Spring released so that the knife on it may mark the disjunctor.

underneath (*see* Fig. 8 and a Plate II, Fig. (*B*)). This disc is attached to the end of a small lever, *ab*, which pivots about an axis through its centre; the other end of the lever is provided with a catch, which holds back a spring furnished with a little knife *e* (Plate II, Fig. (*B*)), capable of reaching and marking the copper cover of the chronometer, when the spring is released by pressing down the disc *a*, and consequently jerking up the catch on *b*. The lever, spring, and knife constitute "the trigger."

Suppose both circuits completed, if the chronometer and registrar are suspended, and the trigger is cocked, the registrar current only is broken, and the registrar falls, because the magnetism is destroyed which held it up; the trigger is set in action, and a mark would be made on the chronometer: this is called the *zero*, found once for all by the instrument maker.

But before using the instrument, certain points must be attended to

(1.) The magnetism must be only just sufficient to sustain each rod: this is effected by moving the dial hands of the rheostats, thus interposing more or less resistance in each circuit, until each rod is only just supported. If the magnetism were stronger, the rod would not fall readily when the current is broken, on account of remaining magnetism, which takes an appreciable length of time to disappear, and the results would consequently be unreliable.

(2.) The instrument must be properly levelled with the stem vertical.

(3.) The time taken for the registrar to fall and for the trigger to act must be determined.

The disjunctor. For this last purpose the **disjunctor** is used (*see* Plate II, Fig. *C*); both currents enter it, but they are insulated from each other. It consists of a wooden base, on which is a trapezium *a*, *a*, carried on strong springs *b*, *b*, which tend to press the parts above it against the bridge *g*: the spring is pressed and kept down by the spring-catch *h*: two light springs *d*, *d* are fixed on the trapezium, they carry at their free ends small weights *e*, *e*, which are thus pressed *down* on the two insulated contact points *f f*, one of which is in each circuit of the currents to be broken.

On releasing the catch *h*, the trapezium is set at liberty, and the springs *b*, *b*, move it quickly, but it is suddenly stopped by the stop *g*: the weights *e*, *e*, however, continue their movement in virtue of their momentum, and thus both currents are simultaneously broken at the points *f f*.

Suppose the circuits completed, rods suspended, and trigger cocked, and that both currents are then broken at the same instant by the disjunctor (*see* Fig. 10 (*b*), p. 126), the chronometer and registrar fall together; the latter, pressing on the disc of the trigger, releases the catch, and permits the knife on the spring to mark the falling chronometer at a certain distance (h) from the zero mark. We can now tell how long it takes for the registrar and trigger to act, for the distance (h) must be due to a time (τ), and the relation between them is expresssed by the formula—

$$h = \tfrac{1}{2}g\tau^2;$$

thus, if $h = 4{\cdot}345$ inches, it will be found that $\tau = 0{\cdot}15$ second. This is called the "disjunctor reading" (*see* "disjonction," Plate II, Fig. (*D*)).

To avoid calculation and to enable a previously-prepared scale to be used, it is desirable that the disjunctor reading should always correspond to a time of 0·15 second, or be 4·345 inches distant from the

zero mark. Before taking a velocity the disjunctor reading should always be found, and if it is not 4·345 inches (from alteration in the stiffness of the spring, or from any other accidental cause), it should be made so; this is done by altering the height through which the registrar falls, by raising or lowering the electro-magnet of the registrar by means of the thumb screw H (*see* Plate II, Fig. (A)), and thus the time of disjunctor reading can be adjusted by trial and error to be 0·15 second.

If now the disjunctor is closed, the rods hung up again, and the trigger cocked, the instrument is ready for use.

The gun is then fired, and the projectile, in passing through the first screen, cuts the wire, breaks a circuit, the upper electro-magnet is demagnetized, and the chronometer falls (*see* Fig. 7 and Fig. 10 (c)). The projectile soon breaks the circuit at the second screen (*see* Fig. 10 (d)), and then the registrar falls on to the trigger, which makes a mark on the still falling chronometer. The distance (H) of this last mark from the zero is of course greater than h (the disjunctor reading previously obtained), as it is due to a time (T) which is equal to the time (t) taken by the projectile to pass from one screen to the other, as well as to the time τ (generally 0·15 second) taken for the registrar and trigger to act.

As before, the total time T is found from the equation—

$$H = \tfrac{1}{2}gT^2,$$

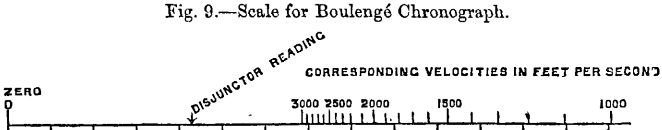
whence T is easily found;

to find the time t taken to pass from one screen to the other, we must deduct the disjunctor reading from T, or $t = T - \tau$.

Thus, suppose the value of H is 10·217 inches $= 0·8514$ feet, from the above relation of time to space, we find $T = 0·23$ second, and if from this 0·15 second is deducted, we have the time taken to pass from one screen to the other $= 0·08$ second; and the mean velocity must be $\frac{120}{0·08} = 1500$ f.s. This is assumed to be the *actual* velocity of the middle point between the screens, and it is true (*see* p. 255) if the cubic law holds good, which can safely be assumed for the short distance of 120 feet. Example 1.

The distance H is carefully measured by a brass scale, with a vernier reading to thousandths of an inch; or if the disjunctor reading is adjusted to a time of 0·15 second, the mean velocity between the screens may be found at once by applying another brass scale, which is similar to that shown in the accompanying sketch, Fig. 9 (one-fourth of the real size). (*See* also Plate II, Fig. (D)).

Fig. 9.—Scale for Boulengé Chronograph.



Thus (*see* Fig. 9) suppose the second screen mark or chronometer reading is at 8·518 inches from zero, the velocity is 2000 f.s. at the middle point between the screens. Example 2.

The distance of the middle point between the screens from the muzzle being known: the velocity *at* the muzzle can be found by employing Bashforth's tables, as will afterwards be described (*see* Chapter XII, Example 9, p. 147): the difference to be added to the mean velocity between the screens is always a small quantity, as the distance of the screen from the gun is small, and only a slight retardation has been effected in the velocity of the projectile, consequently when measuring velocities of similar projectiles fired with charges of equal weight for the proof of powder, it is usual to find this difference once for all by calculation, it is then doubled, as the flat-headed projectiles used at the proof butts experience about double as much resistance from the air (*see* p. 139) as the ogival-headed ones for which the tables are made out, and the amount obtained is then added to the velocity found each time by the chronograph in order to determine the *M.V.*

Fig. 10 shows the chronograph under different circumstances.

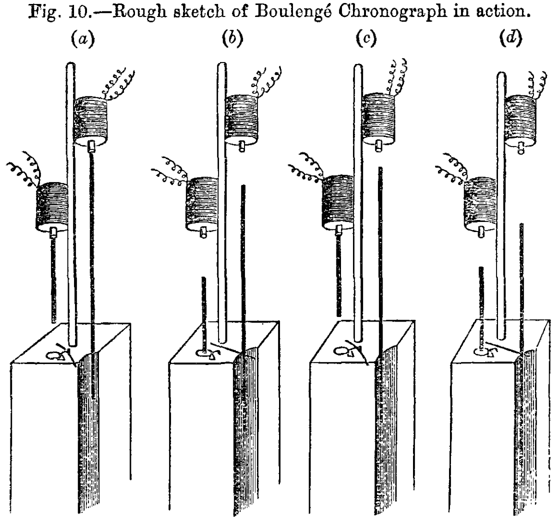
(*a.*) Both circuits complete, both rods suspended, and trigger cocked.

(*b.*) Both currents broken at the same time to obtain "disjunctor reading," both rods falling, and "trigger" acting.

(*c.*) Gun fired; the projectile has passed through one screen, but not yet through the other. "Chronometer" falling, "registrar" hanging, the "trigger" cocked. (*See* also Fig. 7.)

(*d.*) This is a small fraction of a second later than (*c*); the projectile has passed the second screen, the registrar has fallen, and the knife of the "trigger" is just marking the falling "chronometer."

Fig. 10.—Rough sketch of Boulengé Chronograph in action.



NOTE.—Details such as holding tube, &c., omitted for the sake of simplicity.

Plate II.

BOULENGÉ CHRONOGRAPH.

Modified by Captn. Bréger.

FIG. (A)
ELEVATIONS.

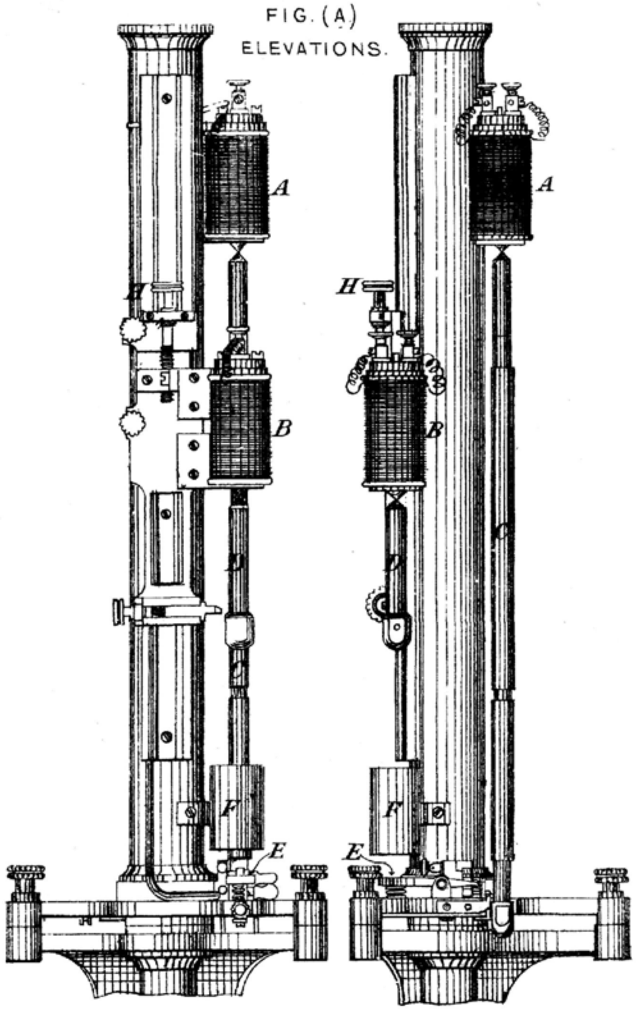


FIG. (A.)
PLAN & TWO ELEVATIONS (AT RIGHT ANGLES TO EACH OTHER) OF THE INSTRUMENT.
Scale $\frac{1}{6}$th

A. B. Electro Magnets.
C. Chronometer Rod.
D. Registrar.
E. Trigger.
F. Holder to retain registrar after its fall.
H. Screw for regulating the height of Electro Magnet B.

PLAN

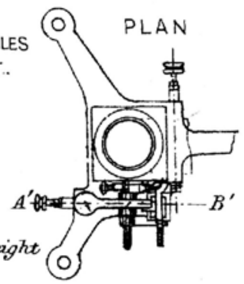
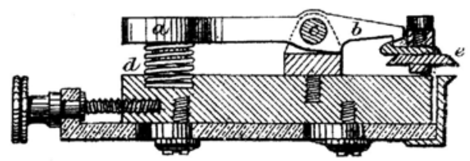
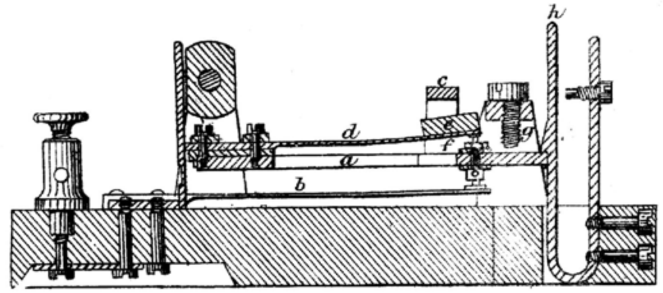


FIG. (B)
TRIGGER
ELEVATION ON A'B' OF FIG (A) SCALE $\frac{1}{2}$.

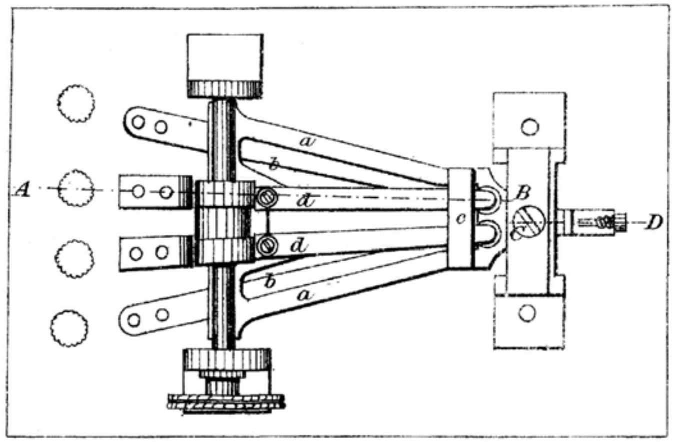


a. b. Lever. | *d. Spiral Spring to press up the end a*
c. Pivot. |
b. Catch. | *e Knife on Spring.*

FIG. (C)
DISJUNCTOR. SCALE $\frac{1}{2}$
SECTIONAL ELEVATION ON A.B.C.D



PLAN



NOTE. *The Disjunctor is not shewn in Fig. (A), a contact point f is under each weight e. e.*

FIG. (E)
CIRCULAR RHEOSTAT
PLAN_SCALE $\frac{1}{4}$

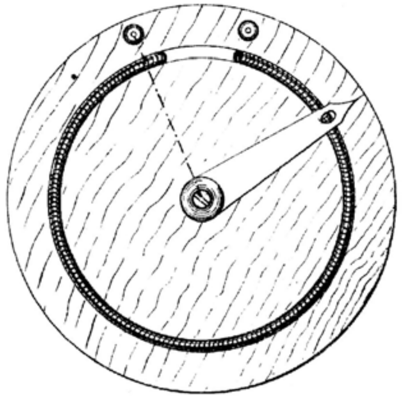
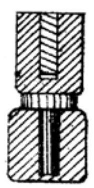
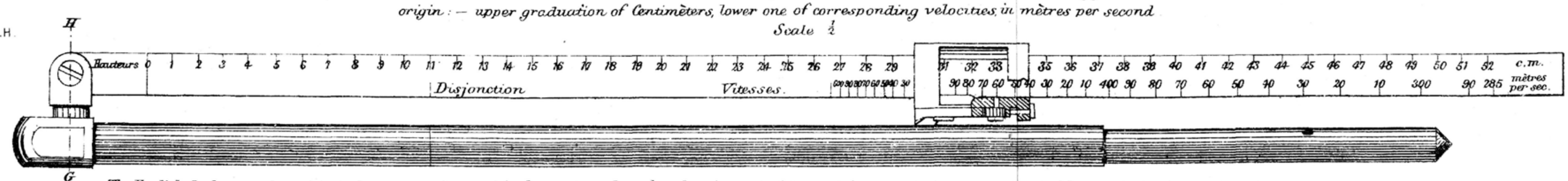


FIG. (D)

Chronometer Rod, with Scale attached for measuring the distance of the mark made by the trigger knife from the origin: — upper graduation of Centimeters, lower one of corresponding velocities in mètres per second
Scale $\frac{1}{2}$

SECTION ON G.H.





The English Scale is graduated in inches on one edge, and in feet per second on the other; instead of in centimètres and mètres per second. The vernier is clamped by a screw near the chronometer reading; ready and accurate adjustment is obtained by a slow motion screw. (neither of these screws are however shewn in the drawing.)

To face p. 126.

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CHAPTER XI.—TWIST OF RIFLING AND DRIFT.

(*See* also Part II, Chapter I, p. 240, and Table III, p. 280).

Rotation is always practically **effected** by fitting or forcing projections (studs, lead coat, gas check or rotating ring) on the projectile into spiral grooves, which are cut in the bore of the gun: and no plan has succeeded of giving rotation in any other way, except in the case of Hale's rockets.

As mentioned on p. 77, when considering the stresses in the bore of a gun, the spiral grooving or twist is either—

(1.) Uniform;
(2.) Increasing.

A progressive groove is employed with the Martini rifle; on the explosion of the charge, the leaden bullet is expanded into the grooves which become shallower towards the muzzle, consequently the metal is tightly compressed in each groove, as the bullet passes down the bore, and rotation is given with great steadiness, and centring is very fully attained.

Uniform twist.

A **uniform twist** is one which resembles a common screw in having always the same pitch. It can be formed by wrapping a right-angled triangle ABC (Fig. 1), round a cylinder in such a way that one side BC is parallel to the axis; the hypotheneuse AB then represents the direction of the groove of the rifling; and the angle of rifling or twist is that between the line BC and the hypotheneuse AB. Let the other side, AC, be of such a length that A and C will just meet round the cylinder (Fig. 2). Then AB will make one revolution when the triangle is wrapped round the cylinder, and AC is the circumference $= \pi d$, if d is the diameter or calibre in inches; and if $CB = nd$, we have a spiral AB which makes one turn in n calibres or nd inches.

Fig. 1.

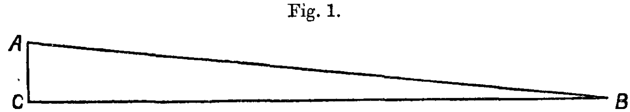
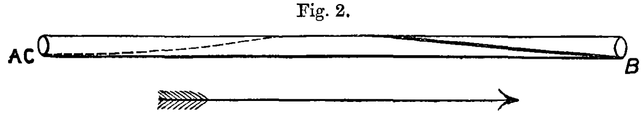


Fig. 2.



Direction of axis — to muzzle.

Right-angled triangle of Fig. 1, wrapped round a cylinder, thus making a uniform twist of one turn in 30 calibres; A and C of Fig. 1 meet round the cylinder in Fig. 2.

Let ABC be the angle of rifling $= \alpha$.

$$\text{Then } \tan \alpha = \frac{AC}{CB},$$
$$= \frac{\pi d}{nd},$$
$$= \frac{\pi}{n}. \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(i)}$$

Fig. 3.

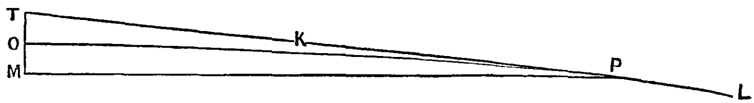


Fig. 4.

Parabola of Fig. 3 wrapped round a cylinder, making a twist increasing from 0 at breech to one turn in 30 calibres at the muzzle. O and M of Fig. 3 are at opposite ends of a diameter of the cylinder in Fig. 4.

Increasing twist.

Suppose a uniformly **increasing twist** is used: instead of an inclined straight line, a part of a parabola (Fig. 3) must be wrapped round the cylinder or inside the bore (Fig. 4).

For let OPL be a parabola (Fig. 3), its vertex being at O, and equation—

$$x^2 = 4ay.$$

Let α be the angle which the tangent at P makes with the ordinate of y; the equation to this tangent is

$$x = \frac{2a}{x'}(y + y'),$$

$$\text{when } \tan PTM = \frac{2a}{x'},$$

$$\text{and consequently } \tan TPM = \tan \alpha = \frac{PM}{2a}.$$

Hence the tangent of the angle of rifling α increases with PM, or we may say for small angles, that the angle of rifling itself increases uniformly as the distance down the bore increases.

The twist of this groove at any point is the inclination of a tangent to the curve to the ordinate at that point. Some guns employ a uniformly increasing twist, which is 0 at the breech, and becomes one turn in n calibres at the muzzle. Others start with a slight twist at the breech of one turn in n calibres, increasing to one in n' at the muzzle. For the former of these (Fig. 3) the curve from the vertex O to some point P must be wrapped round the bore; for the latter the curve from some point K to P must be employed, the axis of x being in each case parallel to the axis of the bore.

With the more recent B.L. guns an increasing twist is employed,

but at the muzzle it becomes uniform with the intention of giving steadiness of rotation to the projectile.

With the 80-ton gun a twist has been used which is an increasing one, but the increase is accelerated with the intention of adjusting the pressures on the grooves advantageously. The curve which is employed (developed on to a flat surface) is a semi-cubical parabola.

The uniform twist is the simplest, and was the one first adopted in rifled guns. Other things being equal the muzzle velocity is a little greater with the uniform twist, probably because the maximum pressure of the powder gas has more time to act in consequence of the increased resistance at the breech end of the bore.

Velocity of Rotation.

Let ω be the **angular velocity** of the projectile, *i.e.*, the number of unit angles of circular measure it revolves through in a second; if the projectile revolves once in a second the angular velocity $\omega = 2\pi$, since there are 2π angles of circular measure in the whole circle. Measure of the velocity of rotation.

Consequently, if the projectile revolves m times in a second we must have—

$$\omega = 2\pi m, \dots\dots\dots\dots \text{(ii)}$$

Also if the distance (of translation) moved in each revolution, as already noted, is nd, the whole distance moved in a second is mnd *inches* (since d is in inches).

But in one second the projectile moves V *feet*, and if V is the muzzle velocity, which is supposed uniform for a short space of time,*

$$\therefore 12V = mnd \dots\dots\dots\dots \text{(iii)}$$

Hence from (ii) and (iii) we easily obtain

$$\omega = 2\pi \frac{12V}{nd}, \dots\dots\dots\dots \text{(iv)}$$

whence we see that the angular velocity increases directly with the muzzle velocity, and decreases with an increase of calibre or of n.

The **linear velocity of rotation** is the product of the distance r (in inches) of the point considered, from the axis, multiplied by the angular velocity; Linear velocity of rotation.

$$\text{or linear velocity} = \omega r \textit{ inches} \text{ per second}$$

$$= 2\pi \frac{Vr}{nd} \textit{ feet} \text{ per second.} \dots\dots \text{(v)}$$

For the linear velocity of a point on the circumference

$$2r = d,$$

hence linear circumferential velocity

$$= \frac{\pi V}{n} \text{ f.s.} \dots\dots\dots\dots \text{(vi)}$$

Examples.

Compare the velocity of rotation of the 9-pr. R.M.L., and the 9-inch R.M.L. guns; the following data being given:— Example 1.

9-pr. Twist, one turn in 30 cals. Calibre 3″. *M.V.* 1400 f.s.

9-inch. Twist, one turn in 45 cals. Calibre 9″. *M.V.* 1400 f.s.

---

* The factor 12 is inserted in (iii) to reduce V, which is expressed in feet to inches, since d is in inches.

For 9-pr.	For 9-inch.
$\omega = 2\pi \dfrac{12V}{nd}$.	$\omega_1 = 2\pi \dfrac{12V}{n_1 d_1}$.

Substituting and cancelling equal terms, we have—

$$\omega : \omega_1 :: 9 : 2,$$

or the 9-pr. rotates $4\frac{1}{2}$ times as rapidly as the 9-inch.

Example 2. What is the linear velocity of rotation of a stud on a 9-pr. shell, supposing $M.V. = 1390$ f.s., twist one turn in 30 calibres?

By formula above, the linear velocity of rotation

$$= 2\pi \frac{Vr}{nd} \text{ f.s.} \qquad \text{here } 2r = d;$$

$$\therefore = \frac{\pi V}{n} \text{ f.s.}$$

$$= \frac{3{\cdot}14159 \times 1390}{30} \text{ f.s.}$$

$$= 145{\cdot}4 \text{ f.s.}$$

Example 3. Suppose the tangent of the angle of rifling $= \frac{1}{10}$, in how many calibres does the projectile make one turn?

$$\text{Given } \tan \alpha = \frac{1}{10},$$

$$\text{We know } \tan \alpha = \frac{\pi}{n}; \textit{ see } \text{equation (i)}$$

$$\therefore \frac{\pi}{n} = \frac{1}{10};$$

$$\text{Hence } n = 31{\cdot}4159,$$

or the projectile makes one turn in 31·4159 calibres.

Example 4. On firing a gun having a uniform twist, does the projectile rotate as rapidly when half way down the bore as at the muzzle?

No. The velocity of translation is increasing all the way down the bore, and the angular velocity increases with it, for

$$\omega = 2\pi \frac{12V}{nd},$$

and $2\pi\dfrac{12}{nd}$ is a constant in the uniform twist: therefore the angular velocity continues to increase as the velocity of translation down the bore increases.

Example 5. What is the angular velocity of a 10-inch projectile: twist, one turn in 40 calibres. $M.V. = 1500$ f.s.?

$$\omega = 2\pi\frac{12V}{nd},$$

$$= \frac{2 \times 3{\cdot}14159 \times 12 \times 1500}{40 \times 10}$$

$= 282{\cdot}7$ units of circular measure per second.

Similar projectiles are fired from two guns having the same calibre and length of bore; their muzzle velocities differ, but their velocities of rotation are the same. Explain how this is possible. Example 6.

As the value of ω is the same in each case we must have—

$$2\pi\frac{12V}{nd} = 2\pi\frac{12V_1}{n_1 d},$$

d is the same in each.

$$\therefore \frac{n_1}{n} = \frac{V_1}{V},$$

or the twists of rifling must be proportionate to the muzzle velocities.

DRIFT.

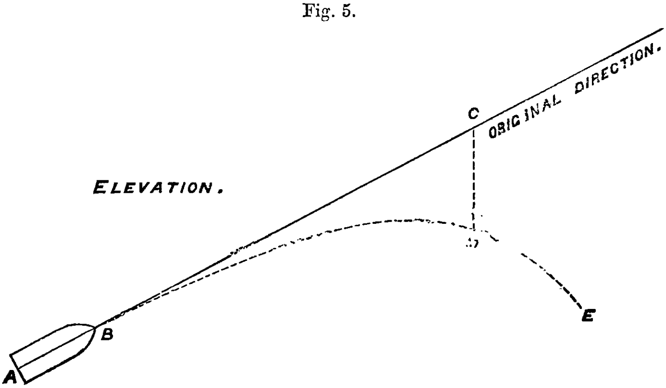
Speaking generally a body moves most easily through a resisting medium when the front is sharp and the cross sectional area small, as thus the opposing particles are most easily pushed asunder, and the resistance, and consequently the retardation, is small. Thus, the swiftest birds and fishes have a small cross sectional area, and the surface they present to the air or water in a forward direction is narrow and sharp. The same thing is also seen in ships and spears, which are sharp in front in order to pierce an opposing medium; if the sides are very long, the resistances increase; the shape of the rear part also greatly influences the result. Shape of head.

It is thus evidently an advantage to employ an elongated pointed projectile; but with S.B. guns this cannot be done, since the resistance of the air not only gives rise to a retarding effect, but also produces a **couple** which causes the projectile to turn over in flight, and thus sometimes to present its side surface, whence extreme irregularity in flight ensues from changing and unequal pressures.

The couple is caused thus:—

Let ABC (Fig. 5) be the initial direction of a projectile fired from a gun seen in side elevation; it never will reach C, but will Couple.

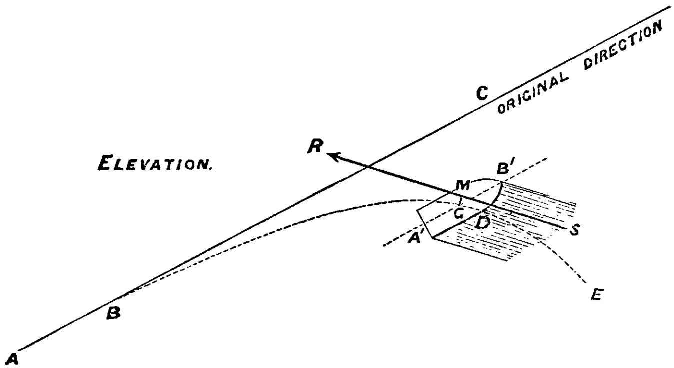
Fig. 5.



pass at some point D below it, such that CD is very approximately equal to the distance the body would fall (under the influence of gravity alone) in the time taken to pass over ABD. Thus the path of the centre of gravity of the projectile is some curved line, ABDE.

Now, let us imagine that the projectile has reached D, and that its axis A′B′ still remains pointing in the same or a parallel direction (*see* Fig. 6), for we have not yet shown any reason that it should be altered.

Fig. 6.



NOTE.—R is represented as acting above G, because from the construction of artillery projectiles the centre of gravity is always nearer to the base than the point; also R is shown nearer the point than the base, since the resistance of the air is greater on the side of the head, which is across the direction of motion, than on the side of the body which is more obliquely placed. The angle between A′B′ and the tangent to the trajectory would never really be as large as shown in Fig. 6; but it is drawn thus for the sake of showing the couple clearly.

The resistances of the air act in a contrary direction to motion, and they are parallel to a tangent to the curve at D, as shown by the shaded lines. These resistances can be represented by some resultant R, parallel to all of them acting along some line SR.

From G, the centre of gravity, draw GM perpendicular to SR. The effect of R not only tends to stop the onward motion of translation of the centre of gravity of the projectile, but it also produces a couple tending to cause rotation round a short axis passing through the centre of gravity.

It may be asked, could not a projectile be constructed in which the resultant resistance of the air would act through its centre of gravity, and thus prevent the formation of the couple? Practically this cannot be done. If it is the case at one angle of inclination of the axis to the trajectory, it will not be so when gravity has increased that angle, since fresh surfaces will be presented to the resistance of the air, and thus the resultant of the resistances will no longer act through the centre of gravity. But if it *could* be done, the full advantage of the pointed head would not be gained at even ordinary angles of elevation, since gravity soon causes the axis of the projectile to make a considerable angle with the trajectory, and consequently it would not travel point first.

When the resultant of the resistance is a good way behind the centre of gravity, a non-rotating elongated projectile can be employed when the velocities are moderate, as in—

(1.) A rocket with stick.
(2.) An arrow.

In the first case the centre of gravity is very far forward, and the resultant resistance acting at, say, three-fourths of the length from the rear, is well behind the centre of gravity. In the case of the arrow, the centre of gravity is only a very little in front of the middle; but the feathers offer such considerable resistance that the resultant must be towards the rear, and consequently behind the centre of gravity.

In these cases a couple is produced, but it pushes the point of the missile *into* the direction of the trajectory, and not further away from it, as would be the case in an elongated projectile from a S.B. gun. An arrow shot into the air at a considerable angle of elevation is observed to keep its axis very nearly a tangent to its trajectory. These shapes would of course be inconvenient for gun projectiles, and possibly at high velocities the couple might push the point below the trajectory, when it would be forced up again; and so a swinging motion might be set up, which would lead to inaccuracy in flight.

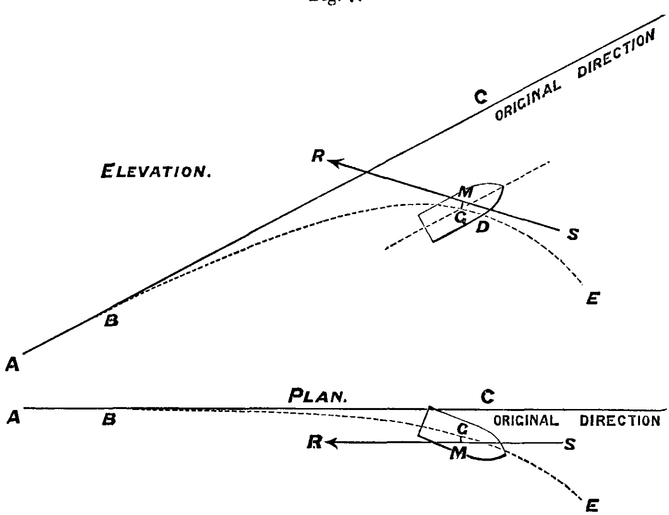
The overturning couple cannot be overcome with smooth bores, and consequently elongated projectiles cannot be used with them, since there is no means of making the point always go first; but spherical projectiles are employed, since whatever accidental rotation takes place, a similar surface and a similar sectional area are presented to the resisting air, which opposes its onward motion.

Effect of couple on a rifled projectile.

Let us now consider why **a rifled projectile**, which rotates about its longer axis, **can** be made to **travel** through the air with its **point** always approximately **first.**

Suppose that the projectile has reached some position, D (*see* Fig. 7), in its flight, where gravity has caused the axis to make a certain angle with the trajectory; as before we may suppose the resultant resistance of the air to act along SR. But the effect of the couple R.GM

Fig. 7.



in this case is not (as before) to raise the point of the projectile and give rotation round the centre of gravity, on an axis at right angles to the plane of the trajectory; but it causes a slow movement of the point laterally, with service projectiles having right-handed rotation, to the right of an observer stationed behind the gun, and who looks down the range: with left-handed rotation the movement would be to the left. This is shown in the plan, and the projectile sets laterally across the original direction.

As seen in the plan, the couple R.GM, instead of turning the point still more to the right, gives it a slow downward movement. The point is thus approximately kept down to the trajectory.

The resultant resistance, as seen in the plan, also exercises a retarding effect, and as the direction of the projectile is inclined to this resultant a lateral force is exerted, which causes a tolerably uniform lateral acceleration, giving rise to **drift** to the right.

In a similar way a flying bridge is impelled across a river when it is attached to a buoy rope, and kept at an angle to the current by the rudder.

The projectile moves and the air is still, while the boat does not move forward, but the water rushes past; the same resistances, however, occur, since in both cases there is relative motion between the body and the resisting medium, and it is immaterial (as far as pressures are concerned) which actually moves. The energy of the projectile causes the resistance of the air to continue, and so it corresponds to the rope; and the slow, gyratory movement causes the projectile to set across the range like the rudder of the boat.

The plan of the trajectory is thus seen to be a curved line and not a straight one, and the trajectory itself has a double curvature, and is not contained in one plane.

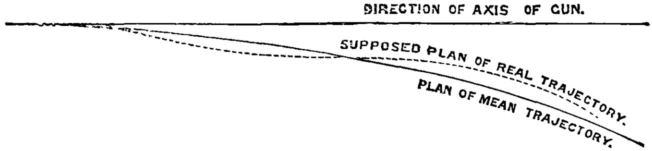
The effect of a couple causing the axis of a rotating body to move in a plane perpendicular to that in which it is itself acting, is illustrated in a common top. If placed on its point unspun in a slightly inclined position it falls over, the resistance of the ground against the point forming a couple. But if it is placed spinning in the same position, it no longer falls over, but a slow movement in a plane at right angles to this couple is observed, and the axis of the top moves slowly round in circles or gyrations. The cause of this couple is different from that with a rifled projectile in flight; but the effect is similar, as in both cases a slow motion of the axis is produced in a plane at right angles to the couple; a gyroscope illustrates this fact most plainly. (*See* Part II, Chapter I, p. 242.)

Possible gyrations

It is most probable that the point does not move in a straight line diagonally downwards and to the right, but in a more or less gyrating or curved manner.

Drift is thus seen to be a differential effect, due to several causes. Though the plan of a trajectory may be represented by a curve of a nearly regular parabolic character, the minor causes just mentioned may create a kind of undulation* superposed on what may be called the plan of the mean trajectory. Thus, in Fig. 8, the general or mean direction of the trajectory is represented in plan; but the minor causes *may* make the real path to vary from this as shown by the dotted line, and thus the increase of drift may not be perfectly regular with the increase of range; practically, however, it may be considered so except at very short ranges.

Fig. 8.



Plan of a trajectory, drift exaggerated.

Other explanations of drift.

It must be stated that opinions differ greatly as to the explanation of this difficult problem in rigid dynamics. Lt.-Colonel Sladen, in his "Principles of Gunnery," considers that the axis of the projectile makes one or more complete gyrations round the trajectory, and that

* *See* diagrams, Mayevski's "Traité de Balistique."

the drift with service projectiles is always to the right, because the first movement of gyration is in that direction.

A very short explanation of drift sometimes given is, that the projectile rolls on a dense layer of air underneath it to the right; in agreement with this theory are the facts that the more rapid the twist the greater is the drift, and the comparatively rough studded M.L. projectiles drift more than smoother B.L. shell; but this simple explanation will hardly account for the facts that rifled projectiles are not overturned in flight, and that their axes are kept nearly tangential to the trajectory. In the present state of our knowledge it is impossible to give reasons for drift which will be received by all, as different explanations have been given by various authorities.

When a **projectile** is properly **centered**, its axis of revolution at starting coincides with the initial direction of its motion of translation; but when, from defective arrangements for giving rotation, this is not the case, the axis is set in some accidental direction across the trajectory at the very first, before gravity has had time to act, and produce an angle between the two in a vertical plane. Hence we see that an accidental precessional or gyrating movement is set up, affecting the drift and causing inaccuracy; but this generally becomes less sensible as the range increases, when the projectile steadies down in flight from the friction of the air; for the same reason, that a top carelessly spun with its axis not vertical at first, steadies itself and goes to sleep from the friction of its point on the ground. Centring.

A practical question is, how much **velocity of rotation** should be imparted? If too little is given, the projectile is unsteady in flight, and if there is too much, the axis sets more across the range, thus increasing the the drift—a fact well established by experiment; thus, compare the long and short 8-in. howitzer; the former has a twist of rifling 1 in 35, the latter is much sharper—1 in 16 calibres. The following is taken from the range tables (2000 yards in both cases):— Velocity of rotation.

Howitzer.	Elevation.	Charge.	Weight of projectile.	Drift.	Time of flight.
		lbs.	lbs.	yards, right.	secs.
8-in. of 46 cwt.	13° 5′	10	185	27·6	9·6
8-in. of 70 cwt.	13° 14′	6½	180	18·0	9·0

The charges are so different in the two cases, because the longer bore consumes the powder more profitably than the other, as more expansions are allowed for it in the piece.

The velocity necessary for stability of rotation has been investigated mathematically by Professor Greenhill, who has obtained a formula,* from which Table III, *see* p. 280, has been calculated for various projectiles; the twist, however, for Service projectiles has been determined practically by trial and error, as very many causes contribute to make its calculation extremely difficult; but this table has been of assistance, particularly now that long projectiles are being employed, requiring a greater twist than the shorter ones.

* *See* Pro. R.A.I., vol. x, No. 7, and vol. xi, No. 2.

Energy of Rotation.

Energy of rotation.

The **energy of rotation** is $\frac{w(k\omega)^2}{2g}$, in which w is the weight of the projectile in lbs., k is the radius of gyration in feet, and ω is the angular velocity: from this expression it is apparent that a shell possesses more energy than a shot of the same weight rotating at the same speed, since its radius of gyration is greater; for the same reason a flywheel, which stores up energy, is provided with a heavy rim; thus great energy can be put into it, while the angular velocity is moderate, since its radius of gyration is considerable.

We have seen (p. 129) that the initial velocity of rotation depends directly on the muzzle velocity of translation; but the two do not decrease together in flight, the rotation being very slightly decreased even at long ranges; this has been found by noting scorings on graze.

Magnitude of couple formed by the resistance of the air.

The **overturning moment** depends on the magnitude of the couple formed by the resistance R.GM (*see* Fig. 6, p. 132)—the resultant of the resistances multiplied by its distance from the centre of gravity. Evidently if either the resultant resistance or the arm are increased, the moment will be increased, provided the other conditions remain unaltered.

The resultant resistance may be increased—

1. By increasing the diameter.
2. By increasing the velocity.
3. By alteration of shape of head and roughness of surface.

(1.) When the diameter is increased, the arm of the couple is lengthened if the proportions of the projectile remain the same; but if the weight remains constant the projectile will be shorter, and the arm may not be increased in length. If the weight is increased the energy of rotation, which resists the overturning tendency, is increased.

(2.) When the M.V. is increased, the angular velocity increases, but the energy of rotation increases as the *square* of the angular velocity.

(3.) Changes in the exterior shape, &c., cause differences in the magnitude of P; a smooth surface is preferable.

The chief practical alteration in the value of the couple is due to the length of the arm; this is less when the centre of gravity is far forward, and greater when it is far back. Speaking generally, the length of the arm of the couple increases with the length of the projectile.

The length of the arm is altered by the surface of the projectile being slightly roughened towards the base by the rotating ring, causing the centre of resistance to act further back than would otherwise be the case: this is the principle of the feathers on arrows mentioned previously.

Practical summary.

Disregarding theories and explanations, the established **facts connected with drift** are as follows:—With projectiles having service heads and right-handed rotation it is to the right; other things remaining unchanged, it is found that the greater the twist the greater the drift; the new smooth and well centred B.L. projectiles drift less than those M.L. shells which are roughened by studs; at extreme ranges the drift always rapidly increases, and the projectiles become

unsteady in flight, owing to the greater curvature of the trajectory and consequent increase of deflecting forces.

The minor effects of the resistance of the air and the rotation of the projectile are that the axis of the latter remains nearly tangential to the trajectory; this is estimated from the fact that the holes made in wooden targets are as nearly as possible circular, even when the angle of descent is considerable, and also from watching by eye the appearance of projectiles fired with low velocities at a considerable elevation. The point of the projectile is believed to be a little above the trajectory, and to the right of it.

CHAPTER XII.—THE RESISTANCE OF THE AIR.

(*See* also Part II, Chapter III, and Tables IV to XI, pp. 281 to 305.)

THE **resistance of the air** to a projectile in flight is of great importance, as it causes great decrease in range and accuracy, as well as loss of energy in the projectile. Were it not for this pressure or resistance, a bird could not fly, and feathers would fall to the ground as fast as heavy shot.

It is immaterial, as far as this pressure is concerned, whether air moving at a certain rate impinges on a body at rest, or whether the body moves with an equal velocity through still air; for instance, the pressure produced on the face on a still day by meeting the atmosphere when riding fast, is just the same as if you stand still, and the wind blows with an equal velocity, since the *relative* motions of the air and the face remain unchanged. The powerful effects of wind pressures are well known: the maximum (and that only in short gusts) registered at the Royal Observatory, Greenwich, is 53 lbs. on the square foot, due to a velocity of about 61 miles an hour, on the 14th October, 1881 (or 0·368 lb. on the square inch and a velocity of 90 feet per second).

But rifled projectiles frequently attain a velocity of 2200 f.s. when the resistance of the air is calculated to exert a pressure of some 28 lb. per square inch; about 76 times as much as the storm pressure noted above. It is thus evident that a very considerable resistance immediately starts up in a still atmosphere to hinder the onward progress of a projectile in flight.

This resistance depends on—

1. The sectional area of the projectile.
2. The velocity of the projectile.
3. The shape of the projectile and its steadiness in flight.
4. The density of the atmosphere.

Sectional area.

(1.) Other things remaining the same, it is found, as might be expected, that the greater the **sectional area** exposed, the greater is the resistance: the sectional area of an elongated projectile is circular, and it increases as the square of the diameter, or we may say the resistance varies as d^2 (d = diameter in inches).

Velocity of the projectile.

(2.) No law has been discovered which tells how the resistance is affected by the **velocity**; the increase is very considerable and fairly regular.

Very many formulæ founded on the results of experiments have been put forward by the many eminent men of various nations, who have at different times given their attention to this difficult subject; but it is only within the last 20 years, when readily recording electric instruments for finding velocities (and hence resistances) have been employed, that sufficient experimental data have been accumulated, from which to deduce results useful for general application.

The plan at present* adopted is that of Mr. Bashforth, founded

* *See* Pro. R.A.I., 1866, p. 161.

on elaborate experiments with an instrument of his own invention, in which the times taken by a projectile to pass over *several* consecutive intervals between screens in *each* trajectory are recorded; "in this way each experiment supplies the means of testing the accuracy of the results, which are wholly wanting, when only two intervals are measured, and that by two different instruments."* This latter plan had previously been adopted.

It is found as a first approximation that the resistance of the air varies as the cube of the velocity; this is nearly true for a certain range of velocity (but not generally); the exact amount of the resistance at any velocity, is obtained by the use of a coefficient *K*, which varies with the velocity. The value of *K* has been found by calculation from the results of many experiments at various velocities between 100 f.s. and 2900 f.s. (*vide* Table IV, p. 281).

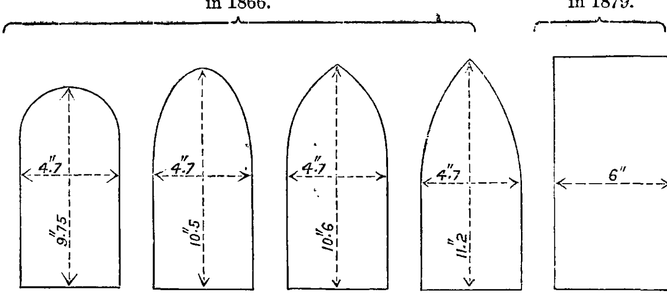
(3.) If the projectile presents a pointed or curved head towards the opposing air the resistance is less than if it is flat. An instance of this is afforded by the ordinary cup wind gauge used at observatories and elsewhere, which owes its motion solely to the difference of pressure, which the same wind causes upon similar sectional areas at the ends of equal arms. Since the cups are concave to the wind on one side of the axis and convex to it on the other side, a difference of pressure is felt on the opposite arms of the instrument, and rotation ensues. Mr. Bashforth has found that at a certain velocity, if the resistance to a (1) hemispherical head is represented by unity, the resistances to others of the same diameter but with heads of different shapes are as under:—

Shape of projectile and its steadiness in flight.

Fig. 1.

Forms of Shot experimented with

in 1866. in 1879.



(1) Hemispherical head. (2) Hemispheroidal head. (3) Ogival head (1 diam.) (4) Ogival head (2 diams.) (5) Flat head.

	(2) Hemispheroidal head proportions shown in Fig. 1 ..	0·78
	(3) Ogival head, curve of head, one diameter of projectile	0·83
	(4) ,, ,, two diameters ,,	0·78
and	(5) A flat head, according to recent experiments, encounters about twice the resistance opposed to an ogival of two diameters	1·53

The proportionate resistance offered to different shaped heads varies with the velocity, thus the advantage of the ogival over the hemispherical head is a little

---

* Report of the Committe of Reference Bashforth Chronograph.

greater at a velocity of 1870 f.s. than at a velocity of 1640 f.s. *See* "Supplement to Motion of Projectiles." Bashforth, 1881. Phil. Trans., R. S., 1868, p. 438; and Reports on Experiments with the Bashforth Chronograph, p. 10, and Final Report Bashforth Chronograph.

For a theoretical investigation of the resistance of the air to heads of different shapes, *see* Exterior Ballistics, by Captain M. Ingalls, U.S. Artillery.

From this it will be noticed that the hemispheroidal and the ogival of two diameters experience about the same resistance, both being less than the ogival of one diameter, whose point is sharper than that of the hemispheroidal. Practically it is always found that the resistance of the air is more affected by the slope of the surface joining the head and the body (which should be as gentle as possible), than by the sharpness or bluntness of the actual point.

It has often been found that the results of experiment have not agreed closely with those furnished by the use of the Ballistic Tables VI and VII, pp. 283 and 288, and this is not surprising, because all projectiles have not the same **smoothness of surface, shape of head** and base, and **steadiness in flight,** each of which conditions modifies the resistance of the air. The tables were chiefly made out from experiments with studded M.L. projectiles, but modern smooth B.L. shells are much less retarded in flight. Very approximately correct results may, however, be obtained by the use of a factor, which is obtained as follows:—Series of rounds are carefully fired at several elevations, the ranges are measured, and the times of flight noted as accurately as possible, which is generally to about the tenth of a second. Knowing the elevation and jump of the gun, and the M.V., weight, and diameter of the projectile, the corresponding ranges and times of flight are calculated by the use of the Ballistic Tables VI and VII, pp. 283 and 288, and it is found that a nearly constant proportion, σ, exists in the difference between the results of experiment and calculation. The mean of the factors thus found may be applied in estimating resistances and ranges for guns similar to that experimented with, and a fair approximation to accuracy is thus obtained; the factor σ has been found to be approximately for*

Light R.M.L. shells	0·95
Heavy " "	1·00
Light B.L. "	0·85 to 0·90
Medium " "	0·90 to 0·95
Heavy " "	0·95

A correction is seldom necessary with R.M.L. shells; experiments are now (1886) in progress to test this method of employing the tables with the new B.L. projectiles.

Theoretically the base of a projectile should be much sloped away to lessen resistance and prevent the formation of a vacuum, or partial vacuum in rear, but practically the bases of all shells are nearly flat for convenience of stowage, capacity for bursting charge, or attachment of the gas check.

Density of the air.

(4.) Variation in the **density of the air** causes variation in the resistance.

Table XI, p. 305, gives the factor τ, which must be applied when the atmospheric conditions differ from the normal (thermometer 60° F., barometer 30 inches, and atmosphere two-thirds saturated with

* *Vide* Notes on the Compilation of a Range Table, by Commander May, R.N., Pro. R.A.I., vol. xiv, No. 10.

moisture). It is calculated to give the weight of a cubic foot of air under the altered circumstances $\div 534{\cdot}22$, since $534{\cdot}22$ grains is the weight of a cubic foot under normal conditions.

RESISTANCE.

From what has been said above, it will be seen that the total **resistance** (R) of the air may be represented in mathematical symbols by the expession,— Expression for resistance.

$$R = d^2 \frac{K}{g}\left(\frac{v}{1000}\right)^3 \text{ lbs. avoirdupois, } \ldots\ldots\ldots\ldots \text{(i)}$$

in which

d = diameter of the projectile in inches.
g = acceleration of gravity in f.s.
v = velocity in f.s.
K is a coefficient varying with the velocity, and tabulated in Table IV, page 281.

d^2 is employed since the sectional area varies as the *square* of the diameter; the variable coefficient K allows the use of the simple relation that the resistance varies as v^3, the constant g appears in order to reduce absolute to ordinary gravitation units,* and $(1000)^3$ is inserted simply for convenience, to avoid an unnecessary number of cyphers in the value of K, as otherwise it would be a very small decimal. It is not usual to have different measures of length in the same expression as in the above, in which d is in *inches* and g and v are in *feet* per second, but here it is convenient, and K has been calculated on this understanding.

If p is the pressure in lbs. on what is called the circular inch, *i.e.*, a circle of 1 inch diameter, (i) becomes

$$p = \frac{K}{g}\left(\frac{v}{1000}\right)^3 \text{ lbs. } \ldots\ldots\ldots\ldots \text{(ii)}$$

From this formula Table V, p. 282, has been calculated; the resistance on a projectile of any other diameter can readily be found by multiplying by the square of the diameter in inches.

Example 1

> Thus find the resistance of the air to a 7-inch projectile moving at a velocity of 1230 f.s.
>
> From the table the resistance to a 1-inch projectile moving at that velocity is $6{\cdot}331$ lbs., multiply this by 7^2 or 49, and we have $310{\cdot}22$ lbs. as the resistance to the 7-inch projectile.

Taking into account σ, a factor depending on the shape of the projectile and its steadiness in flight, and τ, a factor depending on the density of the air, *see* Table XI, p. 305, formula (i) becomes—

$$R = d^2\sigma\tau\frac{K}{g}\left(\frac{v}{1000}\right)^3 \text{ lbs. avoirdupois, } \ldots\ldots\ldots\ldots \text{(iii)}$$

the product $\sigma\tau$ is sometimes called the coefficient of reduction.

RETARDATION.

The *effect* of a pressure or resistance (R) acting on a body is to cause acceleration or **retardation** (f) in velocity, and the *amount* of it is known from the elementary law in dynamics expressed by the proportion Expression for retardation.

---

* The absolute unit of force, called the poundal, is that which acting for one second on one pound generates a velocity of one foot per second; the gravitation unit of force contains g poundals.

$$R : w :: -f : g,$$

where w = weight of projectile in lbs.,

$$\therefore \text{ retardation } f = -\frac{R}{w}g.$$

Substituting the value of (R) from (iii), we have

$$f = -\frac{d^2}{w}\sigma\tau K\left(\frac{v}{1000}\right)^3 \quad \ldots\ldots\ldots\ldots \text{(iv)}$$

which is the *rate of loss of velocity* in feet per second, *caused* by the resistance (R) in pounds.

We notice from (i, ii, and iii) that the *resistance* of the air is irrespective of the weight of the projectile; but the *retardation* (*see* iv) is inversely as the weight.

Power of projectiles to withstand the resistance of the air.

Hence with projectiles of the same diameter, moving at the same velocities, the resistances are equal, but the retardation on the heavier is less than on the lighter one. Speaking generally, the weight increases as the length of an elongated projectile; therefore at the same velocity, a long projectile is less retarded, or better *able to maintain its velocity* than a short one of the same diameter. It must be remembered, however, that it takes more work in the bore of the gun to give the same velocity to a heavy than to a light projectile.

If we have two projectiles of the same weight moving at the same velocity, but of different diameters, we notice from (iv) that the shell of smaller diameter is subject to a less retardation than the other. But on the other hand, if the guns are of about the same length, the projectile of larger diameter receives its energy from a smaller charge, since a larger proportion of the total work of the powder is utilised, because more expansions are allowed to it in the larger bore, whose capacity must be greater than that of the piece of smaller calibre,—unless the latter is made longer.

We can now find the resistance and the retardation to projectiles, at any ordinary velocities, by the use of the formulæ (i), to (iv), and the values of K in Table IV, page 281, of σ on page 140, and of τ in Table XI, page 305.

Example 2.

Find the resistance of the air and the retardation to a 12·5-inch R.M.L. Palliser shell moving at a velocity of 1400 f.s. Atmospheric conditions normal.

Take $w = 802{\cdot}25$ lbs.,
$d = 12{\cdot}5$ inches.

Substituting in (i) we have (finding the value of K for a velocity of 1400 f.s. from Table IV, p. 281).

$$\text{Resistance} = (12{\cdot}5)^2\frac{104{\cdot}7}{32{\cdot}19}\left(\frac{1400}{1000}\right)^3\text{lbs.}$$
$$= 1395 \text{ lbs.}$$

Again, substituting in (iv) we have

$$\text{Retardation} = \frac{(12{\cdot}5)^2}{802{\cdot}25}(104{\cdot}7)\left(\frac{1400}{1000}\right)^3$$
$$= 55{\cdot}96 \text{ feet per second.}$$

We have here obtained the resistance and retardation *at the instant* that the velocity is 1400 f.s. The resistance of the air lessens that velocity, and hence it follows that the resistance and retardation do not remain constant, but decrease rapidly as they both contain v^3, *vide* (i) and (iv); it would evidently therefore be false to assume that

the 12·5-inch projectile would be deprived of 1395 × 100 = 139,500 ft. lbs. of energy in passing over 100 feet, or that it would lose 55·96 f.s. of velocity in one second. The amount really lost is less in each case.

Ballistic Tables.

(*See* Tables VI to X, pp. 283—304.)

It is seldom that a knowledge of the resistance and retardation experienced by a projectile when moving at any given velocity is of much practical use, because they do not remain constant; the problem generally required is to find the loss of velocity during a given time or over a given distance under varying resistances and retardations, and this can be done by the use of Tables VI and VII, pp. 283, 288, deduced by the aid of the coefficient K. *Vide* Part II, Chapter III.

Time and Velocity Table.

This table gives the relation between time and velocity, thus on reference to Table VI, page 284, we find that it takes 222·7806 seconds for a certain elongated *unit projectile,* with ogival head $1\frac{1}{2}$ diameters radius, to attain a **velocity** of 700 f.s.* (denoted by T_{700}) from a certain initial **time**. We also find that it takes 231·1367 seconds to attain a velocity of 1200 f.s. (denoted by T_{1200}) from the same initial time, and under the influence of the same variable acceleration: therefore it follows that the time (t) taken to rise from 700 f.s. to 1200 f.s. will be the *difference* of the tabulated time quantities, *i.e.*, 231·1367 − 222·7806 = 8·3561 seconds, and this is of course the same time as it will take for the velocity of the same projectile to fall from 1200 to 700 f.s. under the influence of a *retardation* of equal magnitude.

Since only the *differences* between the times taken from starting are considered, it is immaterial *when* the start actually was made, for it is the *same* for each velocity tabulated in Table VI, under the influence of a variable acceleration equal in magnitude to the retardation caused by the resistance of the air.

It is readily seen that this is expressed generally thus: time of flight, $t = T_V - T_v$. T_V, T_v meaning the time in seconds for unit projectile to attain a velocity of V, or v, f.s., under a variable acceleration equal in magnitude to the retardation caused by the resistance of the air from some initial time.

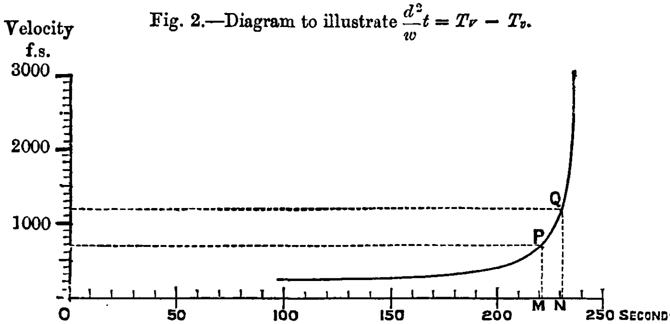
These tables were at first made out on the simpler supposition that a *retardation* was acting; this, however, caused the tabulated numbers to decrease as the velocities increased, which led to possible mistakes in taking differences. Hence the present arrangement, which is somewhat complicated to explain, but just as simple to work with as the other. Foreign tables, which are derived from the English ones, are made out on the original plan. *See* a paper on Gunnery Tables by the author, Pro. R.A.I., vol. xiv, No. 1.

The diagram, Fig. 2, may perhaps render this notation clearer. It consists of a curve, which is obtained by plotting Table VI. The horizontal line is a scale of equal parts, representing seconds, and the vertical ordinates at any point are proportional to the velocity

* 222·7806 seconds are found recorded in the second column of Table VI: to find the corresponding velocity in f.s. the first figures of it (70) are found in the first column headed v, and the last figures (0) is at the head of the column in which the time is recorded.

attained in the time indicated (from a certain initial time) on the scale, under the influence of the variable acceleration f, which is equal and opposite to the retardation.

Fig. 2.—Diagram to illustrate $\frac{d^2}{w}t = T_V - T_v$.



Times of flight.

Thus suppose the *unit projectile* has a velocity of 700 f.s. Draw a horizontal line through 700 on the velocity scale, cutting the curve in some point P, and from P drop a perpendicular cutting the time scale at M, this gives the time as 222·7806 seconds; which means that the projectile attains the stated velocity in this time (counting from some initial time) under the influence of the variable acceleration f.

If we now wish to find how long it will take to attain a velocity of 1200 f.s., we draw a horizontal line through 1200 on the velocity scale, to cut the curve in Q, and drop QN a perpendicular giving a result of 231·1367 seconds on the time scale. Evidently MN represents the time it takes for the velocity to rise from 700 to 1200 f.s. under the influence of the acceleration f, or to fall from 1,200 to 700 f.s. under an equal and opposite retardation $-f$.

If O be the starting point, from which all the Table is calculated;

$$MN = ON - OM,$$

Corresponding to $t = T_V - T_v$,

$$\therefore \text{ time required } t = \text{time over } ON - \text{time over } OM,$$
$$= 231{\cdot}1367 - 222{\cdot}7806,$$
$$= 8{\cdot}3561 \text{ seconds.}$$

For the purposes of Table VI *unit projectile* is one for which $\frac{d^2}{w} = 1$, d being the diameter in inches and w the weight in pounds—say a 9-pr. of 3 inches diameter.

Example 3. How long will it take for the velocity of unit projectile to fall from 1200 f.s. to 700 f.s., atmospheric and other conditions normal?

$$t = T_{1200} - T_{700}$$
$$= 231{\cdot}1367 - 222{\cdot}7806 \text{ secs.}$$
$$= 8{\cdot}3561 \text{ seconds.}$$

Table VI can be used for any other projectile of similar form by employing the value of its $\frac{d^2}{w}$, which is called the ballistic coefficient; for many service projectiles these values are tabulated in Table XVI, p. 313.

Find the velocity of a 40-pr. R.M.L. shell after two seconds. Atmospheric and other conditions normal. Example 4.

$$\text{Given that } M.V. = 1293 \text{ f.s.}$$
$$w = 38{\cdot}09 \text{ lbs.}$$
$$d = 4{\cdot}69 \text{ inches.}$$
$$\text{Consequently } \frac{d^2}{w} = 0{\cdot}5775.$$
$$\text{Substituting in formula } \frac{d^2}{w}t = T_V - T_v;$$
$$\therefore\ 0{\cdot}5775 \times 2 = T_{1293} - T_v.$$

From Table VI $T_{1293} = 231{\cdot}5775$ secs.; for 1293, find 129 under column headed v, and as the last figure is 3, look under column headed 3, and 1·5775 is recorded; 23 in column headed 0 belongs to all, but is not repeated in order to save room; the total amount is therefore 231·5775.

$$\therefore\ T_v = 231{\cdot}5775 - 0{\cdot}5772 \times 2 \text{ secs.}$$
$$= 230{\cdot}4225 \text{ secs.}$$

From Table VI we find this corresponds to

$$v = 1085{\cdot}1 \text{ f.s.}$$

Find the value of $\frac{d^2}{w}$ of a projectile having the same M.V. and time of flight as that in the last example, but with a remaining velocity of 1110 f.s. Example 5.

The formula becomes

$$\frac{d^2}{w} \times 2 = T_{1293} - T_{1110}$$
$$= 231{\cdot}5775 - 230{\cdot}6004$$
$$\therefore\ \frac{d^2}{w} = \frac{0{\cdot}9771}{2}$$
$$= 0{\cdot}4885.$$

If the M.V. of the 10-in. R.M.L. projectile is 1364 f.s. and the final velocity 1200 f.s., find the time of flight. Take $\frac{d^2}{w} = 0{\cdot}242$, atmospheric conditions normal. Example 6.

The formula becomes

$$0{\cdot}242\, t = T_{1364} - T_{1200}$$
$$= 231{\cdot}8591 - 231{\cdot}1367$$
$$\therefore\ t = \frac{0{\cdot}7224}{0{\cdot}242}$$
$$= 2{\cdot}9851 \text{ seconds.}$$

If in the last example the time of flight is 3·2 seconds, what will be the M.V. if the final velocity is still 1200 f.s.? Example 7.

The formula becomes

$$0{\cdot}242 \times 3{\cdot}2 = T_V - T_{1200}$$
$$\therefore\ T_V = 231{\cdot}1367 + {\cdot}7744$$
$$= 231{\cdot}9111$$

whence $V = 1378{\cdot}2$ f.s. This result is obtained by making use of Table IV in an inverse manner: search is made for the tabulated quantity nearest in amount to 231·9111, it is found to be 231·9103, which corresponds to a velocity of 1378 f.s. Working more exactly by taking the difference 231·9111 − 231·9103 = 0·0008 and dividing it by the tabulated difference 0·0036, which corresponds to 1 f.s., we obtain 0·2 f.s. to be added; and we have a total 1378·2 f.s., but it is generally sufficiently accurate to work to the nearest whole number.

The direct and inverse methods of using this table are just the same as with ordinary logarithms.

We thus see from the last four examples that if any three of the four quantities $\frac{d^2}{w}$, t, V, or v, are known, the fourth can be found by aid of Table VI.

Any of the problems of $\frac{d^2}{w}t$ can be done by means of Table IX, p. 297, which is due to Professor Greenhill, who has recalculated Table VI, together with two others, to be hereafter described (pp. 149, 167), in a different manner from the original data. He has welded the three tables into one, and thus gets rid of repetitions; the tabulated quantities only for each change of *ten* feet in the velocity with the "difference" (Δ) in a separate column are given; the effect of this is much the same as employing 4-figure logarithms instead of 6 figures, *i.e.*, a certain loss of accuracy; but this is practically very slight, as the differences (except at the lower limit) vary slowly, and after all the system only claims to be an approximation to the truth. The advantage of Table IX is its shortness, as it only takes five pages, while the longer ones take fourteen, with many more figures in each line. As the tabulated quantities are only given for each change of 10 f.s. in velocity instead of 1 f.s., differences must be taken for intermediate amounts, and thus a little longer time is taken in the computation. Foreign tables are made out for each change of one mètre per second alteration of velocity; as a mètre = 3·2809 feet, they may be said to be graduated to a degree of accuracy intermediate between those of Bashforth and Greenhill.

For instance, the last example worked out by Table IX becomes—

$$\begin{aligned} 0{\cdot}242 \times 3{\cdot}2 &= T_V - T_{1200} \\ \therefore T_V &= 155{\cdot}5858 + 0{\cdot}7744 \\ &= 156{\cdot}3602 \end{aligned}$$

From Table IX the nearest tabulated quantity which is less is ..	156·3294 corresponding to 1370 f.s.
Difference ..	0·0308
From Table IX the difference for 10 f.s. is	0·0363

$\therefore \frac{0 \cdot 0308}{0 \cdot 0363}$ of 10 f.s. $=$ 8·5 f.s. must be added.

Consequently V $= 1378 \cdot 5$ f.s.,
a result almost identical with that given by the other table.

Distance (or Space) and Velocity Table.

Table VII, p. 288, shows the relation between **distance** (or space s) and **velocity**, and is made out on the same plan as Table VI. The tabulated numbers refer to the *distance* (or space) in feet,* over which *unit* projectile must travel from a certain starting point, for its velocity to become v under the influence of a variable acceleration equal to the retardation caused by the resistance of the air.

We could also represent this table graphically by substituting a scale of feet for distances, instead of a time scale of seconds: the curve somewhat resembling that shown in Fig. 2. The notation employed is similar: S_V S_v meaning the space for unit projectile to pass over, to attain a velocity of V, or v, f.s., under the variable acceleration, equal in magnitude to the retardation caused by the resistance of the air f, from some starting point.

As before, the employment of the coefficient $\frac{d^2}{w}$ enables the table to be applied to other projectiles; the factors σ and τ must be employed if necessary in both Tables.

In what *distance* will the velocity of the 40-pr. R.M.L. projectile $\frac{d^2}{w} = 0 \cdot 5775$ fall from 1293 f.s. to 1085 f.s.; atmospheric conditions normal? **Example 8.**

Substituting in the formula

$$\frac{d^2}{w} s = S_V - S_v$$

$$(0 \cdot 5775) s = S_{1293} - S_{1085}.$$

From Table VII

$$s = \frac{42140 \cdot 5 - 40778 \cdot 4}{0 \cdot 5775}$$

$$= 2358 \cdot 6 \text{ feet},$$

$$= 786 \cdot 2 \text{ yards}.$$

In the equation employed in this example we again have four quantities, $\frac{d^2}{w}$, s, V, and v, any three of which being known, the fourth can be found: it is unnecessary to give an example of each case which may occur, as this expression is treated exactly like that containing $\frac{d^2}{w} t$, and a reference to Examples 4, 5, or 7 will show how the other unknown quantities can each be found in different cases.

Suppose the velocity at the middle point between the screens is found to be 1500 f.s. by the Boulengé chronograph, find the $M.V.$ for a flat-headed projectile. The first screen is 150 feet from the muzzle, and the screens are 120 feet apart. $d = 12 \cdot 5$ inches, $w = 815$ lbs. **Example 9.**

* It should be remembered that ordinary ranges are expressed in yards, which must be reduced to feet in order to make use of this table.

he middle point between the screens is evidently distant from the muzzle $150 + \frac{120}{2} = 210$ feet.

Hence the problem reduces to finding the *M.V.* when the remaining velocity and range are known.

$$\frac{d^2}{w} = \frac{(12\cdot5)^2}{817}$$
$$= 0\cdot191$$

Hence

$$0\cdot191 \times 210 = S_V - S_{1500}$$
$$40\cdot11 = S_V - 43162\cdot0$$
$$\therefore S_V = 43202\cdot1$$
$$\text{and } V = 1509 \text{ f.s.}$$

a difference of 9 feet from the velocity at the middle point between the screens: as the projectile is a *flat-headed* one, which experiences more resistance than the projectiles for which the table was made out, it is usual to double this amount, = 18 f.s. (*see* (5), p. 139); this is added to the velocity, 1500 f.s. first found, and we have 1518 f.s. for the real *M.V.*

Having investigated the simpler problems we will consider the effects of variation in the density of the air applicable to both Tables VI and VII.

Example 10. Find the range for the 40-pr. projectile in Example 8, if the thermometer is 35° F. and barometer 30·2 inches, other conditions normal.

On reference to Table XI, p. 305, we find τ, the correction for the altered density of the air, is $1\cdot053 + \frac{2}{10} \times 0\cdot035 = 1\cdot06$.

The formula now becomes

$$\frac{d^2}{w}\tau s = S_V - S_v$$
$$0\cdot5775 \times 1\cdot06 s = S_{1293} - S_{1085}$$
$$\therefore s = \frac{42140\cdot5 - 40778\cdot4}{0\cdot5775 \times 1\cdot06}$$
$$= 2225\cdot1 \text{ feet}$$
$$= 741\cdot7 \text{ yards}$$

a less result than in Example 8, as the air is considerably denser than the normal.

We will now consider also the effect of change in smoothness, shape of head, and steadiness in flight in the case of new type B.L. shell, applicable to both Tables VI and VII.

Example 11. In what distance will the *M.V.* of the 6-inch B.L. projectile fall from 1875 f.s. to 1570 f.s. $\frac{d^2}{w} = 0\cdot45$? Thermometer 45° F., barometer 29·9 inches, and a factor $\sigma = 0\cdot88$.

On reference to Table XI we find τ for the density of the air is $0\cdot997 + \frac{9}{10} \times 0\cdot034 = 1\cdot028$.

The formula now becomes

$$\frac{d^2}{w}\sigma\tau s = S_V - S_v$$

$$0\cdot45 \times 0\cdot88 \times 1\cdot028s = S_{1875} - S_{1570}$$

$$\therefore s = \frac{44735\cdot7 - 43475\cdot}{0\cdot45 \times 0\cdot88 \times 1\cdot028}$$
$$= 3096\cdot5 \text{ feet}$$
$$= 1032\cdot2 \text{ yards.}$$

Any of the problems of $\frac{d^2}{w}s$ can be done by means of Table IX, p. 297, a part of which* resembles Table VII. Differences must generally be taken, as the tabulated quantities are only given for each change of 10 f.s. in velocity.

For instance, the last example worked out by it becomes—

$$0\cdot45 \times 0\cdot88 \times 1\cdot028s = S_{1875} - S_{1570}$$

$$\therefore s = \frac{(43637\cdot61 + \frac{5}{10} \times 38\cdot7) - 42396\cdot40}{0\cdot45 \times 0\cdot88 \times 1\cdot028}$$
$$= 3096\cdot3 \text{ feet}$$
$$= 1032\cdot1 \text{ yards.}$$

A result almost identical with that given by the other Table.

Time and Distance.

No general table connecting **time** and **distance** can be constructed; they can only be linked together by the relation of each to **velocity**, *i.e.*, both Tables VI and VII must be used. Relation between time and distance.

Thus, suppose it is required to find the time of flight of a 64-pr. shell for a range of 1000 yards, $M.V. = 1457$ f.s., $w = 64$ lbs., $d = 6\cdot3$ inches: atmospheric and other conditions normal. Example 12.

We have $\frac{d^2}{w} = 0\cdot6201,$

We must here intermediately find the final velocity from the $\frac{d^2}{w}s$ table, before we can find the time of flight from the $\frac{d^2}{w}t$ table.

Taking the formula—

$$\frac{d^2}{w}s = S_V - S_v,$$

and remembering that s is 3,000 feet as Table IV is made out for *feet*,

$$0\cdot6201 \times 3000 = S_{1457} - S_v.$$

Whence we obtain from Table IV,

$$v = 1127\cdot7 \text{ f.s.}$$

Making use of this value of v in the formula—

$$\frac{d^2}{w}t = T_V - T_v,$$

* For the other parts *see* pp. 146 and 167.

we have

$$0{\cdot}6201 \times t = T_{1457} - T_{1127{\cdot}7},$$

Whence we obtain from Table VI,

$$t = 2{\cdot}3575 \text{ seconds.}$$

Example 13. Suppose in the last example that the thermometer is 80° F. and the barometer 29·5 inches, find the time of flight.

On reference to Table XI we find τ for the density of the air is $0{\cdot}927 + \frac{5}{10} \times 0{\cdot}032 = 0{\cdot}943$.

The formula to employ is

$$\frac{d^2}{w}\tau s = S_V - S_v$$

$$0{\cdot}6201 \times 0{\cdot}943 \times 3000 = S_{1457} - S_v$$

$$\text{whence } v = 1143 \text{ f.s.},$$

shewing a greater remaining velocity than in the previous example, as the atmosphere is less dense than before.

To find the time of flight—
the formula to employ is

$$\frac{d^2}{w}\tau t = T_V - T_v$$

$$0{\cdot}6201 \times 0{\cdot}943t = T_{1457} - T_{1143}$$

$$\text{whence } t = 2{\cdot}337 \text{ seconds,}$$

shewing a less result than in the previous example, as the mean velocity is greater.

If a new type projectile had been employed a suitable value for the factor σ would be employed in making use of both Tables IV and V.

If only the space and time are given for any projectile, the velocity at the beginning and end can only be found with accuracy from the formula and Tables (VI and VII) $\frac{d^2}{w}t$ and $\frac{d^2}{w}s$, by a system of approximation.

Example 14. Thus suppose the range (1032·1 yards) and the corresponding time of flight (1·8067 secs.) of the 6-inch projectile in Example 11 are given, find the *M.V.*

Assume as a first approximation that the mean velocity is the *actual* velocity at the middle point of this range; this would be really true if the cubic law of resistance holds good.

The mean velocity at $\frac{1032{\cdot}1}{2} \times 3$ ft. is then $\frac{3096{\cdot}3}{1{\cdot}8067} = 1714$ f.s.

$\frac{d^2}{w}$ multiplied by the coefficient of reduction is $0{\cdot}45 \times 0{\cdot}88 \times 1{\cdot}028 = 0{\cdot}4071$.

The corresponding *M.V.* is obtained from

$$0{\cdot}4071 \times \frac{3096{\cdot}3}{2} = S \quad - S_{1714}$$

$$\text{whence } V = 1871{\cdot}3 \text{ f.s.}$$

And the final velocity at end of the range is obtained from

$$0{\cdot}4071 \times 3096{\cdot}6 = S_{1871{\cdot}3} - S_v,$$

$$\text{whence } v = 1566{\cdot}7 \text{ f.s.}$$

The time of flight corresponding to these velocities is obtained from

$$0{\cdot}4071\, t = T_{1871\cdot3} - T_{1566\cdot7},$$
$$\text{whence } t = 1{\cdot}8106 \text{ secs., which is } 0{\cdot}0039 \text{ sec. } \textit{too much.}$$

Next assume the mean velocity to be at some other point, say at 550 yards, we obtain on this supposition in the same manner

$$V = 1882 \text{ f.s.}$$
$$\text{and } t = 1{\cdot}8000, \text{ which is } 0{\cdot}0067 \text{ sec. } \textit{too little.}$$

By proportional parts we can now obtain a very close approximation to the real muzzle velocity: thus if m is the amount to be added to the lesser *M.V.*, 1871·2 f.s., we have

$$m : 1882 - 1871{\cdot}3 :: 0{\cdot}0039 : 0{\cdot}0039 + 0{\cdot}0067,$$
$$\text{whence } m = 3{\cdot}9 \text{ f.s. to be added.}$$

And the approximate $M.V. = 1871{\cdot}2 + 3{\cdot}9$
$= 1875{\cdot}1$ f.s., which is almost identical with the *M.V.* given in Example 11.

This plan cannot, however, practically be employed with much exactness over ordinary ranges on account of the difficulty of measuring the time of flight with sufficient accuracy.

A diagram used with a wooden scale derived from Bashforth's tables solves many gunnery problems; no calculations are required, except to find the values of $\frac{d^2}{w}$ (multiplied by the coefficient of reduction $\sigma\tau$ if necessary); it is described on p. 215.

The tables and calculations used in this chapter are based on the assumptions that the projectile travels in a straight line through the resisting air, and point foremost; but neither of these suppositions is really quite correct, for the acceleration of gravity causes the trajectory to curve, and the projectile moves only approximately point first. However, the results obtained by these means are very near approximations to the truth for flat trajectories.

This system was devised by the Rev. F. Bashforth, who accumulated the necessary data from some 500 rounds, and calculated the tables. Experiments and tables on the same method have since been made in Russia and by Krupp, and tables on the same plan have also appeared in Italy, Spain, and in the United States.*

Inclination and Velocity Table.

Table VIII, p. 293, which shows the relation between the **inclination of the trajectory** to the horizontal in degrees and **velocity,** is made out on the same plan as Tables VI and VII. The tabulated numbers refer to the number of degrees through which the inclination to the horizontal must be altered from a certain starting point for the velocity of *unit* projectile to become v under the influence of a variable acceleration, equal to the retardation caused by the resistance of the air.

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* Mr. Bashforth has pointed out in R.A.I. Proceedings, vol. xiii, No. 10, how much Krupp's tables differ from his own at velocities of about 1100 f.s.; at higher velocities the discrepancy between the two tables is not quite so great. *See* p. 4, "Expériences de tir des Acières de Fried Krupp exécutées au polygone de Meppen."

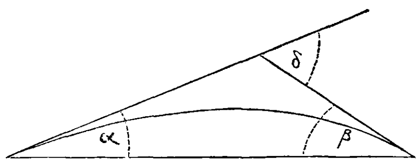
The formula employed is $\frac{d^2}{w}\delta = D_V - D_v$, in which δ (the difference between two tabulated quantities) represents the alteration in degrees in the inclination of a tangent to the trajectory at two points. Thus (Fig. 3) if α and β are the angles of departure and descent respectively, δ the difference between the inclination of the tangents at these points is the angle found by means of this table; and since—

$$\beta = \delta - \alpha,$$

we have a means of finding the angle of descent if the angle of departure and the muzzle and remaining velocities are known; the latter can be found by use of $\frac{d^2}{w}s$ table.

As before, the ballistic coefficient $\frac{d^2}{w}$ enables this table to be of general application: the coefficient of reduction $\sigma\tau$ must also be employed when necessary.

Fig. 3.



Example 15. Find the angle of descent of the 12·5-inch projectile, range 1500 yards, angle of departure 1° 51′. *M.V.* 1575 f.s., $\frac{d^2}{w} =$ 0·1907, atmospheric conditions normal.

From Table VII of $\frac{d^2}{w}s$ we find the remaining velocity at 1500 yards is 1389 f.s.

Employing Table VIII—

$$\begin{aligned} 0{\cdot}1907\ \delta &= D_{1575} - D_{1389}. \\ &= 84{\cdot}1976 - 83{\cdot}4723. \\ &= 0{\cdot}7253. \\ \therefore\ \delta &= 3{\cdot}803^\circ, \\ &= 3^\circ\ 48'. \\ \text{and angle of descent } \beta &= \delta - \alpha \\ &= 3^\circ\ 48' - 1^\circ\ 51', \\ &= 1^\circ\ 57'. \end{aligned}$$

It is sometimes necessary in calculating a trajectory to find the remaining velocity at the vertex, where the inclination to the horizontal is 0.

Example 16. What is the remaining velocity v_0 at the vertex of the 6-inch gun, Mark III, *M.V.* 1850 f.s.?

$d =$ 6 inches, $w =$ 100 lbs., angle of departure 1° 58′,
$\sigma =$ 0·9, atmospheric conditions normal.

$$\therefore \frac{d^2}{w} \times \sigma = 0{\cdot}324,$$

And
$$\begin{aligned} \delta &= 1^\circ\ 58', \\ &= 1{\cdot}967^\circ. \\ \therefore 0{\cdot}324 \times 1{\cdot}967^\circ &= D_{1850} - D_{v0}, \\ \therefore D_{v0} &= 84{\cdot}9226 - D_{v0}, \\ \therefore D_{v0} &= 84{\cdot}2853, \\ \therefore v_0 &= 1602{\cdot}4 \text{ f.s.} \end{aligned}$$

A table connecting altitude with velocity has been devised and calculated by the Italian Captain Siacci; it has been successfully employed in working out trajectories, but it is not given in this book.

CHAPTER XIII.—TRAJECTORIES.

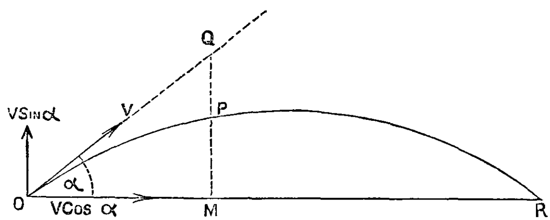
(*See* also Part II, Chapter IV.)

In vacuo.

In vacuo. LET α be the angle of departure, V the muzzle velocity in f.s., then $V \cos \alpha$ and $V \sin \alpha$ are its components in a horizontal and vertical direction respectively.

Suppose the projectile has started from O in direction OQ, and has moved for t seconds, reaching some point P below Q (Fig. 1), and let x and y be the coordinates of P.

Fig. 1.



The horizontal velocity remains uniform (for gravity only acts at right angles to it); and therefore the horizontal distance ($OM = x$) passed over in time t is—

$$x = Vt \cos \alpha. \quad \text{(i)}$$

The vertical distance passed over in the same time is that, which is described with a uniform velocity $V \sin \alpha$, minus the distance $QP = \frac{1}{2}gt^2$, which a body falls in the same time t, under the action of gravity, hence $y = PM = QM - QP$,

$$\text{or } y = Vt \sin \alpha - \tfrac{1}{2}gt^2. \quad \text{(ii)}$$

From these two simple equations, (i) and (ii), everything about the trajectory *in vacuo* may easily be deduced.

We notice that they contain five quantities which may be unknown, viz., x, y, V, α, and t. Any three of these being given, the other two can be found.

Substitute the value of t from (i) in (ii), and we have the equation

$$y = x \tan \alpha - \frac{gx^2}{2V^2 \cos^2 \alpha}$$

Trajectory a parabola. which shows that the **trajectory is a parabola**, with its axis parallel to that of y, and origin at some point O below the vertex, as drawn in Fig. 1. From the symmetrical properties of this well-known curve we can tell that the highest point must be at half the range, and, as the horizontal velocity is uniform, it must be at half the time of

flight, and, also, by symmetry, the angle of descent must be equal to the angle of departure at the same horizontal level.

Expressions are readily calculated for—

$$\text{Total range (in feet) } R = \frac{V^2 \sin 2\alpha}{g}. \quad \ldots\ldots\ldots\ldots\ldots \text{(iii)}$$

$$\text{Total time of flight (in seconds) } T = \frac{2V \sin \alpha}{g}. \quad \ldots\ldots\ldots\ldots\ldots \text{(iv)}$$

$$\text{Greatest height (in feet) } H = \frac{V^2 \sin^2 \alpha}{2g}. \quad \ldots\ldots\ldots\ldots\ldots \text{(v)}$$

These are deduced at once from (i) and (ii)

For (iii) eliminate t and put $y = 0$.
(iv) „ x „ $y = 0$.
(v) „ t „ $x = \frac{1}{2}R$

But these expressions may not be remembered, and it will be best to use only the two fundamental equations (i and ii), when working from memory.

It often seems, in a problem, that only two of the quantities x, y, V, α, t are given instead of the three which are necessary.

Thus, giving the angle of departure 30°, and time of flight 10 seconds— Example 1.

Find $M.V.$; Range; and greatest height.

A moment's consideration shows that in the first two cases $y = 0$, and in the third $t =$ half the total time or $\frac{1}{2}10 = 5$ seconds.

Substitute in equations (ii) and (i) and we have for the first two cases, taking $g = 32{\cdot}2$.

$$0 = V \times 10 \times \tfrac{1}{2} - \tfrac{1}{2} \times 32{\cdot}2 \times 10^2,$$
$$R = V \times 10 \times \frac{\sqrt{3}}{2}$$

whence we obtain

$$V = 322 \text{ f.s.},$$
$$\text{and range } R = 2788{\cdot}6 \text{ feet}$$
$$= 929{\cdot}5 \text{ yards.}$$

For the third case, substitute in (ii) $t = 5$ secs. and $V = 322$ f.s.

$$\therefore \mathrm{H} = y = 322 \times \tfrac{1}{2} \times 5 - \tfrac{1}{2} \times 32{\cdot}2 \times 5^2$$
$$= 402{\cdot}5 \text{ feet.}$$

When the muzzle velocity is very low, as with a mortar or howitzer using a small charge, an approximate result may be obtained at high angles by adopting the parabolic theory and neglecting the resistance of the air, but the range and height obtained will be too large.

Flat Trajectories in Air.

Approximations to flat trajectories in air can readily be obtained, and although the methods are often laborious, no difficult calculations are involved; it need hardly be said that the calculation of a trajectory is a matter of considerable importance.

Time of flight or range known.

Substitute the value $V = \dfrac{gT}{2 \sin \alpha}$ from (iv) in (ii), and we obtain

$$h = \tfrac{1}{2}gt(T - t), \quad \ldots\ldots\ldots\ldots\ldots\ldots \text{(vi)}$$

a useful formula (due to Lt.-Col. Sladen) which, though deduced from the conditions *in vacuo*, gives an approximation for the flatter trajectories in the air, the assumption being that the **vertical component of the velocity is not affected by the resistance of the air.** T is the total time of flight over the whole trajectory, or time to descend again to the horizontal level from which the projectile started; t is the time of flight to some point on the trajectory whose vertical ordinate is h. With guns at ordinary low angles of elevation, even when the *M.V.* is considerable, the vertical component of the velocity is but small, and the retardation caused by the resistance of the air may be neglected in that direction, when close accuracy is not necessary.

With small angles of elevation up to, say, 4° (and more with less accuracy), we may suppose the horizontal component of the muzzle velocity to be the same as the muzzle velocity itself, since $V = V \cos \alpha$ nearly, when α is a small angle.

If the total time of flight is known, we can by aid of equation (vi) find the height above plane of the projectile at any time t, or at any distance s, from the muzzle due to the time t; for, by means of Bashforth's $\dfrac{d^2}{w}t$ table, the remaining velocity (v) is found at time (t); and this enables the distance (s) to be found by the $\dfrac{d^2}{w}s$ table.

If the range is known and not the total time of flight, the latter can be found; for the $\dfrac{d^2}{w}s$ table will give the remaining velocity as before, and then the $\dfrac{d^2}{w}t$ table can be used to find the time of flight.

The greatest height can be found approximately by making $t = \frac{1}{2}T$ in (vi); or by considering that the projectile falls for half the total time of flight, whence—

$$H = \tfrac{1}{2}g\left(\frac{T}{2}\right)^2 \quad \ldots\ldots\ldots\ldots\ldots\ldots \text{(vii)}$$

$$\text{or } H = \frac{gT^2}{8}.$$

$$= 4T^2 \text{ approximately assuming } g = 32 \quad \text{(viii)}$$

This plan gives too great a height, especially for light projectiles; it can only be looked upon as a fair approximation.

This method may be illustrated by examples.

Example 2.

At a range of 1200 yards a 64-pr. R.M.L. shell grazes the top of a traverse 8 feet high, how far beyond will it strike the ground?

M.V. = 1260 f.s., $\dfrac{d^2}{w} = 0{\cdot}595$; atmospheric conditions normal.

If OM (Fig. 2) is 1200 yards, and PM is 8 feet; it is required to find the distance MR.

(1.) Employ $\frac{d^2}{w}s$ formula and Table VII to obtain the velocity at 1200 yards (at P);

this is found to be 998 f.s.

(2.) Employ $\frac{d^2}{w}t$ formula and Table VI to obtain the time (t) to travel 1200 yards (to P);

this is found to be 3·257 seconds.

(3). Employ formula (vi) $h = \frac{1}{2}gt(T-t)$ to find time of flight (T) to end of trajectory (R);

this is found to be 3·410 seconds.

(4.) Employ $\frac{d^2}{w}t$ formula and Table VII, to find the velocity at the end of 3·410 seconds;

this is found to be 991 f.s.

(5.) Employ $\frac{d^2}{w}s$ and Table VII to find (MR), over which the velocity changes from 998 fs. to 991 f.s.;

this is found to be 153 feet,

or 51 yards, the result required.

The reasons for these steps are as follows:—The $\frac{d^2}{w}s$ table must be employed in (5) to find the distance required, but this cannot be done till the final velocity at the end of the range is known from (4). Again, the final velocity can only be found when T, the total time of flight, has been obtained from (3), and this in its turn depends on t, the time to the traverse, from (2), which can be found when the velocity at the traverse is known from (1).

If, instead of a traverse in the last example, a horseman (8 feet high) is supposed to be advancing towards the gun which continues firing at the same elevation, he or his horse may be struck by a direct hit whilst moving over the space from R to M; this is consequently called the **dangerous distance** in gunnery, but the term *margin* is applied in musketry fire. Evidently the flatter the trajectory the

Fig. 2.

greater the dangerous distance, and the greater the probability of hitting, if the range is not accurately estimated, and if consequently the correct elevation has not been given on the tangent scale.

The angle of descent (β) at various ranges is generally known, as it is recorded in range tables (*see* pp. 308, 309); when this is the case the dangerous distance for a height h feet can very readily be found, for it is approximately $h \cot \beta$.

Example 3.

What is the greatest height to which the projectile in the last question will rise above plane? The total time of flight was found to be 3·410 secs.

$\therefore$ Greatest height $= 4(3{\cdot}410)^2$ approximately *see* (viii)
$= 46{\cdot}5$ feet.

Example 4. The 16-pr. R.M.L. has M.V. 1355 f.s. $\frac{d^2}{w} = 0{\cdot}77$

13-pr. R.M.L. „ M.V. 1560 f.s. $\frac{d^2}{w} = 0{\cdot}69$

Find the greatest height of trajectory in each case over a range of 1200 yards.

Find the final velocity from Table VII of $\frac{d^2}{w} s$.

For the 16-pr. it is 989 f.s.; for the 13-pr. it is 1106 f.s.

Employ these values with Table VI of $\frac{d^2}{w} t$ to find time of flight.

For the 16-pr. it is 3·187 secs.; for the 13-pr. it is 2·775 secs.

Substitute these values of T in equation (viii) $H = 4T^2$, and we find the greatest heights are—

For the 16-pr. 40·6 feet: for the 13-pr. 30·8 feet.

This example demonstrates the superior flatness of a high velocity projectile, which has a long dangerous distance and increased probability of hitting the target.

Example 5. A 38-ton gun whose muzzle is 15 feet above the surface of the water is aimed and fired at the middle of the side of a ship 30 feet high and 1000 yards distant. By mistake the range is estimated to be 1100 yards, and elevation on the tangent scale is given accordingly. Will the ship's side be hit, and, if so, where? Take

$$\frac{d^2}{w} = 0{\cdot}193. \qquad \text{M.V.} = 1575 \text{ f.s.}$$

This is the same as finding the height of the ordinate at 1000 yards, when the total range is 1100 yards.

First, find the whole time of flight (T) from the $\frac{d^2}{w} s$ and $\frac{d^2}{w} t$ tables for a range of 1100 yards.

$$0{\cdot}193 \times 1100 \times 3 = S_{1575} - S_v, \text{ whence } v = 1435 \text{ f.s.}$$
$$0{\cdot}193t = T_{1575} - T_{1435}, \text{ whence } T = 2{\cdot}1948 \text{ secs.}$$

Next find the time (t) to travel 1000 yards.

$$0{\cdot}193 \times 1000 \times 3 = S_{1575} - S_v, \text{ whence } v = 1447 \text{ f.s.}$$
$$0{\cdot}193t = T_{1575} - T_{1447}, \text{ whence } t = 1{\cdot}9912 \text{ secs.}$$

Then from equation (vi)—

$$h = \tfrac{1}{2} \times 32{\cdot}19 \times 1{\cdot}9912(2{\cdot}1948 - 1{\cdot}9912$$
$$= 6{\cdot}525 \text{ feet,}$$

or the side of the ship will be hit at a point 6·525 feet above the middle (which is level with the muzzle of the gun firing), or at 15 − 6·525 feet = 8·475 feet below the top.

This may also be found approximately by supposing $h = 300 \tan \beta$, if β is the angle of descent; but this will give rather too large a result, as it supposes the projectile to travel in a straight line for the last 100 yards of its course.

A whole trajectory may be plotted out by finding the heights of ordinates at various distances from the muzzle, and sketching in the curve.

Example 6. Thus, find the height of the trajectory of the Martini-Henry bullet, at intervals of 100 yards for a range of 500 yards.

Weight of bullet = 480 grains = 0·06857 lb., since 7000 grains troy = 1 lb. avoirdupois. d = 0·45 inch, $M.V.$ = 1353 f.s., which is about the maximum velocity attained with the service charge,

$$\text{consequently } \frac{d^2}{w} = 2{\cdot}9532.$$

First, find the time over the whole range from the $\frac{d^2}{w}s$ and $\frac{d^2}{w}t$ tables.

$$2{\cdot}9532 \times 500 \times 3 = S_{1353} - S_v, \text{ whence } v = 880{\cdot}2 \text{ f.s.}$$
$$2{\cdot}9532t = T_{1353} - T_{880{\cdot}2}, \text{ whence } t = 1{\cdot}4355 \text{ secs.}$$

The time, t_1, over 100 yards is then found thus—

$$2{\cdot}9532 \times 100 \times 3 = S_{1353} - S_{v1}, \text{ whence } v_1 = 1196{\cdot}9 \text{ f.s.}$$
$$2{\cdot}9532t_1 = T_{1353} - T_{1196{\cdot}9}, \text{ whence } t_1 = 0{\cdot}2362 \text{ secs.}$$

And the height of the trajectory at 100 yards is found from (vi), p. 156, thus—

$$h = \tfrac{1}{2} \times 32{\cdot}19 \times 0{\cdot}2362\,(1{\cdot}4355 - 0{\cdot}2362)$$
$$= 4{\cdot}56 \text{ feet.}$$

In a similar way the heights at 200, 300, &c., yards for ranges of 500 and 1000 yards have been calculated and recorded in the annexed table.

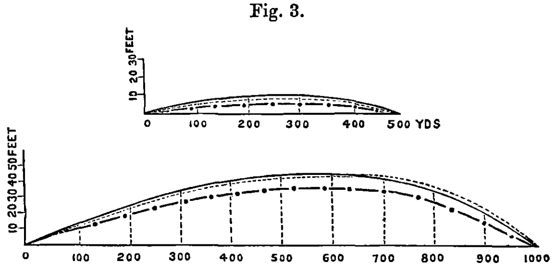
For purposes of comparison, the same has been done for the Mauser rifle, which fires a bullet of 382 grains, diameter 0·435 inch, and M.V. 1526 f.s.

TABLE A.

Distance.	Martini-Henry.				Mauser.			
	Velocity.	Time of flight.	Height of trajectory.		Velocity.	Time of flight.	Height of trajectory.	
			Range 500 yds.	Range 1000 yds.			Range 500 yds.	Range 1000 yds.
	f.s.	secs.	ft.	ft.	f.s.	secs.	ft.	ft.
0	1353·0	0·0000	0·00	0·00	1526·0	0·0000	0·00	0·00
100	1196·9	0·2362	4·56	12·01	1311·6	0·2124	3·96	10·79
200	1074·1	0·5015	7·53	23·35	1141·7	0·4584	6·73	21·49
300	996·8	0·7923	8·20	33·11	1024·8	0·7372	7·51	31·25
400	934·9	1·1037	5·89	40·70	949·1	1·0418	5·51	39·05
500	880·2	1·4354	0·00	45·27	883·8	1·3704	0·00	44·13
600	831·6	1·7866	—	45·25	826·9	1·7213	—	45·70
700	787·7	2·1572	—	42·98	776·2	2·0963	—	43·00
800	746·9	2·5481	—	34·74	729·6	2·4943	—	35·19
900	708·6	2·9602	—	20·72	685·9	2·9193	—	21·22
1000	672·2	3·3951	—	0·00	644·7	3·3710	—	0·00

Fig. 3 represents the above plotted to scale, with the trajectories sketched in; the continuous line represents the Martini-Henry, the plan dotted the Mauser, and in addition the chain dotted line is the proposed Enfield-Martini. (The vertical scale is ten times greater than the horizontal.)

Fig. 3.



From the above it appears that at the shorter ranges the Mauser has the advantage in flatness of trajectory, length of "margin," and smallness of angle of descent, owing to its high muzzle velocity.

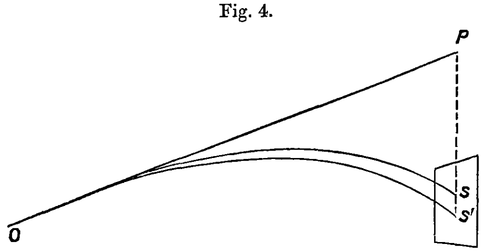
But at longer ranges the advantage lies with the Martini, which fires the heavier bullet ($w \div d^2$ or the *sectional density* being greater) and it, consequently, is less retarded by the air; at 1000 yards the "margin" is longer, and the angle of descent smaller with the Martini than with the Mauser.

The proposed Enfield-Martini rifle has M.V., 1550 f.s., and, as the bore is small (0·4 inches), with a bullet of 384 grains, the sectional density is considerable, and the trajectory is flatter at all ranges than with either of the other rifles.

Further progress has been made in flatness of trajectory by many modern rifles; for instance the Hebler has a small bore of 0·307 inches and a bullet of only 225 grains, but the M.V. is 1942 f.s.; the greatest height over a range of 500 yards is only 4·6 feet, while at 1,000 yards it is but 30 feet.

Example 7 The Martini-Henry rifle bullet strikes a vertical target at 500 yards at a certain spot when the M.V. is 1353 f.s., how much lower will the point of mean impact be, if the M.V. is only 1300 f.s., the elevation and other conditions being the same in both cases?

Fig. 4.



Find the striking velocity at the target in each case by use of Table VII of $\frac{d^2}{w}s$;

for the high velocity it is 880 f.s.; for the low velocity 864 f.s.

Employ these values with Table VI of $\frac{d^2}{w}t$ to find the times of flight; they are respectively

1·4369 secs. and 1·4747 secs.

If gravity does not act in flight, each bullet would reach the point P (Fig. 4); but as gravity *does* act, they will hit at points S and S' such that PS and PS' are the heights fallen through during the times of flight; find PS and PS' by the use of the formula $h = \frac{1}{2}gt^2$, and we have—

$$PS' = 35{\cdot}00 \text{ feet.}$$
$$PS = 33{\cdot}22 \quad ,,$$
$$\therefore \text{ the } \textit{difference } SS' = 1{\cdot}78 \quad ,,$$
$$= 21{\cdot}36 \text{ inches}$$ is the height of one point of mean impact above the other.

On the 21st July, 1881, two rounds of Palliser shell were fired at Shoeburyness from the 12-inch B.L. guns of 43 tons at an elevation of 3° 7′, when the mean time of flight was found to be 6·2 seconds, and the mean range was 3274 yards. Example 8.

What time of flight and range would be expected from calculation? The muzzle velocity was known to be 1855 f.s., the weight of the shell 715 lbs., and its diameter 12 inches. Hence $\frac{d^2}{w} = 0{\cdot}2014$.

Assume the jump to be 5 minutes, the vertical component of velocity is 1855 sin 3° 12′ = 103·5 f.s. This is entirely destroyed by gravity in time $\frac{103{\cdot}5}{g} = 3{\cdot}217$ seconds = time of ascent; and supposing the time of descent to be equal to this, the total time of flight = 6·434 seconds. This method of solution can only be applied with approximation to accuracy when the vertical component of velocity is small, as the resistance of the air in a vertical direction is disregarded; and better results are obtained with the larger projectiles than with those of smaller calibre, as the retardation is, in general, much less.

The horizontal component of velocity = 1855 cos 3° 12′ = 1852 f.s. and the horizontal final velocity is found from,—

$$0{\cdot}2014 \times 6{\cdot}434 = T_{1852} - T_v,$$

whence v, the final horizontal velocity = 1377 f.s.

And the range is found from

$$0{\cdot}2014\, s = S_{1852} - S_{1377},$$
$$\text{whence range } s = 10{,}261 \text{ feet}$$
$$= 3{,}420 \text{ yards,}$$

which is some 146 yards in excess of the mean obtained by experiment:—thus an approximation to the true range can be calculated.

Niven's Method.

The before-mentioned methods do not give very exact results, as the resistance of the air *does* act in a vertical direction upon even the heaviest projectiles. Niven's plan, which is described in Part II, Chapter IV, p. 269, is more accurate, and should be employed for angles of departure above 15°; for lower angles good results can be obtained from the following abbreviated method:—

Having given the angle of departure, the muzzle velocity, and value of $\frac{d^2}{w}$ of the projectile; it is required to find the range, velocity, angle of descent, and time of flight.

Fig. 5.

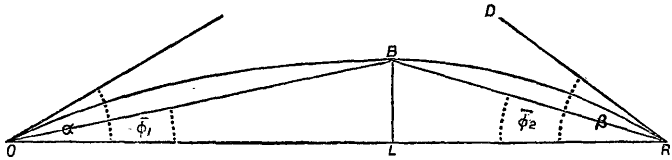


Fig. 6.

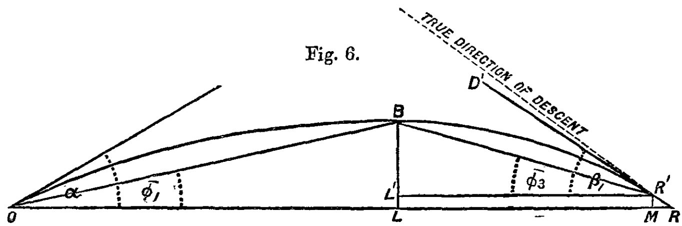
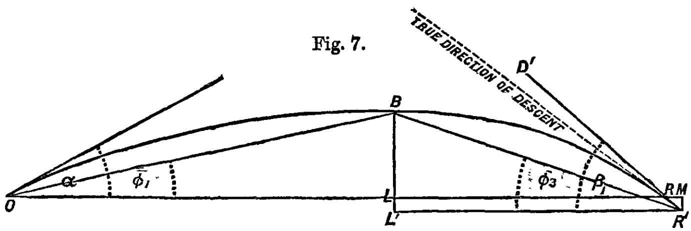


Fig. 7.



Let OBR be the true trajectory (Fig. 5) with angles of departure and descent α and β respectively; from the vertex draw BL perpendicularly and join OB, BR.

Consider the trajectory in two parts, one on either side of the vertex, one ascending and the other descending. As the angle of departure is known the ascending part is readily calculated, and the height of the vertex can then be found: for purposes of calculation an angle of descent β_1 must be *assumed*, which will be either *less*, as in Fig. 6, or *more*, as in Fig. 7, than the true angle (the trajectories are the same in Figs. 5, 6, and 7). The range from O and the distance below the vertex of R′, where the tangent to the trajectory is inclined at the assumed angle of descent, can then be found by calculation: it will be near to R, the true end of the range.

The distance of R below the vertex being known: the projectile is supposed to travel in a straight line, inclined at the assumed angle of descent for the short distance RR′ (Figs. 6 and 7), until it is at the

correct distance below the vertex, and the range of R is thus determined, being very approximately OL + LM $\pm$ MR.

The remaining velocity, true angle of descent, and time of flight can then be found.

Let angle BOL	$= \bar{\phi}_1$.	Figs. 5, 6, and 7.
BRL	$= \bar{\phi}_2$.	Fig. 5.
BR′L′	$= \bar{\phi}_3$.	Figs. 6 and 7.
D′R′L′	$= \beta_1$.	Figs. 6 and 7 (assumed angle of descent).
OL	$= x_1$.	Figs. 5, 6, and 7.
LM	$= x_2$.	Figs. 6 and 7.
MR	$= x_3$.	Figs. 6 and 7.
BL	$= y_1$.	Figs. 5, 6, and 7.
BL′	$= y_2$.	Figs. 6 and 7.
LL′	$= y_3$.	Figs. 6 and 7.
OB	$= s_1$.	Figs. 5, 6, and 7.
BR′	$= s_2$.	Figs. 6 and 7.
RR′	$= s_3$.	Figs. 6 and 7.

The steps of the method are as follows :—

(1.) (Fig. 5.) Find the remaining velocity v_0 at the vertex from

$$\frac{d^2}{w}\delta = D \quad - D_v,$$

δ being the angle of departure in degrees,
V the M.V.,
and v the horizontal velocity v_0 at vertex.

(2.) Find the distance s_1 from

$$\frac{d^2}{w}s = S_V - S_v.$$

(3.) Find the value of $\bar{\phi}_1$ from the approximation

$$\bar{\phi}_1 = \frac{\alpha}{2} + \frac{1}{3} \cdot \frac{V - v_0}{V + v_0} \cdot \alpha.$$

(4.) Find the distance x_1 and the height y_1 from the relations

$$x_1 = s_1 \cos \bar{\phi}_1. \qquad y_1 = s_1 \sin \bar{\phi}_1.$$

(5.) (Fig. 6 or 7.) *Assume* an angle of descent β_1 about $\frac{1}{3}$ greater than the angle of departure. If the projectile is one from a heavy gun with a small value for $\frac{d^2}{w}$, or if the range is short, or the muzzle velocity low, a less value should be taken: under opposite conditions a rather greater value should be assumed for the angle of descent.

(6.) (Fig. 6 or 7.) Find the remaining velocity at R′ with this assumed angle of descent from

$$\frac{d^2}{w}\delta = D_V - D_r,$$

δ being the angle of descent β_1,
V the horizontal velocity v_0 at vertex,
v the remaining velocity at R′.

(7.) (Fig. 6 or 7.) Find the distance BR′ or s_2 from

$$\frac{d^2}{w}s = S_V - S_v.$$

(8.) (Fig. 6 or 7.) Find the value of $\bar{\phi}_3$ from the approximation

$$\bar{\phi}_3 = \frac{\beta_1}{2} - \frac{1}{3}\frac{v_0 - v}{v_0 + v}\beta_1.$$

(9.) (Fig. 6 or 7.) Find the distance x_2 and the height y_2 from the relation

$$x_2 = s_2 \cos \bar{\phi}_3. \qquad\qquad y_2 = s_2 \sin \bar{\phi}_3.$$

(10.) The value of y_2, furnished by (9), will differ a little from that given for y_1 by (4), which is the true height: the difference between them, y_3 (Fig. 6 or 7), is the distance of R′ above (or below) R, the true end of the range: if the projectile is supposed to move in a straight line for the short distance s_3, inclined at an angle β_1 to the horizontal,, an amount $y_3 \cot \beta_1$ must be added to (or subtracted from) $x_1 + x_2$ for the true range. If R and O are at different levels, the difference between their heights must be added to (or subtracted) from y_3.

The remaining velocity at the end of the true range R, the angle of descent and time of flight are then readily found.

(11.) To find the remaining velocity: knowing the velocity at R′ and the distance s_3 (Fig. 6 or 7), the remaining velocity, v, is found from

$$\frac{d^2}{w}s = \text{[illegible]} - S_v.$$

(12.) To find the angle of descent, knowing the velocities at R and R′ (Fig. 6 or 7), find the change in inclination between these points from

$$\frac{d^2}{w}\delta = D_V - D_v,$$

and the result δ is added to (or subtracted from) the angle of descent β_1 *assumed* in (5), in order to find the true angle of descent β at R.

(13.) To find the time of flight: knowing the velocities at O and at R, find the time from

$$\frac{d^2}{w}t = T_V - T_v.$$

Example 9. If Example 8 is worked out by this method the steps are as follows:—

(1.) Find remaining velocity at vertex from

$$0{\cdot}2014 \times 3{\cdot}2 = D_{1855} - D_v,$$
whence $\qquad v_0 = 1603{\cdot}5$ f.s.

(2.) Find the distance s_1 from

$$0{\cdot}2014 \times s = S_{1855} - S_{1603{\cdot}5},$$
whence $\qquad s_1 = 5145$ feet.

(3.) Find the value of $\bar{\phi}_1$ from the approximation

$$\bar{\phi}_1 = \frac{3\cdot2}{} + \frac{1}{3} \cdot \frac{1855 - 1603\cdot5}{1855 + 1603\cdot5} \times 3\cdot2^\circ.$$

whence $\bar{\phi}_1 = 1^\circ\ 41'$.

(4.) Find the distance x_1 and the height y_1 from the relations

$$x_1 = 5145 \cos 1^\circ\ 41'. \qquad y_1 = 5145 \sin 1^\circ\ 41'.$$

whence $x_1 = 5142$ feet. and $y_1 = 151\cdot1$ feet.

(5.) *Assume* an angle of descent (β_1) of $3^\circ\ 51' = 3\cdot85^\circ$.

(6.) Find the remaining velocity of R′ with this assumed angle of descent from

$$0\cdot2014 \times 3\cdot85 = D_{1603\cdot5} - D_v.$$

whence $v = 1397\cdot8$ f.s.

(7.) Find the distance s_2 from

$$0\cdot2014\, s = S_{1603\cdot5} - S_{1397\cdot8},$$

whence $s_2 = 4666$ feet.

(8.) Find the value of $\bar{\phi}_3$ from the approximation

$$\bar{\phi}_3 = \frac{3\cdot85}{2} - \frac{1}{3} \cdot \frac{1603\cdot5 - 1397\cdot8}{1603\cdot5 + 1397\cdot8} \times 3\cdot85.$$

whence $\bar{\phi}_3 = 1^\circ\ 50'$.

(9.) Find the distance x_2 and the height y_2 from the relations

$$x_2 = 4666 \cos 1^\circ\ 50'. \qquad y_2 = 4666 \sin 1^\circ\ 50'.$$

whence $x_2 = 4664$ feet. and $y_2 = 149\cdot3$ feet.

(10.) Find the difference (1·8 feet) between heights y_1 (151·1 feet), and y_2 (149·3 feet). An amount $y_3 \cot \beta_1$ $(1\cdot8 \cot 3^\circ\ 51') = 26\cdot7$ feet, must be added to the sum of $x_1 + x$ (5142 + 4664) for the range; this is

$$5142 + 4664 + 26\cdot7 = 9832\cdot7 \text{ feet range.}$$
$$= 3277\cdot6 \text{ yards range.}$$

(11.) Find the remaining velocity at R from

$$0\cdot2014 \times 26\cdot7 = S_{1397\cdot8} - S_v.$$

whence $v = 1396\cdot7$ f.s.

(12.) Find the angle of descent from

$$0\cdot2014\delta = D_{1397\cdot8} - D_{1396\cdot7},$$

whence $\delta = 1\cdot5'$.

adding this amount to the assumed angle of descent $3^\circ\ 51'$, we obtain $3^\circ\ 52\cdot5'$ for the true angle of descent.

(13.) Find the time of flight from

$$0\cdot2014t = T_{1855} - T_{1396\cdot7},$$

whence $t = 6\cdot1241$ seconds.

Thus collecting the results we have—

Range, **3277·6 yards.**
Remaining velocity, **1396·7 f.s.**
Angle of descent, **3° 52·5′.**
Time of flight, **6·1241 seconds.**

The results obtained by this method are more accurate, and agree more nearly with the results of actual practice than those furnished by the shorter plan adopted in Example 8, p. 161.

In practical calculations, account should be taken of the value of σ, and also for τ, the factor for atmospheric conditions, in connection with $\frac{d^2}{w}$, as previously explained; in the example just considered the coefficient of reduction $\sigma\tau$ is evidently about unity. When the projectile rises to a considerable height, good results are obtained by supposing the mean barometric pressure to be that which is at two-thirds of the maximum height of the trajectory: in order to find this an estimation must be made of the total time of flight, and then the greatest height can be found with sufficient accuracy for this purpose from formula (viii) $H = 4T^2$. The height of the barometer may be assumed to decrease 1 inch for each 1000 feet of elevation.

Abbreviated Method.

Endeavours have been made to obtain approximations to the more curved trajectories in a simple manner. The difficulty is to decide what approximate assumptions may be made in the course of the calculations, without causing considerable changes in the results.

Captain MacMahon, R.A., has investigated this subject, and an example of his method of using the tables is now given: this plan will suffice for a fair approximation, but it is not so accurate as the longer plan just described.

Example 10. An 8-inch howitzer shell, $w = 180$ lbs., $d = 8$ inches, is required to have an angle of descent of 15°, and a striking velocity of 600 f.s. Find the M.V., the angle of departure, and the range.

Here $\frac{d^2}{w} = 0{\cdot}3556.$

Hence

$0{\cdot}3556\delta = D_{v0} - D_v$ if v_0 be velocity at vertex
v ,, ,, end of range,

or $0{\cdot}3556 \times 15 = D_{v0} - D_{600}$

whence $D_{v0} = 58{\cdot}3343$
$\therefore v_0 = 641{\cdot}4$ f.s.

Though it is generally sufficiently accurate to work to the nearest whole number, when the velocities are low it may be necessary to take the first place of decimals; this is done as follows:— The tabulated amount next below 58·3343 is 58·2878 (corresponding to 641 f.s. velocity); the difference between these amounts is 0·0465; the difference between 58·2878 and 58·4046 (the tabulated quantities for 641 and 642 f.s. respectively) is 0·1168. Consequently $\frac{0{\cdot}0465}{0{\cdot}1168} = 0{\cdot}4$ is the amount to be added to 641 f.s., and we have 641·4 f.s.

In the ascending branch of the trajectory the projectile is retarded in a vertical direction by gravity and by some mean resistance of the air, while the resistance opposes gravity in

the descending arc, and therefore lengthens the time of descent, but as an approximation we may assume the time to reach the vertex is half the total time of flight, we have

$$\frac{d^2}{w}\,\frac{t}{2} = T_V - Tv_o, \qquad \text{if } V \text{ represents the } \mathit{M.V.}$$

and $$\frac{d^2}{w}\,\frac{t}{2} = Tv_o - T_v,$$

whence $T_V = 2Tv_o - T_v$.

Substituting the values of v_0 and v,

$$T_V = 2T_{641\cdot4} - T_{600},$$
$$= 2\{220\cdot5740 + \tfrac{4}{10}\,0\cdot0403\} - 218\cdot7957$$
$$= 222\cdot3845$$

The nearest tabulated quantity which is less is 222·3614 (corresponding to 688 f.s.), and the difference between them is 0·0231; the tabulated difference for one whole number is 0·0358. Consequently $\frac{0\cdot0231}{0\cdot0358} = 0\cdot6$ is the amount to be added to 688 f.s., and we have 688·6 f.s.

whence $V = 688\cdot6$ f.s.

To find the angle of departure

$$0\cdot3556 \times \delta = D_{688\cdot6} - D_{641\cdot4}$$
$$\therefore\ \delta = \frac{63\cdot2519 + \tfrac{6}{10}\,0\cdot0949 - 58\cdot3343}{0\cdot3556}$$
$$= \mathbf{13\cdot99^\circ}$$
$$= \mathbf{13^\circ\ 59'}.$$

To find the range

$$0\cdot3556 = S_{688\cdot6} - S_{600}$$
$$\therefore\ s = \frac{33989\cdot6 + \tfrac{6}{10}\,24\cdot5 - 31701\cdot8}{0\cdot3556}$$

= **6475 feet**

= **2158 yards.**

Any of the problems of $\frac{d^2}{w}\delta$ can be done by means of Table IX, page 297, a part* of which resembles Table VIII. Differences must nearly always be taken, as the tabulated quantities are only given for each change of 10 f.s. in the velocity.

For instance, the last example worked out by it with the same steps becomes

$$0\cdot3556 \times 15 = D_{v0} - 52\cdot9200$$

whence $D_{v0} = 58\cdot2540$

The nearest tabulated amount which is less is 58·0866 (corresponding to 640 f.s.), and the difference from it is 0·1671. The tabulated difference for 10 f.s. is 1·1496. Consequently $\frac{0\cdot1674}{1\cdot1496}$ of 10 f.s. = 1·5 f.s. is the amount to be added to 640 f.s., and we have 641·5 f.s.

= 641·5 f.s.

* For the other parts *see* pp. 146 and 149.

Now assuming, as before, that the time to reach the vertex is half the total time of flight

$$T_V = 2T_{v0} - T_v$$
$$= 2T_{641\cdot5} - T_{600}$$
$$= 2\{145\cdot0012 + 0\cdot15 \times 0\cdot4020\} - 143\cdot2660$$
$$= 146\cdot8570$$

The nearest tabulated amount which is less is 146·5399 (corresponding to 680 f.s.), and the difference from it is 0·3171. The tabulated difference for 10 f.s. is 0·3576. Consequently $\frac{0\cdot3171}{0\cdot3756}$ of 10 f.s. = 8·5 f.s. is the amount to be added to 680 f.s., and we have 688·5 f.s.

whence $V = 688\cdot5$ f.s.

To find the angle of departure

$$0\cdot3556\,\delta = D_{688\cdot5} - D_{641\cdot5}$$
$$\therefore\ \delta = \frac{(62\cdot3902 + 0\cdot85 \times 0\cdot9629) - 58\cdot2540}{0\cdot3556}$$
$$= \mathbf{13\cdot93^\circ}$$
$$= \mathbf{13^\circ\ 56'}.$$

To find the range

$$0\cdot3556s = S_{688\cdot5} - S_{600}$$
$$\therefore\ s = \frac{(32722\cdot10 + 0\cdot85 \times 224\cdot96) - 30631\cdot99}{0\cdot3556}$$
$$= \textbf{6415 feet}$$
$$= \textbf{2129 yards.}$$

The conclusions arrived at by the long and by the abridged tables are thus seen to be nearly the same: this result might be expected, as both are founded on the same data. Too much reliance must not, however, be placed on these calculations, as when the trajectories become considerably curved, as in this example, this method only furnishes a rough approximation. Consulting the range table (*see* p. 309) for the 8-inch howitzer of 70 cwt. (and taking differences), an elevation of 13° 50′ (which with a supposed jump of $7\frac{1}{2}$ minutes would give an angle of departure of 13° 57·5′, which is the mean of the angles furnished by the two methods) and M.V. 688·6 f.s., the range is found to be 1970 yards, and the angle of descent 15° 46′, results from which our calculations differ by about +8·8 and −4·9 per cent. respectively.

Estimates of ranges founded on any calculations should always be corrected from the results of practice.

Rockets.

Cause of motion.

The *motion of a rocket* in the air (like the recoil of a gun) is due to the fact that the momentum of the gas and smoke in one direction causes momentum of the rocket in an opposite direction.

The accuracy of rockets is by no means good, and this nature of fire has hardly improved of late years. Unlike other projectiles, the life-saving rocket (which has a long stick) flies *up* a cross wind—since the light stick is blown more out of the true direction than the heavy head; the rocket then points and flies against the wind.

CHAPTER XIV.—ACCURACY OF FIRE.

(See also Part II, Chapter IV, p. 274, and Tables XII to XV, p. 306.)

ALL authorities are agreed that accuracy of fire is of the greatest importance. Lord Wolseley stated in his orders to the troops previous to the Ashantee War: "Every shot that is not deliberately aimed not only encourages the enemy, who would soon learn to despise a fire that did them no injury, but it seriously affects the efficiency of the force: if ammunition were to run short, a stop would be put to our further advance."

Absolute certainty of hitting the same spot at each round is impossible of attainment, as several causes of error exist, which cannot be got rid of, even under the most favourable conditions. Accuracy of fire is thus a comparative term—it is said to be good when a group of projectiles, fired under as nearly as possible the same conditions, strike the target or the ground close together. The adoption of rifled guns very greatly increased good shooting: this is specially the case with B.L. projectiles and with R.M.L. projectiles provided with gas-checks.

Causes of Inaccuracy.

The **chief causes of inaccuracy**, which may exist on the experimental practice ground, where all the conditions are (generally) most favourable, are as follows:—

1. Want of accuracy in the gun, and variation in the mounting.
2. Want of uniformity in the ammunition.
3. Errors in laying.
4. External causes, such as wind.

(1.) *Want of Accuracy in the Gun and Mounting.*

(i.) Unsteadiness of the mounting on firing, giving rise to a variable "jump," which alters the angle of departure.

(ii.) As the gun becomes hotter by continued firing, more and more of the heat produced in explosion must in each succeeding round be employed in expanding the gases and producing pressure; consequently in rapid firing each round should range a little further than the one preceding it, but this is not found to be the case in practice, as other causes are also at work.

(iii.) The chief cause of inaccuracy is want of centering in the projectile, *i.e.*, it does not revolve about its own longer axis; if this is the case, unexpected lateral pressures are caused by the resistance of the air, and the shooting is greatly impaired.

(2.) *Want of Uniformity in the Ammunition.*

(i.) Certain proportions of length in a *projectile* and distribution of its mass, give rise to irregularity in flight. With the first new type B.L. shells some of the copper of the Vavasseur rotating ring was squeezed back in thin ridges as the projectile passed down the bore, and on leaving the muzzle this often fanned out, *i.e.*, was turned up at right angles to the surface of the shell by the pressure of the powder gas at the muzzle, and thus impaired the accuracy of flight. Driving bands are now provided with canneleurs which give relief to

the metal, and prevent the formation of the ridges in rear. Variations in weight will give different ranges; consequently, all projectiles for the same piece are made as nearly as possible the same weight. Eccentricity from defective casting will also give variable results.

(ii.) The errors due to the charge of powder in the *cartridge* are made as small as possible by—

(*a.*) Using powder of the same brand throughout the experiment.
(*b.*) By giving the same space for each charge, to secure uniformity of gravimetric density.

(3.) *Errors in Laying.*

These are rendered very small by always employing the same trained men to lay the guns at experimental practice, and by using sights capable of fine laying in connection with well-defined targets.

(4.) *External Causes.*

Wind and difference of barometric pressure on different days, are calculated for, but they give rise to inaccuracy: in squally and gusty weather it is difficult to make good shooting—especially with howitzers.

Range and Accuracy.

With the object of finding out, in a general way, the relative accuracy of service ordnance and ammunition, and also for compiling range tables, a gun of each nature, when introduced into the Service, is sent to Shoeburyness, and series of rounds are fired at several different elevations for **range and accuracy**, with its service ammunition (*see* Table XIII, p. 307). Ten rounds, or sometimes only 5, are fired at each elevation; when firing slowly, or when the wind or other atmospheric conditions are altering rapidly, it seems advisable to split up series of 10 or more into groups of 5 each, and to consider each group separately as regards errors in range and direction.

Mean ranges and deviations are then obtained for each elevation; the difference of each round from these *means* give the *error* and the mean of the *errors* of the series gives an estimate of the accuracy.

To take a practical example. On the 18th December, 1885, the 8-inch B.L. gun, Mark IV, of 14 tons, was fired with a corrected elevation of 5° 36′. Length of bore, 29 calibres; charge, 105 lbs. prism[1] powder; Palliser projectile, 210 lbs.; length of shell, 21 inches; muzzle velocity, 2045 f.s.

Table A.

No. of round.	Range.	Difference from mean.	Deviation right.	Difference from mean.
	yds.	yds.	yds.	yds.
1	4968	22·8	24·4	3·0
2	4954	8·8	21·6	0·2
2	4962	16·8	22·8	1·4
4	4908	37·2	20·0	1·4
5	4934	11·2	18·4	3·0
	24726	96·8	107·2	9·0
Mean ..	4945·2	19·4	21·4	1·8

To explain—the second column in the above table gives the actual ranges. The mean range is obtained by adding all together and dividing by 5, since 5 rounds were fired.

The third column contains the *difference* of each round, irrespective of sign, from the mean range just found. The mean of these differences is then obtained, and called the mean error in range or mean longitudinal error. Evidently, if all the projectiles fall nearly at the same range, this mean error must be small. Mean errors. Longitudinal.

The fourth column gives the lateral deviation from the direction in which the line of sight points; the mean deviation is at the bottom of this column.

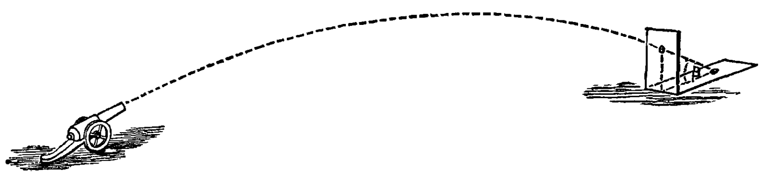
And the fifth column gives the differences from this mean, with a mean at the bottom called the mean error in deviation or mean lateral error. Lateral.

Collecting the results from the Table A we have—

Mean range	**4945·2 yards.**
Mean longitudinal error	**19·4 ,,**
Mean deviation right	**21·4 ,,**
Mean lateral error	**1·8 ,,**

When the position of the point of mean impact on the horizontal plane is known, Fig. 1 shows how the magnitude of the angle of descent determines the position of the point of mean impact on a vertical target. Thus if β be the angle of descent, and if the horizontal target is struck at a distance l from the vertical one, the latter will be struck at a height which equals $l \tan \beta$.

Fig. 1.



The angle of descent of the 8-inch projectile at 4945 yards is known to be 7° 25′.

$$\therefore \textbf{Mean vertical error} \ldots\ldots\ldots = 19{\cdot}4 \tan 7^\circ 25'$$
$$= \mathbf{2{\cdot}5 \text{ yards.}}$$

The calculation of the value of 19·4 tan 7° 25′ can easily be effected by the employment of the four-figure logarithms of numbers and tangents (*see* Table XIX, pp. 318 to 321).

Vertical targets are employed at the shorter ranges, because they may then be of moderate size, and errors due to inequalities of the ground or sand are eliminated, but at long ranges targets cannot generally be made large enough to catch all the rounds. Vertical targets.

The **point of mean impact** on a horizontal target is the intersection of the lines of mean range and mean lateral deviation, and on a vertical target it is the intersection of the lines of mean vertical and lateral deviation.

The **mean trajectory** is that which strikes the point of mean impact: it is the central one of all the trajectories fired at the same elevation. Point of mean impact and mean trajectory.

Fig. 2.

In Fig. 2 the central white line represents the mean trajectory, the dark band is that in which 50 per cent. of the trajectories lie; the shaded band is that which contains 75 per cent., while the outer band contains the remainder. The width of these bands is exaggerated in Fig. 2, for the sake of showing them clearly.

A practical illustration of dispersion or want of accuracy is given by a fire-hose, in which the stream of water is more separated at the end than at the beginning of its course through the air: the whole trajectory being a kind of bent cone, with its apex at the nozzle. The greater the range the greater is the loss of accuracy.

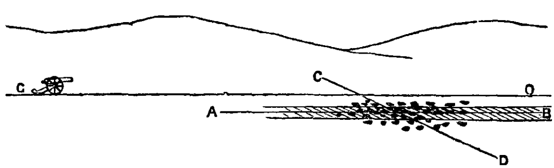
Probable Zones.

It can be shown by the theory of probabilities (*see* Part II, Chapter IV, p. 274), that if each of the three mean errors is multiplied by the factor 1·69, the breadths of zones (of infinite length), which will contain 50 per cent. of the hits, are obtained.

The mean longitudinal error × 1·69 gives the width of the **length zone**; the mean lateral error × 1·69 gives the width of the **breadth zone**; the mean vertical error × 1·69 gives the width of the **height zone**.

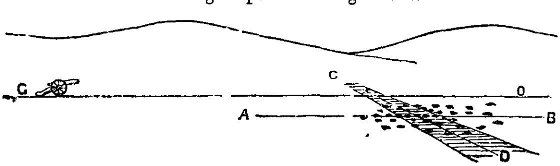
Thus, if GO, Figs. 3 and 4, represents the direction of the gun, and AB be a straight line parallel to it, at a distance equal to the mean

Fig. 3.
Showing 50 per cent. breadth zone.



lateral deviation, and CD be a straight line at right angles, at a distance from the muzzle equal to the mean range; then if the zone in Fig. 3, called the breadth zone, and that in Fig. 4, called the length

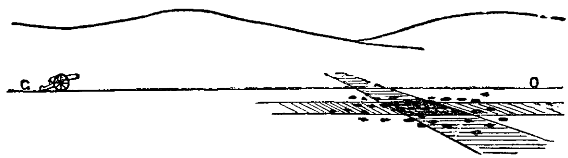
Fig. 4.
Showing 50 per cent. length zone.



zone, each contains 50 per cent. of the hits on the surface of the ground, their widths must be 1·69 times the mean lateral error, and

Fig. 5.

Showing 50 per cent. length zone and 50 per ceut. breadth zone intersecting and forming 25 per cent. rectangle.



1·69 times the mean longitudinal error respectively. AB and CD are the central lines of these zones.

If now we look at Fig. 5, where these zones are superposed, we see a rectangle which must contain 50 per cent. of 50 per cent., or 25 per cent. of the total number of hits. In a similar manner the 25 per cent. rectangle on a vertical target is made up of the intersection of the 50 per cent. breadth and height zones. Probable rectangle.

The relative accuracy of different guns at different ranges is estimated by the dimensions of this rectangle, which is called the 25 per cent. probable rectangle, or more shortly **the probable rectangle.**

At each range there is a horizontal and a vertical probable rectangle; the width of each is the same, as each has the same breadth zone, but the relation of the length of one to the height of the other depends on the angle of descent.

Employment of the Probability Table.

Supposing we know the width of a 50 per cent. zone; one which contains only 20 per cent. will not be exactly $\frac{2}{5}$ or 0·4 of its width, but it will be only 0·38 of it (*see* Table XII, p. 306, in which a **probability factor** 0·38 corresponds to 20 per cent. ;—this means that the 20 per cent. zone is 0·38 of the width of the 50 per cent. zone). Also, a zone double as wide (or one which has a factor 2) as that of 50 per cent. will not contain all the hits, but only a little over 82 per cent.

If the mean errors in two directions are given, we can find two 50 per cent. zones, and hence the probable number of hits, on a rectangular target of any dimensions, in the plane of the zones.

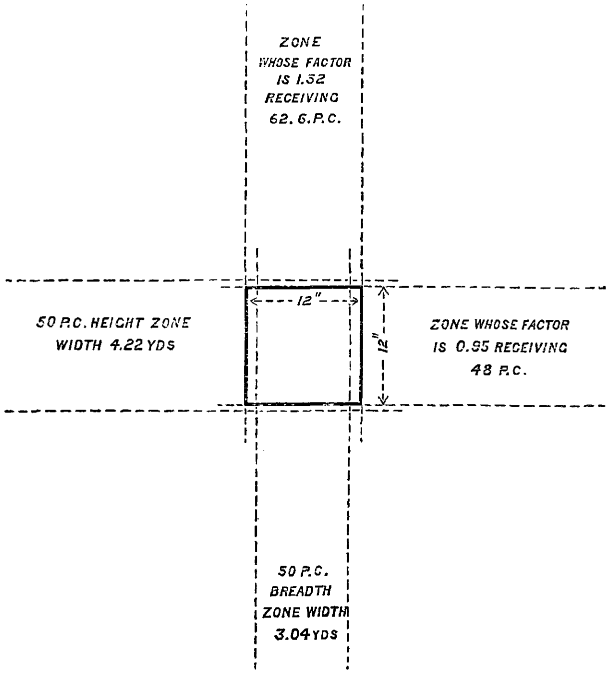
These principles can best be explained by examples.

What percentage of rounds from the 8-in. B.L. gun may be expected to hit a 12 feet square vertical target at a range of 4945 yards, when the range is accurately known? Example 1.

We find (*see* p. 171) the mean lateral and vertical errors are 1·8 and 2·5 yards wide respectively; consequently the 50 per cent. breadth and height zones are respectively

$1{\cdot}8 \times 1{\cdot}69 = 3{\cdot}04$ yards, and $2{\cdot}5 \times 1{\cdot}69 = 4{\cdot}22$ yards (Fig. 6).

Fig. 6.



The width of the breadth zone which just contains the target is 12 feet or 4 yards, and the proportion of its width to that of the 50 per cent. zone is $\frac{4}{3{\cdot}04} = 1{\cdot}32$—a result *greater* than unity, as the zone is *wider* than the standard one of 50 per cent. This proportion 1·32 is the *factor* of the new zone, and from Table XII, page 306, we find that it will receive 62·6 per cent. of the total fired.

The width of the height zone which just contains the target is also 12 feet or 4 yards, and the proportion of its width to that of the 50 per cent. zone is $\frac{4}{4{\cdot}22} = 0{\cdot}95$—a result *less* than unity, as the zone is *narrower* than the standard one of 50 per cent. This proportion 0·95 is the *factor* for the new zone, and from Table XII we find that it will receive 48 per cent. of the total fired.

Consequently the target, which is the rectangle formed by the intersection of the two new zones, will receive 62·6 per cent. of 48 per cent. or $\frac{62{\cdot}6}{100}$ of $\frac{48}{100}$ of the total fired = 30 per cent.; in other words, at the long range of 4945 yards 30 shots in 100 will hit the target.

Example 2.

What would be the maximum number of hits on a long wall 12 feet high, if it is fired at by the 8-in. howitzer of 70 cwt., at a range of 1600 yards, with a charge of $10\frac{1}{2}$ lbs.?

The range table (Table XIV, p. 309) gives the 50 per cent. height zone as 2·31 yards.

The proportionate width of the zone formed by the wall to one of 50 per cent. is, in this case—

$$\frac{4}{2 \cdot 31} = 1 \cdot 73.$$

From Table XII we find that this factor corresponds to 75·6 per cent., which is the result required; as it is a *long* wall, we suppose that no misses occur from lateral deviation.

Example 3.

What length of wall in the last question would practically not fail to be hit on account of lateral errors?

We find from the range table that the width of the 50 per cent. breadth zone is 0·62 yard, and therefore one which is 3·82 times as wide will take 99 per cent., or practically we may say that one which is four times as wide will take all; or $0 \cdot 62 \times 4 = 2 \cdot 48$ yards length of wall, will be long enough to receive all lateral errors, provided that the point of mean impact is in the middle of it.

Example 4.

If a zone of a certain width catches 20 per cent. of the rounds fired, how much wider must another be to catch 80 per cent.

From Table XII we find that the 20 per cent. zone is 0·38 of the width of the 50 per cent. zone; and also that the 80 per cent. zone is 1·9 times the width of the same standard.

Consequently the widths of the zones in question must be to each other as

$$0 \cdot 38 : 1 \cdot 9, \text{ or as } 1 : 5 \cdot 0,$$

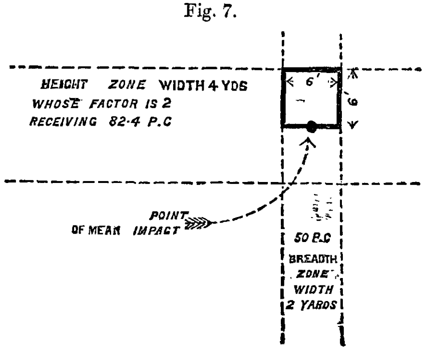
or the 80 per cent. zone must be five times as wide as the 20 per cent. zone.

Example 5.

If the breadth and height zones are each 2 yards wide, what percentage of hits may be expected on a target 6 feet square, if the point of mean impact is in the middle of the lower edge?

The 50 per cent. breadth zone just includes the target (Fig. 7).

Fig. 7.



The height zone to be employed must be one which is double the height of the target, for then the point of mean impact will be in the middle of the zone, and the whole of the target will be included. The factor for this zone is evidently 2, corresponding from Table XII to a percentage of 82·4: but as the target only lies on one-half side, we must take half the percentage or 41·2 per cent.

Consequently on the target we have

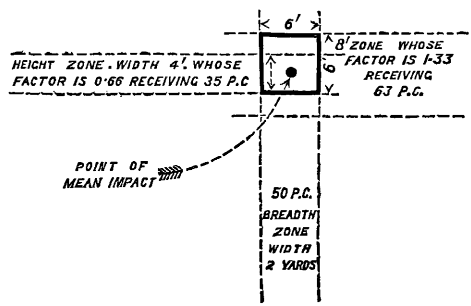
50 per cent. of 41·2 per cent. = 20·6 per cent.

Example 6.

If in the last example the point of mean impact is raised 2 feet, what improvement may be expected in the shooting?

As before, the 50 per cent. breadth zone just includes the target (Fig. 8).

Fig. 8.



For the height zones—take one 4 feet wide and another 8 feet wide. Then the target will be contained in the lower half of the 4-feet zone and in the upper half of the 8-feet height zone.

The 4-ft zone has a factor $\frac{4}{6} = 0{\cdot}67$, and it receives, 35 per cent. hits, according to Table XII.

The 8-ft. zone has a factor $\frac{8}{6} = 1{\cdot}33$, and it receives, 63 per cent. hits, according to Table XII.

As the height band, which just contains the target, is composed of the halves of these two zones, it must receive $\frac{1}{2} \times 35 + \frac{1}{2} \times 63 = 49$ per cent. hits; and the whole target has

50 per cent. of 49 per cent. = 24·5 per cent.

—an improvement of 3·9 per cent. of the total fired, or 19 per cent. more hits on the target than in the last case, for $\frac{3{\cdot}9}{20{\cdot}6} = \frac{19}{100}$ nearly.

Example 7.

Suppose there are two targets, 6 feet wide, and of the same height, 3 feet apart, fired at by a gun at a certain range; the width of the 50 per cent. breadth zone being 6 feet.

Which plan will give the most hits on the target?*

(1.) To aim at the middle of one?

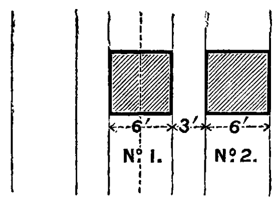
(2.) To aim midway between the two?

Taking the first supposition (Fig. 9)—

(Neglecting height errors which bear the same proportion throughout.)

50 per cent. must fall on the target (No. 1) aimed at.

Fig. 9.



To find out how many fall on the other (No. 2), take a zone just to include No. 2 target, the centre being the middle of No. 1. This zone must be 24 feet wide.

The factor for this zone is $\frac{24}{6} = 4$, corresponding to 100 per cent. from Table XII. Now take a zone, having the same centre, which will just *not* include the second target; this must be 12 feet wide, and the factor is $\frac{12}{6} = 2$, corresponding to 82·4 per cent. from Table XII.

Hence (100 − 82·4) per cent. fall in the spaces between the two zones, but since there is a target in only one of these spaces, we must divide by two to find out how many fall on the second target.

$$\frac{100-82{\cdot}4}{2} = 8{\cdot}6 \text{ per cent.}$$

In this case then we have—

On No. 1 target	50	per cent.
On No. 2 target	8·6	„
Total	58·6	per cent. on both.

On the second supposition (Fig. 10)—

Take a zone to include both targets, this must be 15 feet wide.

And the factor is $\frac{15}{6} = 2{\cdot}5$ or 90·8 per cent. We must subtract from this the numbers which fall in the zone between the targets 3 feet wide, and are lost.

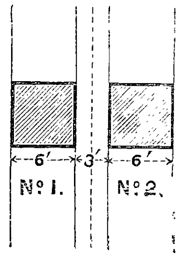
Here the factor is—

$$\frac{3}{6} = 0{\cdot}5, \text{ corresponding to } 26{\cdot}5 \text{ per cent.}$$

* This of course is a most exceptional case; it is similar to a problem given in the Revue d'Artillerie.

The difference of these two percentages, *i.e.*, 90·8 − 26·5 = 64·3 per cent., fall on the two targets in this case, which is more than on the first supposition.

Fig. 10.



Example 8.

If the bull's-eye of a target is struck at the first round, does it prove, that of necessity the correct elevation is exactly given?

No; since the first round may happen to be one which falls shorter or further than the average. Of course it cannot be far out; but deductions drawn from individual rounds cannot be depended on for exactness.

Small Arms.

Small arms.

With **small arms** very large numbers of rounds are manufactured, and a certain proportion are fired from standard rifles to test the accuracy of the ammunition; the powder is first tested separately.

A different method is followed in this case.

Rifles in rests are laid on large vertical targets at 500 yards, and series of 20 rounds are fired from each under as nearly as possible the same conditions. The vertical targets are 24 feet square, and are divided into smaller squares of 3 feet side, and these again into smaller ones 6 inches square. At the firing point, a man with a telescope notes the point of impact of each round, and plots it down at once on a diagram, as shown on Plate III. When all the rounds have been fired, he measures the horizontal and vertical distance of each hit, from some vertical and from some horizontal line, obtains the mean of each of these distances, and by their intersection, finds the point of mean impact; thus far the plan resembles that previously described; but after this the plan adopted for small arms differs from the other; the *radial* distance of each hit from the point of mean impact is measured, and the mean of these radial distances gives the **figure of merit** furnished by the particular sample of ammunition employed; thus in Plate III the figures of merit of 20 shots from a Snider rifle on 22nd December, 1884, was 1 foot 1·35 inches, while the figures of merit of a sample of ammunition for the Martini rifle on the same day was 9·25 inches; as these were average samples of ammunition in each case, the figures may be taken as a fair comparison of the accuracy of the two rifles.

Figure of merit of ammunition.

If a steady wind is blowing it makes but little difference, as though the point of mean impact is altered, the radial distances from this point remain unchanged, or nearly so. Gusts of wind, however, spoil the shooting.

Plate III.

PROOF OF AMMUNITION.

COMPARISON OF ACCURACY OF SNIDER & MARTINI-HENRY RIFLES.

No. 8. Diagram.

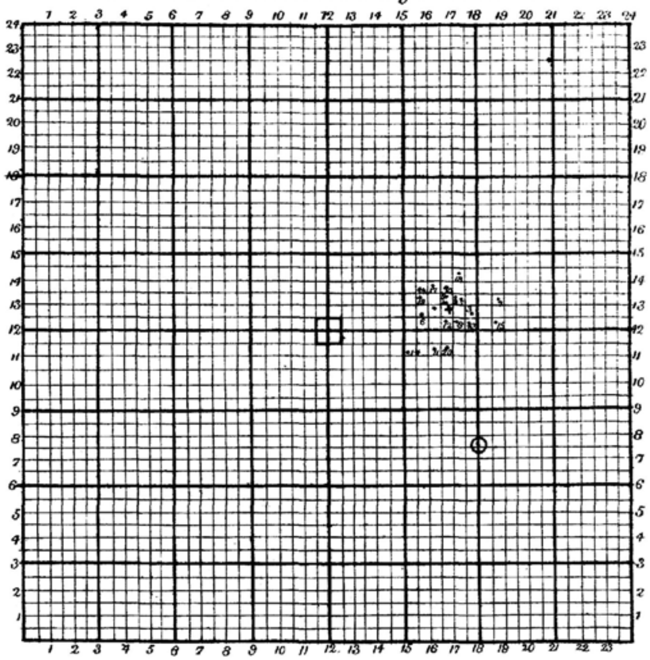
L. C. TARGET. C.

DATA.

Rifle 36^a description of S. Enfield Rifle
Powder 70 Grains R. F. G
Bullets ·573 Clay Plug.
Lubrication Wax
Cartridge Mark IX
Fired from Fixed Rest
Hits 20
Missed
Mean Absolute Deviation 13·35 Inches
Number of Shots 41 to 40
Stripped
Fouled
Range in Yards 500
Elevation 1° 35' 1° 13' 20'
Point Aimed at O

Direction of Range N. N. E.
Wind { *Direction of N. E.* / *Strength of 0 to 1 lb.* / *Character of Steady*
Thermometer 39°
Barometer 30·284
Degree of Humidity 92
Solar Heat

Record of 20 Shots from 33 Guager work of the 20th



SNIDER RIFLE

ROYAL LABORATORY WOOLWICH.

22nd DECEMBER 1884.

Showing the Deviation of each Shot.

No. OF SHOT	HORIZONTAL MEASUREMENT		VERTICAL MEASUREMENT		ABSOLUTE DEVIATION FROM POINT OF MEAN IMPACT	
1 H	1	9	4	1	0	3
2 H	2	3	4	3	0	6
3 H	2	9	3	7	0	11
4 H	0	9	4	7	1	3
5 H	1	9	4	5	0	7
6 H	0	9	3	7	1	1
7 H	3	10	4	4	2	1
8 H	2	2	3	5	0	6
9 H	2	2	4	5	0	8
10 H	1	9	4	8	0	11
11 H	1	3	2	4	1	7
12 H	1	9	3	5	0	5
13 H	2	7	3	3	0	11
14 H	0	1	2	1	2	5
15 H	3	8	3	4	1	11
16 H	1	3	3	11	0	7
17 H	1	3	4	9	1	2
18 H	2	3	5	3	1	7
19 H	0	7	4	4	1	4
20 H	1	9	2	5	1	5
	36	4	76	5	22	3
	1	9·8	3	9·86	1	1·35

No. 10 Diagram

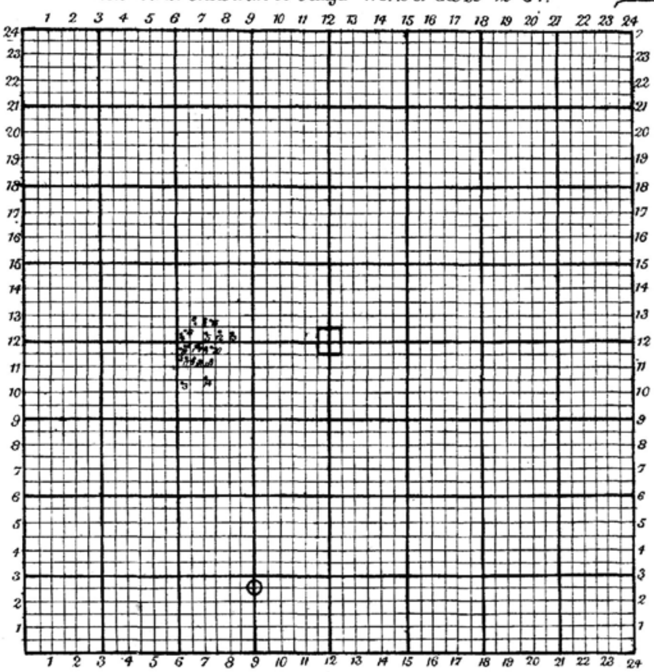
Left TARGET G.

DATA.

Rifle 28 description of M. Henry Rifle
Powder 85 Grains R. F. G.²
Bullets 480 Grains
Lubrication Wax
Cartridge Mark III
Fixed from Fixed Rest
Hits 20
Missed
Mean Absolute Deviation 9·25 Inches
Number of Shots 61 to 80
Stripped
Fouled
Range in Yards 500
Elevation 1° 22' 1° 2' 40
Point Aimed at O

Direction of Range N. N. E
Wind { *Direction of N. E.* / *Strength of ½ to 1½ lbs* / *Character of Steady*
Thermometer 43°
Barometer 30·284
Degree of Humidity 78
Solar Heat

Record of 20 Shots from 53 Gauger Work of the 20-12-84.



Targets 24 × 24 feet.
Squares 6 × 6 inches.

MARTINI-HENRY RIFLE.

ROYAL LABORATORY WOOLWICH

22nd DECEMBER 1884.

Showing the Deviation of each Shot.

No. OF SHOT	HORIZONTAL MEASUREMENT		VERTICAL MEASUREMENT		ABSOLUTE DEVIATION FROM POINT OF MEAN IMPACT	
1 H	0	10	2	11	0	1
2 H	0	1	2	3	0	11
3 H	0	2	1	5	1	7
4 H	0	3	3	3	0	10
5 H	1	2	3	3	0	6
6 H	0	9	3	7	0	9
7 H	1	3	2	10	0	3
8 H	1	2	3	10	1	1
9 H	0	9	2	5	0	5
10 H	0	2	2	9	0	9
11 H	0	3	2	11	0	7
12 H	1	9	3	5	1	0
13 H	2	0	3	4	1	3
14 H	1	2	1	7	1	2
15 H	1	4	3	10	1	2
16 H	0	10	2	4	0	6
17 H	0	5	2	5	0	8
18 H	0	4	3	5	0	10
19 H	1	5	2	4	0	8
20 H	1	3	2	9	0	5
	17	4	56°	10	15	5
	0	10·10	2	10·1		9·25

To face p. 178.

DANGERFIELD, LITH. 22, BEDFORD ST. COVENT GARDEN. 12/85. 10943.

The figure of merit is affected by various details in the manufacture; the chief, perhaps, being inequalities in the weight of the charges.

Cartridges containing inferior powder, which causes fouling, will often give a good figure of merit for the first 20 rounds or so; but if the firing is continued without cleaning the rifles, the hits are much scattered, and the figure of merit falls off.

The point of mean impact is found to be higher up on the target in summer than in winter; taking the average of one year, it was found in the Royal Laboratory that the mean M.V. of the Martini was 1323 f.s. from April to September, and 1300 f.s. from October to March; the difference is doubtless due to the different states of dryness of the powder and variation in the density of the air.

The figure of merit of regiments refers to the accuracy of the men's shooting. Figure of merit of regiments.

Accuracy on Service.

The range and accuracy performances of a particular gun at experimental practice cannot be depended on to give certain indications of what another gun of the same nature will do under **service conditions**, which are not so favourable; but as before stated, the results obtained are valuable, and they afford a basis of comparison between guns, and a good estimate may be formed of what to expect under various conditions. Accuracy on service.

The published range tables can only give the average performance of the guns to which they refer, they must not, therefore, be blindly followed; but the shooting should always be intelligently corrected from careful observation.

On service, in addition to the causes of inaccuracy already mentioned, and which cannot be so carefully controlled, the following may be added:— Additional causes of inaccuracy on service.

(i.) Guns cannot be made exactly alike; sometimes one gun in a battery will carry a little farther than others, on account of some slight difference in the sighting, or difference in gauge of bore, owing to wear; wet sponges should be avoided with howitzers, especially when small charges are employed. Guns.

(ii.) Small-arm bullets may become dented before firing, this seldom, however, makes much difference in the shooting. Ammunition.

Powder alters considerably in keeping; this is particularly the case if it is exposed to a dry heat or to damp; in the former case its strength is increased, in the latter case it is diminished. Different brands of powder will also give different results, the muzzle velocity of the 5-inch B.L. gun with 16 lbs. P. powder has been found to vary between 1830 and 1730 f.s. In recent specifications for powder the limits of velocity to be given in the proof gun have been much narrowed. A charge of selected P powder should give from 1780 to 1810 f.s. velocity to the 5-inch projectile, the variation between high and low brands being only about 30 f.s; an alteration of 100 f.s. in the M.V. alters the range (roughly) by about 8 per cent. In sieges, where large quantities of powder are employed, and there is time at disposal, barrels of different powders have been mixed together, and thus a mixture of a large quantity of fairly uniform quality has been obtained; but this would perhaps not be necessary at the present time, owing to greater care in the manufacture of late years.

(iii.) The errors due to laying are very considerable; in order to reduce them as much as possible the plan now adopted is to cause a few men only, who show special aptitude, to be specially exercised in Errors due to laying.

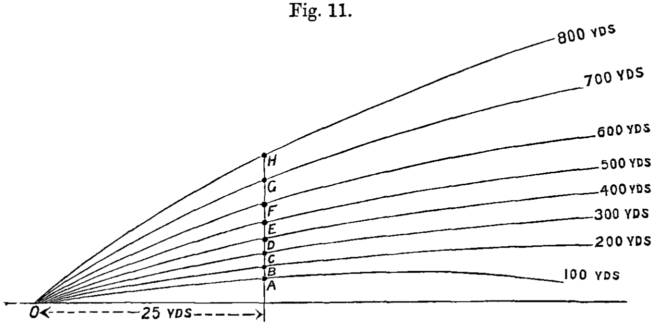
laying guns. Nearly every gunner can be trained to lay a piece of ordnance accurately; but only a few can do so with rapidity. A land service gun is, however, the easiest piece to lay, as the gun and platform are firm, differing from a small arm, which has to be held steadily, and from a naval gun, which moves with the ship and must be fired carefully at the right time, as the vessel rolls.

At a German experiment with a squad of ordinary gunners, who fired before a special course of instruction, and again just afterwards, the accuracy was very greatly increased, being about doubled. The instructional target now generally used helps to teach aiming: the gun is laid with the ordinary sights on a small triangle placed on a large target at some 100 yards, or less distance; after its position has been marked with pencil or with a slight chalk mark the triangle is removed, and the aimer at the gun looks over his sights which have not been moved, and directs another man to replace it, and when in the opinion of the aimer the triangle is again aligned with the sight, its position is again marked; the better the aimer, the more nearly will the triangle be replaced in its first position: the exercise is repeated until a group of marks is obtained on the target: the dispersion of the group of marks shows the errors due to laying.

An intelligent man generally learns to aim more readily than a less educated one; but constant practice is needed to maintain skill in shooting; as the annual allowance of gun practice ammunition is necessarily limited from economic reasons, carbine target firing greatly aids in making a recruit a good artillery shot.

Morris tubes, bore 0·230-inch, placed inside ordinary rifles, and firing small bullets of 37 grains with powder charges of only $3\frac{1}{2}$ grains, also aid in the instruction of recruits: as practice can be carried on at a target at a short range, which is easily obtainable, and ricochet of the small bullets can be avoided without difficulty. The principle of the system will be readily understood from Fig. 11, in which a

Fig. 11.



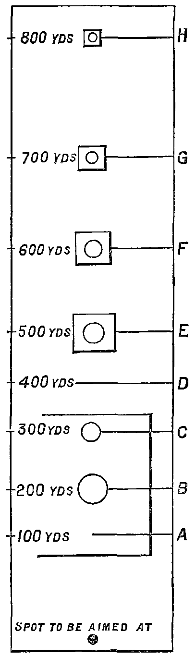
The vertical scale is exaggerated. AH is $\frac{1}{40}$ of real size.

vertical is erected at 25 yards (or even less) from the muzzle, and A, B, C, &c., are the points where the trajectory for 100, 200, and 300, &c., yards intersect it.

A vertical target (Fig. 12) is then set up at a distance of 25 yards having horizontal lines A, B, C, &c., at the same distance apart as A, B, C, &c., in Fig. 10 (which is drawn to $\frac{1}{4}$th the scale of Fig. 11); bull's-eyes and outers are sketched on the horizontal lines to give an idea of distant targets: the sights of the rifle are adjusted for any

desired range, and then they are directed to the aiming spot at the bottom; the bullet should then strike the bull's-eye belonging to the range ordered: thus, if the sights are adjusted for 700 yards and laid on the spot at the bottom, the bullet should hit the centre of the

Fig. 12.

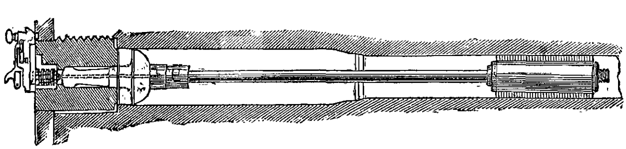


Morris Target for Martini-Henry Rifle. 25 yards from muzzle. $\frac{1}{10}$ real size, four times the scale in Fig. 10.

line G. The accuracy of the tubes at short ranges is said to equal that of the Snider rifle, and shooting can thus be readily taught; no perceptible recoil is experienced on firing.

These tubes are also fitted in breech-loading guns (Fig. 13), and are made to carry the M.H. service rifle cartridge. With the latter

Fig. 13.—Morris Aiming Apparatus for B.L. Guns. Scale $\frac{1}{16}$.



Longitudinal section through chamber and breech mechanism showing apparatus in position with obturator.

excellent practice can be obtained up to 1,000 yards; the Royal Navy fire them at dummy torpedo-boats at sea.

A great cause of inaccuracy on active service is due to a *want of coolness* in those who are themselves being fired at.

Errors due to external causes.

(iv.) Practically it is found that a side wind less affects the accuracy than one up or down the range; the density of the air affects the accuracy. First-rate rifle shots consult pocket barometers, which are compensated for temperature, when target shooting.*

Range Finders.

A cause of inaccuracy on active service which does not generally exist on the experimental practice ground is that the *range* is often *unknown;* it must either be found by some instrument, which is liable to cause delay, or it must be guessed, and the firing corrected by noting the effects of trial rounds with percussion fuzes; but this is difficult to do at long ranges, where the effects of fire cannot readily be distinguished even with the aid of a field-glass or telescope.

Range-finding.

For a long time it seemed doubtful if **range-finders** would find a permanent place in our service, some officers reported favourably, while others again found little good in them; but improvements having been made in cheapness and portability, and more experience in their use having been gained, they are now firmly established in our own and in continental armies, and they doubtless greatly conduce to accuracy of shooting under certain circumstances. On the strong recommendation of the Committee on Range-finders, a School of Instruction has been established at Aldershot, with a definite course, and thorough practice in the use of the Watkin instrument, as it appeared that its use was seldom properly understood, and even when that was the case the necessary facility, only to be developed by constant practice, was not attained. The Committee justly represented that as careful training is required to make army signallers, where those at each end of a range are trying their best to understand each other, so regular training is also necessary in range-finding, where the enemy's constant effort is to conceal what the observer wishes to find out. Care in the use and keeping of these instruments must be insisted on, as sometimes those who use them do not consider that a range-finder should be handled more carefully than a sword or a rammer.

Range-finders will be frequently employed for field service, and in siege and garrison service, where considerable accuracy can be attained, they will be specially used. In the naval service a *good* range-finder is much wanted, as it has been found most difficult to make accurate practice when firing from a rapidly moving ship.

All the systems of range-finding are based upon the principles of surveying, with the exception of the plan for finding the range by the velocity of sound, but this is very inaccurate, and it has the disadvantage that it cannot be employed until the enemy has commenced firing.

The necessary conditions are that ranges must be found with a fair approach to accuracy to be of any real value, say within 1 per cent.: this can be done with comparative ease at a short range, but it is generally more difficult to do so at a longer one, when also *any* error is of more importance in affecting the accuracy of shooting as the trajectory becomes more curved. On the other hand, more time can generally be given to find the longer ranges.

* *Vide* Text-Book for officers at Schools of Musketry, 1884.

All range-finding may be defined as the measurement of an unknown distance in terms of a known one called the base: thus, when finding the distance of the earth from the sun in 1882, at the transit of Venus, a base line nearly the length of the diameter of the earth was employed, the ratio of base to unknown distance was about 1 to 12,000, and the result obtained was supposed to be accurate within about 0·11 per cent. For military range-finding the ratio of base to range should not generally be less than about 1 to 20, but it may be considerably increased if a telescope is used: the shorter the base the more quickly can a range be found, but the longer the base the greater will be the accuracy if other conditions are the same.

Some instruments contain their own bases, they are very quick and simple in their action, and they will give good results at the shorter ranges, but they fail in accuracy at the longer ones, when the variation in the angle subtended by the base becomes too small for accurate measurement. An exception may perhaps be made in favour of the Berdan range-finder; as it has a base line of 6 feet with well mounted telescopes at each end, but it is very expensive and difficult to transport. The fixed base takes no advantage of the circumstance that a longer base can be advantageously employed in finding the longer ranges.

Other instruments, such as Weldon's,* are arranged to take a fixed ratio between base and range, and consequently about the same percentage of accuracy is obtained at all ranges: this system has the advantage of simplicity,† and is specially convenient at short ranges, but it does not readily lend itself to the circumstances which arise, as it may happen on rough ground that it is not possible to obtain the exact length of base desired, and then the range cannot be found at all.

The systems which have given best results under *all* circumstances are those which adapt themselves to the various conditions which may arise, allowing a fairly constant ratio of base to range.

Points of importance are to have the ends of the base sharply defined, and, if possible, the distant object itself should be definite and easily visible, and if the instrument is of the sextant type, the near object should be seen by reflection, and the distant one, which will be less distinct, must be viewed direct: as the measurement of the base, especially for longer ranges, takes some time, a plan has been adopted in both Watkin's and Nolan's for finding its length from the angle subtended by a short sub-base which can be quickly and accurately determined. Calculations must be avoided, and the ranges should be read off the instruments at once.

Depression range-finders from coast forts for distances at sea record the ranges according to the angle of depression at a given height above the water level: as this height forms the base line, the greater it is the greater will be the accuracy in finding the range.

Judging distances.

With regard to **estimating ranges by eye**: distances over a level surface are generally under estimated, and those over undulating ground are often judged to be further off than they really are. Light is often very misleading; for instance, a distant object to the west at sunrise, is strongly illuminated, seems very distinct, and is judged to be nearer than it really is; but in the evening the object, as seen from the same spot, appears under the sun, is

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* A somewhat similar plan had previously been employed by General Drayson, R.A.

† See Journal R.U.S., vol. xxx, No. CXXXIV, "The Weldon Range-finder," by Colonel Richardson, R.A.

indistinct and looks much more distant. In the French night attack on Sfax, the ships lighted up the enemy's position with the electric light; their boats advancing and firing in the dark had a good object to aim at; but they were themselves difficult to see, and consequently to hit.

Table XV, p. 310, gives the alteration in range and vertical accuracy due to various causes for certain guns at specified ranges: and a ready estimate may be formed by its means of alterations in accuracy under other circumstances.

The complaint is frequently made that the complication of the machinery now employed in working guns, opens a door to endless **mistakes**, and there is considerable truth in this assertion. However, as this complication exists, it is practically useful to watch for the mistakes which may occur, as thus they may be avoided; and a list of common errors might usefully be pointed out to prevent their occurrence in future, and to attain accuracy of shooting. Among errors which have been made, and which might occur again unless watched for, may be mentioned the fault of not ramming home a M.L. projectile to its proper place, and thus altering the gravimetric density of the charge, and consequently the muzzle velocity and range of the projectile; in firing with heavy guns and *reduced* charges, the tangent-scale for the *full* charge has, by mistake, been employed, with the natural result that the projectile fell far short of the target. Other mistakes again are likely to arise under certain conditions, and arise as much from weakness of a system as from personal errors; as, for instance, when two men take a range and each observes a different distant object, when they believe they are looking at the same.

Ricochet Fire.

Ricochet fire.

An endeavour to make a direct hit is almost invariably made, but a **ricochet** may be effective if this cannot be done. This kind of fire was employed with smooth bores for dismounting guns behind traverses, but with rifled ordnance the ricochets are uncertain; so much so that it was formerly given (*see* Boxer's "Treatise on Artillery," 1860) as a reason why rifled projectiles could never be generally employed; the drift to the right is considerably increased on ricochet.

The angle of rise is more than that of descent; for instance, if a bullet strikes the ground only a little short of the target, and then ricochets on to it, the mark will be quite high up, and a bullet, striking the ground half way, will pass over the target altogether. If, therefore, aim is taken at a large target whose range is not well ascertained, it is best to aim low and to the left; as in this case, if the range is under estimated, a ricochet may be obtained; and if an over estimate has been made, the top of the target may still be struck.

The ricochets of 13-pr. shrapnel bullets were noted at the Lydd experiments at distances of from 550 to 800 yards from the burst of shell according to the range; but judging from the penetration, the effects of ricochets far beyond the burst at long ranges were small.

Beyond a certain angle the projectile will not ricochet or glance, but will penetrate; hence, at long ranges, with considerable angles of descent, there is no ricochet. The Martini-Henry rifle bullet will not rise after striking level ground at about 2300 to 2400 yards' range.

Shells bursting on Graze.

Quickly acting percussion fuzes are now used for field service, and the shells burst on striking, as is shown by the scoop made being blackened. At the 1500 yards range, when the velocity is high, the rise of the cone of burst is found to be much greater than the angle of descent, but at a longer range there was less difference between the two.*

Accuracy of Shrapnel Shell Fire.

The greatly increased velocity of modern projectiles allows the use of shrapnel shell at greater ranges than formerly; but at the same time it necessitates the employment of more accurate time fuzes, to obtain the burst in the desired position.

For instance, suppose a 9-pr. projectile, moving at a velocity of 900 f.s. The ordinary graduation on the fifteen seconds fuze corresponds to nearly a quarter of a second, in which time the projectile will have travelled 75 yards; but if a new type gun is fired, which has a velocity near the target of some 1200 f.s., the same graduation corresponds to a distance of 100 yards, and thus the adjustment is rougher than with the older projectile.

Effective shrapnel fire with time fuzes requires the maximum of skill, as, in addition to the correct elevation, the length of fuze must be properly given, and this is not always easy to determine, since alterations in rates of burning are caused by changes in the density of the air, and also by the age of the composition.

Before shrapnel with time fuzes are employed at an unknown range, a few rounds should be fired with common shell and percussion fuzes, a little to windward of the target, to find the range; if the smoke of the bursting shell is blown in front of the target the round is short, but if the smoke forms a background to it the shell is over.

The effect of the rounds should be very carefully watched; but as it is difficult to do this from the battery, there should be, if possible, a party at a flank provided with field-glasses or a telescope.

From the battery, the burst of the shell should appear just above the target; but all indications of the striking of fragments should be looked for as well; small clouds of dust or splashes of water may be noticed, and these should be around the target, or a little short of it, as then they will ricochet on to it. But if they are noticed as beyond, the effect of the round is altogether lost, even though the burst of the shell may appear to be just above the target.

Rules are laid down for the best distance short and height above plane, at which shells should be burst at various ranges; but frequently a considerable amount of damage may be done, even if the bursts are much short of the best position.

Shrapnel shells burst at long ranges when the angle of descent is considerable do not sweep the ground so well as when the trajectory is flatter; but cover can be searched in the former case better than in the latter.

The angle of the cone of dispersion of shells burst in the air, is a good deal less than if burst on graze. Shells having the bursting charge in their heads have a larger angle than those with bursters in their bases, especially if they are of steel.

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* Report on the Lydd Experiments, 1879.

The following rule, proposed by Commander May, R.N., to find the angle of opening of the shrapnel shells, has been found to give results approximating to results obtained in practice.

$$\theta = 2\,\alpha\,\frac{d_1}{d}\left(\frac{V}{v}\right)^{0\cdot 7}, \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad \text{(i)}$$

in which θ = angle of opening of shell in degrees.
α = angle of twist at muzzle in degrees.
d = external diameter of shell in inches.
d = internal diameter of shell in inches.
V = muzzle velocity in f.s.
v = remaining velocity in f.s.

Knowing the angle of opening and the number of bullets in a shell, it can be decided how much short the shell should be burst. For example, the 12-pr. B.L. shrapnel shell, containing 180 bullets, has an angle of opening of about 15° at a range of 1500 yards: if burst 50 yards short the cone will cover a front of some 40 feet, and if the bullets are equally distributed there will be one bullet to every 7 square feet of vertical target, exclusive of ricochets. If burst 100 yards short, some 80 feet of front will be covered, but there will only be one bullet to every 28 square feet.

When shrapnel shells are fired with percussion fuzes to burst close in front of the object, much the best results have been obtained by shells having the bursting charge in the head, as the action is quicker than with the other pattern, and consequently the bullets are released before the shells have time to rise; more bullets are also contained.

CHAPTER XV.—RANGE TABLES.

(*See* Tables XIII, XIV, and XV, pp. 307–311.)

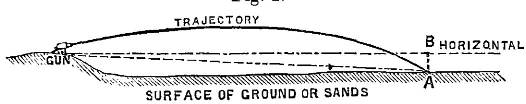
A RANGE table professes to predict the shooting performances of a piece of ordnance, and it is founded on the results of experimental data; from what has been said in the previous chapter, it is evident that it only furnishes an approximation to the truth, as many conditions may exist which differ from those under which the table was compiled. The data are obtained thus; the gun is fired from the service carriage; the bore must be in good order, the charge composed of powder of a mean brand, and the projectile of proper weight; the barometer, thermometer, and degree of moisture of the air should be registered for each day's firing, and the velocity and direction of the wind for each round.

A programme of the firing necessary for the compilation of a good range table is given in Table XIII, p. 307, but many range tables are made from less experimental data, supplemented by calculation of ranges by Niven's method, in which case it is necessary to know the muzzle velocity and the angle of departure, and also the approximate value for σ, to allow for smoothness and steadiness in flight; any rounds which differ a great deal from others are rejected in the compilation of tables; a first round is not unfrequently abnormal.

After the experimental data have been obtained the measured ranges must be corrected for wind, density of the air, and moisture, and also if the gun is warm, or if the powder differs from an average brand.

The elevation must be corrected for the **height of the trunnions** above plane, as the gun is placed at some feet above the level of the sands, which form the range at Shoeburyness. Elevation is given by

Fig. 1.



a quadrant, and it will be seen at a glance (Fig. 1) that the actual ranges are greater than if the point struck were on the same level as the gun. The correction is made with sufficient accuracy by adding the angle subtended at the gun by the height AB to the quadrant elevation, to obtain the elevation really given.

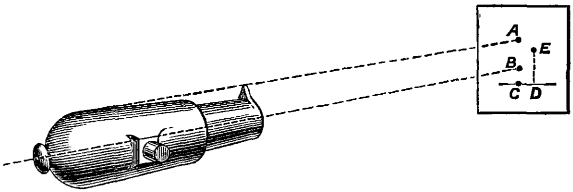
In connection with this subject the question often occurs, "How is the range altered by firing up or down hill, or from a height?" For instance, it was desired to know what would be the range for various quadrant angles of elevation and depression from a battery some 300 feet high. It is generally customary in such cases to assume what is called the "rigidity of the trajectory," the range is assumed to be the same, whether the line of sight is horizontal or

not. Although this is not strictly true, it is a close approximation, unless the ratio of height to range becomes considerable. This subject was fully investigated some years ago in the *Revue d'Artillerie.*

The **jump** at any quadrant angle of elevation is sometimes determined as follows:—

A vertical target (Fig. 2) is placed at a convenient distance from

Fig. 2.



the muzzle of the gun; any desired quadrant elevation being given, the tangent scale is put to zero, and the piece is laid by it on the spot A on the target; AB is measured, equal to the distance of the line of sight from the axis of the piece, and B is marked on the target below A. This is the spot to which the axis of the gun really points. Since the velocity of the projectile and the distance of the target from the muzzle are known, the time of flight (t) can be found; and from the formula $h = \frac{1}{2}gt^2$, the distance BC below B, which the projectile would strike if the elevation remained unaltered, is calculated, and through C a horizontal line CD is drawn on the target. The gun is then fired and the projectile is observed to strike at some point E above the line CD; the small angle which ED (the perpendicular from E on the line CD) subtends at the muzzle of the piece is the increase of elevation called the jump, and is easily calculated from these data.

It is found practically that when several similar rounds are fired at a screen 100 yards distant, that the jump apparently varies a good deal, consequently considerable differences might be expected in range; but this is not found to be the case. At 500 yards the results on a vertical target do not show that the variation in jump has been so great as the nearer screen would indicate; this contradiction is explained as follows:—it is supposed that the projectile makes irregular gyrations round its mean trajectory as it first leaves the muzzle, but that the resistance of the air soon causes it to steady down in flight, and that this is accomplished at about 500 yards range.

It is therefore now a usual plan to find the jump by taking the difference between the calculated angle of elevation for 500 yards and that required practically. A 500 yards range is taken in preference to a longer one, in order to avoid errors arising from wind and resistance of the air as much as possible.

Some half-dozen corrected ranges and corrected elevations having been thus obtained, the ranges OA, OB, OC, &c., are plotted off on some convenient horizontal scale (Fig. 3), and perpendiculars AL, BM, CN, &c., are erected proportional (on some other scale) to the elevations required for the range. A curve is then sketched in through the points L, M, N, &c. (Fig. 4). Perpendiculars are then erected at each 100 yards of range, and their lengths referred to the vertical scale of degrees to find the corresponding elevations; for instance the dotted perpendicular PQ at 1400 yards range is found to correspond to 1° 11′, which is accordingly taken to be the elevation.

Fig. 3.—(For 6-in. Gun, Mark III).

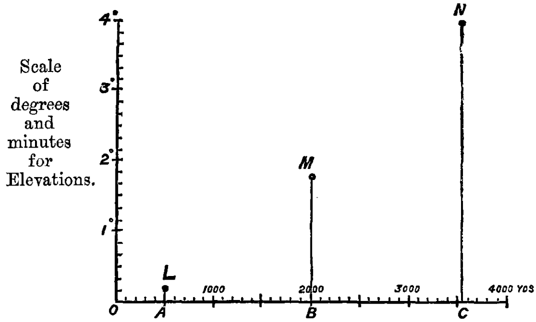
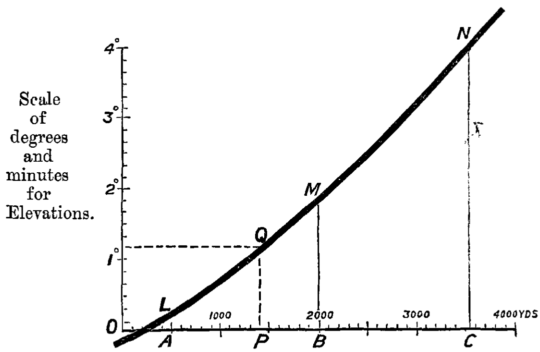


Fig. 4. (For 6-in. Gun, Mark III).



NOTE.—The curve passes below O on account of the jump.

If the data are insufficient but the M.V. known, a few ranges and elevations must be worked out by calculation by Bashforth's and Niven's table with a value for σ deduced from the longest range which has been obtained; a curve can then be plotted, and all the elevations found as before.

If no data at all are available approximate range tables can be made up from calculations, assuming a value for σ; the M.V. can be estimated from the expansions of the charge in the bore; but such tables cannot be trusted, as some unlooked-for condition may exist which would render them inaccurate, and they should be corrected from the results of practice as soon as possible.

Correction of Range.

Admitting the principle that **a range table** is **not an absolute guide,** provision is made for correcting the shooting by a column (*see* Table XIV (B) 8th column) which gives the alteration in range due to five minutes alteration in the elevation at different ranges. This is easily compiled from a knowledge of the ranges and elevations;

thus the elevation for the 8-inch Howitzer of 70 cwt. for 2,400 with the $10\frac{1}{2}$ lb. charge is 9°; for 2500 yards it is 24 minutes *more*, and for 2300 yards it is 24 minutes *less;* consequently 24 minutes added to or taken from the range alters it by 100 yards. Make this proportion

$$24 : 100 :: 5 : \text{alteration in range,}$$

which we find is 20·8 yards. The alteration in range made by 5 minutes alteration in elevation at a few ranges having been found in this manner, a curve is plotted out and other alterations in ranges are measured off from the diagram as was done with the ranges. The lateral alteration given by 5 minutes' deflection is also given at each range; if x is the alteration required at range R, we have $\tan 5' = \frac{x}{R}$, from which x can be found; if $R = 2400$ yards we obtain $x = 3{\cdot}49$ yards.

Example 1. Thus supposing the mean of the first four or five rounds (a single round cannot be relied on) falls 70 yards short, and from the Table XIV (B) we find that at the 2400 yards range 5 minutes additional elevation will increase the range by 20·8 yards, it would be necessary to add $\frac{70}{20{\cdot}8} \times 5 =$ 16·8 minutes to the elevation; and if after four or five more rounds the mean point of impact is still not correct, say 12 yards short, it would be necessary to add some 3·9 minutes more. For 58 yards = (70 − 12 yards) is the increase of range *actually* due to 16·8 minutes increase of elevation, under the conditions at the time of firing, which it will be noticed differ from those given in the range; this second correction is obtained from the proportion

$$58 \text{ yards} : 16{\cdot}8 \text{ minutes} :: 12 \text{ yards} : x \text{ minutes}$$
$$\text{whence } x = 3{\cdot}9 \text{ minutes.}$$

Example 2. Suppose the same projectile strikes 3·28 yards to the right at the same range, how much deflection left must be given?

Inspection of the range tables tells that 5 minutes will alter the point of impact laterally at that range by 3·49 yards.

$$\therefore 3{\cdot}49 \text{ yards} : 5 \text{ minutes} :: 3{\cdot}28 \text{ yards} : x \text{ minutes}$$
$$\text{whence } x = 4{\cdot}7 \text{ minutes, deflection left.}$$

In this way, by carefully observing the results of firing and correcting the errors intelligently, considerable accuracy may be obtained, provided care has been taken to make all the shells up to nearly the same weight, and that the powder charges are uniform.

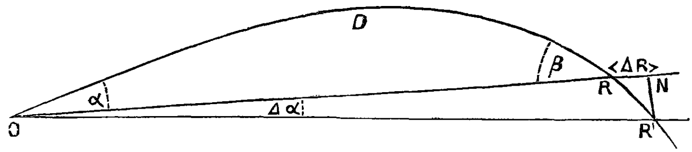
Angle of Descent.

The tangent of the **angle of descent** ($\tan \beta$) can be found up to a range of about 2000 yards by measuring the height (h) of a hole through a vertical target, and the horizontal distance (l) from the target to the point of impact on the ground; at longer ranges it must be found by some indirect means, the following formula gives good results:—

$$\tan \beta = \frac{R \times \text{tangent (of small angle of increase of elevation)}}{\text{Increase of range corresponding to small increase of elevation.}}$$

It may be explained as follows (Fig. 5):—

Fig 5.



Let ODR be a trajectory with range $OR = R$, and angle of departure $= \alpha$.

Increase the elevation by $\Delta\alpha$ and the trajectory becomes ODR', and the range is increased by $RN = \Delta R$ if $R'N$ is drawn perpendicular to ON.

This is on the supposition of the rigidity of the trajectory; the effect of the slight variation in the direction of gravity being neglected.

$$\text{The tangent of the angle of descent} = \frac{R'N}{RN} \text{ approximately.}$$

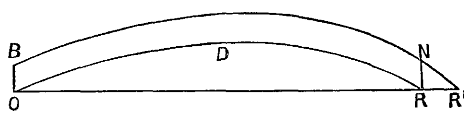
$$\text{and since } R'N = R \tan \Delta\alpha \text{ approximately}$$

$$\text{and } RN = \Delta R$$

$$\therefore \tan \beta = \frac{R \tan \Delta\alpha}{\Delta R}$$

Another plan has been proposed for finding the angle of descent. A gun is supposed to be fired from a certain position O (Fig. 6), and has a trajectory OR. If it be raised a vertical height $h = OB$

Fig. 6.

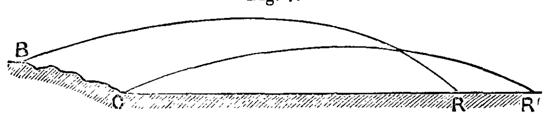


and again fired at the same quadrant elevation over the same horizontal plane, the range will be increased by some distance $RR' = \Delta R$.

From R draw RN vertical, then the tangent of the angle of descent $= \frac{NR}{RR'}$ approximately $= \frac{h}{\Delta R}$.

Practically the gun would not be raised vertically, but the range might first be obtained from the mean of several shots from some position O (Fig. 7) on a horizontal plane, and then the gun could

Fig. 7.



be fired from some slope in rear, and the difference in height and range for both positions could be found, and again $\tan \beta = \frac{h}{\Delta R}$.

Niven's table of $\frac{d^2}{w}\delta$ is also useful as a check on the other method;

it can be used alone to find the angle of descent if the experimental data are insufficient.

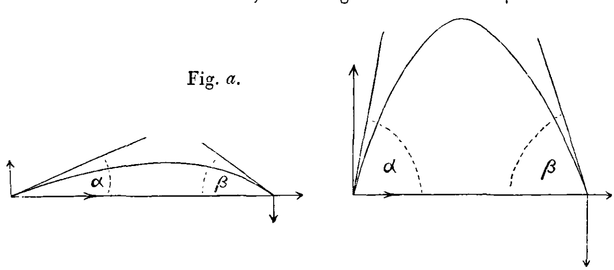
A few angles of descent having been obtained and plotted off as verticals from a horizontal scale of ranges, a curve may be sketched and the magnitude of the angles of descent at other ranges may be found exactly as the elevations were found in Figs. 3 and 4.

It is found, under all service conditions, that the angle of descent is greater than the angle of departure—this may be noticed in the curve described in the air by water projected from a fire hose; the difference between the two increases with the muzzle velocity;* thus the range table for the 12-inch B.L. gun, M.V. 1892 f.s. Table XIV (A), gives an angle of descent of 13° 41′ for an angle of elevation of 9° 12′, but with the charge of $5\frac{1}{2}$ lbs. with the 8-inch howitzer M.V. only 641 f.s., the angle of descent is stated to be 10° 6′ for the same angle of elevation; the projectile with the lower muzzle velocity experiences but little resistance from the air, and thus the conditions approximate to the case of motion in an unresisting medium or in a vacuum, in which, as we know, the angles of departure and descent are equal.

Information depending on the angle of descent is given in some range tables (*see* Table XIV (A)); the slope of descent is the tangent of the angle, and is convenient when estimating the fall of a projectile as it passes over a traverse or crest of the glacis. The dangerous distance for the mean trajectory is the height of the object multiplied by cotangent of the angle of descent. The permissible error in range which allows an object of given height to be struck is $\frac{1}{2}$ dangerous space $= \frac{1}{2}h \cot \beta$. Thus at 1000 yards, unless the range is known within 72 yards with the 6-pr. Hotchkiss quick-firing B.L. gun, a target 10 feet high will not be struck by the mean trajectory, and the same target could not be hit by the 12-inch B.L. gun at 2200 yards, unless the range is known within 44 yards (*see* Table XIV (A), column 5. The remaining velocity is given in the tables at the

* At ordinary angles of elevation the horizontal component of the muzzle velocity is far greater than the vertical: hence the resistance offered by the air is much greater in a horizontal than in a vertical direction; and the loss of horizontal is much greater than that of the vertical velocity at the end of the range, and as $\tan \phi = \frac{\text{vert. component of vel.}}{\text{hor. component of vel.}}$, it would follow that $\tan \beta$ must be greater than $\tan \alpha$ (Fig. *a*), since in the value of $\tan \phi$, the decrease in the denominator is greater than the decrease in the numerator, as the angle varies from α to β.

Fig. *a*.



a Angle of departure. Angle of descent.

But if the conditions are reversed, *i.e.*, if the angle of departure is considerable and the vertical component of velocity is greater than the horizontal, this line of argument would prove the angle of descent to be less than that of ascent, as in this case the vertical component of velocity loses more than the horizontal. A very large angle of departure is not practically employed, though it may be useful under some circumstances.

different ranges; this must be found from calculations by the use of $\frac{d^2}{w}s$, Table VII, employing a value for σ (for form of head, smoothness and steadiness in flight) which agrees with the results of the practice; the probable effect of the blow on striking can then be estimated. In some tables the penetration of wrought iron at various ranges is inserted for a projectile of given weight; this depends chiefly on the square of the striking velocity, the diameter and the weight of the projectile.

The time of flight is always noted when firing rounds for the construction of range tables, and it is recorded in them: it cannot be measured at practice to less than about $\frac{1}{10}$ of a second; but if compared with the result furnished by $\frac{d^2}{w}t$, Table VI, employing the value of σ found from experiment, a useful check is established, and occasional mistakes in range reports may be detected. The time of flight does not appear to bear a fixed relation to the time of burning fuzes, as the rate of combustion varies in different projectiles; thus with a 12·5-inch projectile, M.V. 1575, the 15-second fuze burns out in 11·75 seconds, but in a 64-pr. shell, M.V. 1260, it lasts for nearly a second longer. it is now proposed to graduate fuzes for ranges: a fuze scale is given in range tables. Time of flight.

The width of the three 50 per cent. zones is given at each range from results obtained at only a few ranges, and plotted out from by a curve: the error in range for B.L. guns may be expected to be nearly constant, and not much over 15 to 20 yards up to the range at which the projectile has a tendency to become unsteady in flight. The error in direction is usually less than the error in height at the same range; but both increase with the increase of range. Accuracy.

Further information is sometimes given; as for instance the amount of variation in M.V. to be expected from different brands of powder, and the consequent alteration in ranges: it is also sometimes noted that differences in the mountings will cause change in the jump and necessitate alteration in the elevation.

Howitzer Range Tables.

With howitzer fire there are two main variables; both the *powder charge* and the *elevation* may be altered, and thus the same range may be obtained with a small charge and a high angle of elevation, as with a large charge and a low angle; the other constituents of the trajectory however, viz., angle of descent, remaining velocity, time of flight, and accuracy are not the same.

In an unresisting medium it is easily proved that the range is the same (with the same charge) for angles of elevation of $45° + \alpha$ and $45° - \alpha$, where α is any angle less than $45°$.

In the air, the same range with the same charge can be obtained by an angle of elevation greater than that which gives the maximum range (which is always less than $45°$), and by some lesser angle. Mortars have been fired at very high angles of elevation, thus obtaining great searching power behind cover at all ranges, as the angle of descent must be very great. Practically, however, with rifled howitzers, the angle of elevation is always smaller than that giving the maximum range; the higher angle would necessitate great changes in the construction of the carriage or bed, the strains on it in a vertical direction would be increased, and the accuracy of the shooting would be impaired; on the other hand the striking velocity and

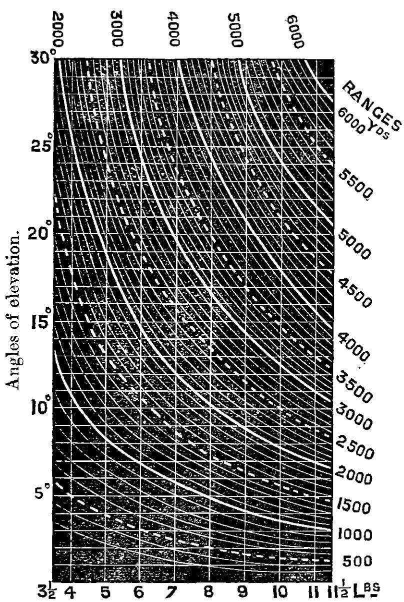
the angle of descent would be increased, and this kind of fire has been proposed against the decks of armour-clad ships.

Range tables, consisting of several pages each, have been constructed for the 6·6-in. and 8-in. howitzers, giving the details of the trajectory for seven different charges respectively, at angles of elevation from 1° to about 30°. (Table XIV, (B), p. 309.)

The same information may be arranged in diagrams (Figs. 8 and 9), which for practical use should be on a larger scale. The

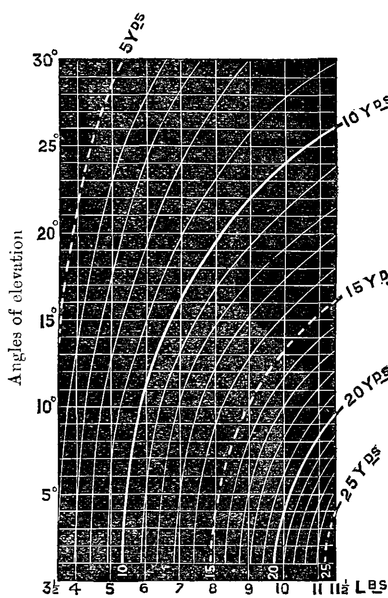
Fig. 8.—Range diagram.

Ranges in yards.



Powder in charge.

Fig. 9.—Range correction diagram, showing the alterations of range in yards due to 5 mins. change of elevation.



Powder in charge.

ranges in Fig. 8 may be read off at once, not only for the ten charges given in the table, but also for all intermediate ones. Each diagram (which is simply plotted from the range table and sketched in) consists of a horizontal scale of equal parts, graduated for charges in pounds, and a vertical scale graduated to degrees, for angles of elevation. Thus (Fig. 8), with the 8-in. howitzer, to find the range given by a charge of 9 lbs. of powder with 10° of elevation, carrying the eye vertically upwards from the 9 on the powder-charge scale, and horizontally from the 10° on the degrees scale, we see these lines intersect at about 2210 yards on the curved lines of ranges. Again, if the range is known to be 1500 yards, and the powder charge is 8 lbs., we see that about 7° 25′ elevation must be given.

If, however, in the last case the point of mean impact is found to be some 29 yards over, we must deduct about 10 minutes from the elevation, as we see from Fig. 9, that at that elevation and with that charge some 14½ yards change in the range is due to every 5 minutes' increase or decrease of elevation.

The angle of descent, time of flight, comparative accuracy, &c., could be denoted on other diagrams constructed on the same plan; the curves for ranges or range corrections in Figs. 8 and 9 being replaced by others for angles of descent, &c.

It is easily seen that the required **angle of descent** (with howitzers) can be obtained either by a *short range* and a *low velocity*, or by a *long range* and a *higher velocity;* the former has generally the advantage in accuracy; the latter in hard hitting. At short ranges, if the twist of rifling admits of it, the force of the blow might be increased by adding to the weight of the projectile, since the striking velocity must be low, if the angle of descent has to be considerable; the howitzer need not be over-strained, as though the projectile would be heavy, the powder charge for the short range must of necessity be small; but this would entail a complication by adding to the numbers of different natures of projectiles.

Krupp has experimented with some very long projectiles, called torpedo shells, 6 calibres in length; they contain large bursting charges, and their effect against earthworks was found to be very destructive. Shells 4 calibres in length, with a twist of one turn in 25 calibres, are now being generally adopted in the German service.

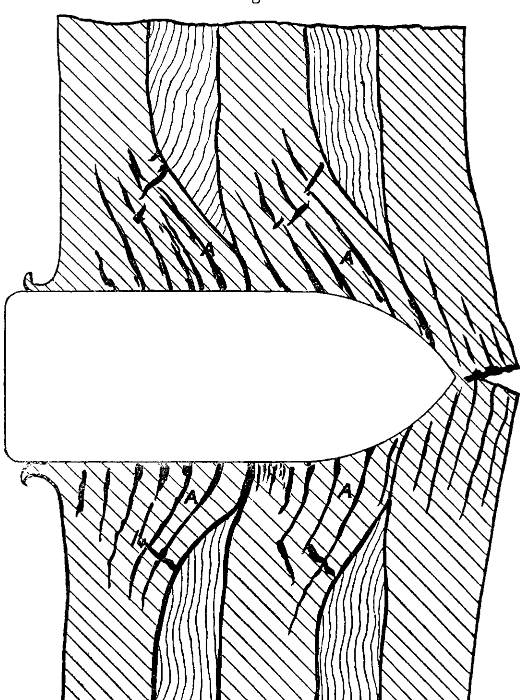
As howitzers are employed for breaching unseen revetments by indirect fire, a correct knowledge of the angle of descent at various ranges and with various charges is of great importance.

The drift to the right is stated in the range tables of howitzers, and also the necessary deflection to the left with the various charges; the tangent scales are put in vertically.

CHAPTER XVI.—ARMOUR.

DURING the last few years the great power of modern heavy guns has called for immense expenditure on armoured defences for ships and forts by all civilised nations We have largely adopted **wrought iron** in rolled plates of considerable size, and all our armoured forts and most of our ships are protected on this plan: and other nations have sent to our manufacturers for plates. As to the way in which they should be applied, whether in very thick masses or in thinner plates upon plates, it has practically been found that the best plan is not to employ a thickness of more than about 8 or 10 inches, even for the strongest armour; but to put several of these together, with a space of about 5 inches between, filled with some elastic substance, such as hard wood, asphalte, bitumen, &c.; a backing of wood with iron stringers, and an inner skin of 1 to $1\frac{1}{2}$ inch of iron is also sometimes added; the latter gives rigidity to the structure, and prevents frag-

Fig. 1.



ments of wood and iron from flying about when the armour is struck with plates upon plates the inner thickness of armour often takes the place of the stringers and skin, and the material put between the plates does duty as backing.

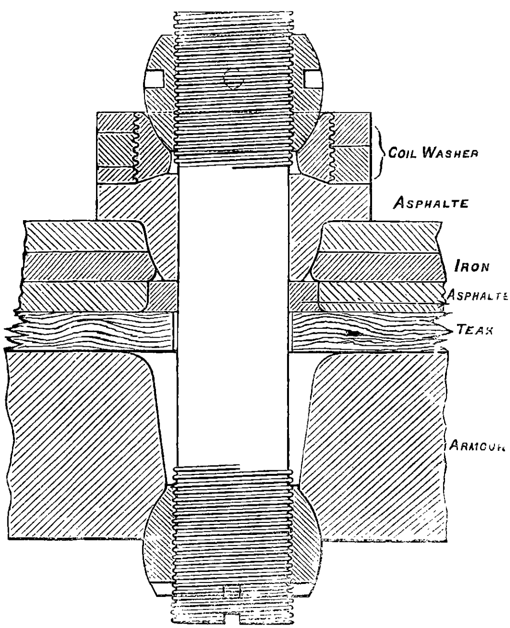
The plate-upon-plate system of armour is not so strong as a solid plate under a single blow; but the elasticity, which it possesses, is said to enable it to offer greater resistance to continuous firing. If very many thin plates are employed the resistance is small, as was shown by a trial of armour made of boiler plates fastened together.

If very thick solid armour is employed the joints must go completely through, but with plates upon plates there are breaks in the joints (as in courses of bricks); it is also difficult to obtain very thick armour of uniform quality throughout. The plates should be firmly attached to each other so that the "back mould" (AA) of one plate may receive resistance from the next, as the projectile pierces the target (Fig. 1); this is done by means of armour bolts.

Armour bolts.

An **armour bolt** for wrought-iron plates (Fig. 2) embodies very fully the principle that metal under tension must not be caught or nipped: the holes in the armour are therefore a good deal larger than

Fig. 2.



the diameter of the bolt, so that however much the target may be twisted or bent, on being struck by a projectile, the entire length of the bolt may be free to extend. In the Palliser-English bolt, spherical nuts seated in a cup-shaped hole in the armour, or in a special cup-shaped washer, allow the bolts to turn in any direction if the plates are moved; and the raised or plus rounded thread of the strong screw prevents the formation of lines of fracture, which were found to be

present with the **V**-shaped screw thread originally cut *into* the bolt. With steel-faced armour, however, where it is essential that the outer surface should be unbroken, the bolts are screwed into the plate and nutted up behind.

Steel and compound plates.

The Spezzia experiments of recent years with the Italian 100-ton gun, have brought out the properties of **steel armour**, which, though it gives way under the repeated blows of comparatively light projectiles, offers much more resistance than wrought iron to a single blow of great violence. When wrought iron is struck with sufficient force a round hole is driven through it, and the metal in the immediate neighbourhood is little injured. If it is not completely penetrated, no damage is practically done, as no cracks extend, and generally the projectile remains in the target, plugging up the hole it has made; but it is otherwise with steel, which cracks to a considerable distance, and continues to do so for many seconds after impact: the actual penetration for each round is, however, considerably less than with wrought iron. These results led to experiments in England, and to the adoption of the so-called **compound armour**, in which an endeavour is made to obtain the good qualities of both steel and wrought iron. It consists of a hard face of steel, to break up the projectile, attached to a *foundation plate* of wrought iron to hold the steel together and prevent the cracks from spreading. Good results have been obtained from this form of armour, which is stronger than wrought iron for single blows (but not for several falling close together) in the proportion of about 4 to 3. It is applied to the heavily-armoured turrets of our most recent armour-clad ships, such as the "Inflexible" and others, in which saving in weight, for equal strength, is a great gain. To obtain the full advantage of compound armour a hard backing such as granite, which prevents bending, very greatly adds to its resisting powers. When the backing has not been very rigid, thick steel plates have lately shown themselves superior to compound armour.*

Iron defences have been applied to a train in Egypt, and the achievements of General Gordon's river steamers on the Nile, protected by wood and thin plates of iron from infantry fire, are well known.

Cast-iron armour.

The great weight of **cast-iron armour** of resisting power equal to that of wrought iron, prevents any nation from using it for naval service: but though we have none on our forts, continental nations have made great use of Grüson's chilled-iron armour for coast defence (*see* Figs. 3 and 4).† About $2\frac{1}{2}$ times as much cast as wrought iron is said to give the same strength; and, although such a large quantity is required, it is cheap, and it can be made in very large masses with few joints and in any desired curved shape.

The general behaviour of cast-iron armour with chilled exterior is somewhat similar to that of an inferior steel plate when struck (Figs. 4 and 10 and 11); large cracks develop, but considerable resistance is offered to penetration by single rounds; the fragments broken off are large and not easily displaced. Hardly any experiments have been made with this armour in England, but good results have been attained elsewhere.

Coal targets.

Good protection against the fire of medium guns and the lightest armour-piercing ordnance has been obtained for the engines, &c., of unarmoured cruisers by placing loose **coal** in large partitions between

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* For a decided preference for steel armour, *see* "Modern Armour for National Defence," by Lieut. W. H. Jaques, U.S. Navy, New York, 1886.

† From the "Revue d'Artillerie."

the outer side of the ship and an inner bulkhead. The greatest penetration obtained at an experimental target by a 7-in. Palliser shell from the 90-cwt. gun, charge 24 lbs. was 8 feet of coal; two of the partitions of $\frac{3}{8}$-inch iron, as well as the side of the vessel, were

Fig. 3.—Interior of St. Marie Battery, Antwerp Defences.

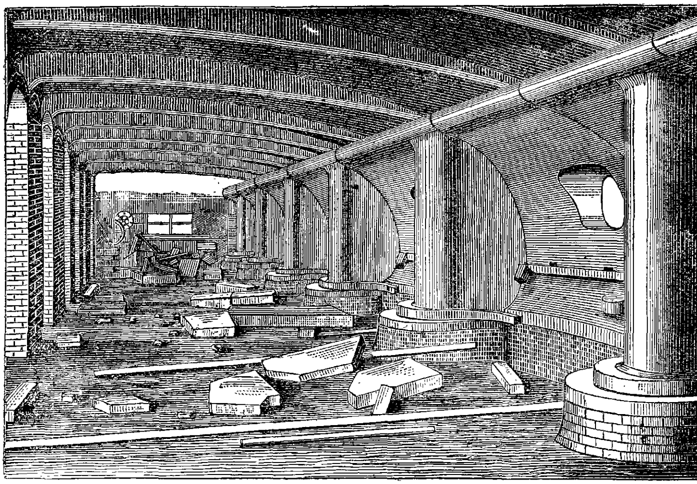
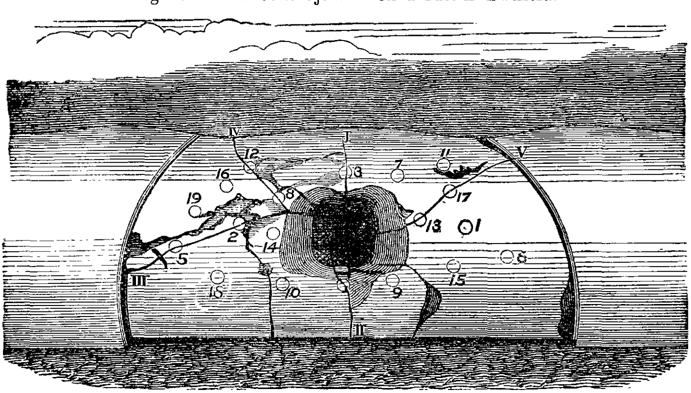


Fig. 4.—Effect of Projectiles on a Cast-iron Shield.



perforated. Some 4 feet of coal and two more partitions remained before the interior could be reached, where the engines were supposed to be situated: the shell burst, but did not ignite the coal, and little harm was done. An evident drawback to this method of defence is that protection is only afforded when the bunkers are full.

Application of Armour to Land Defences.

Many authorities consider that guns may be best employed for coast defence dispersed in earthworks; this plan presents many advantages, but often a restricted site obliges many guns to be collected close together, when **armoured defences** must be employed.

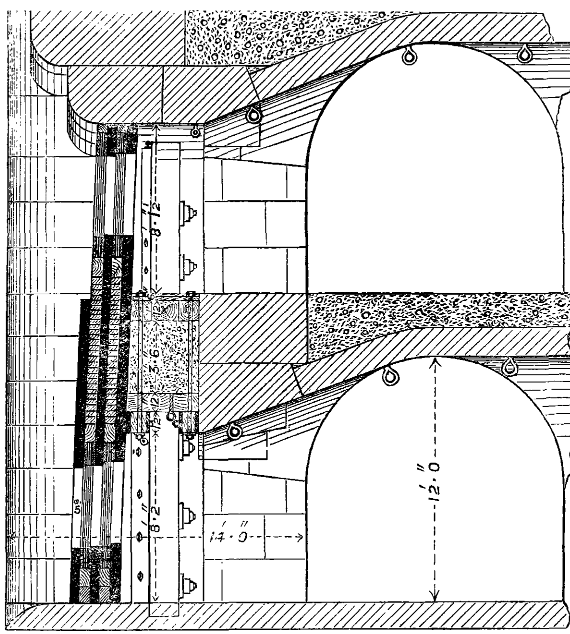
Some years ago it was laid down* from the results of experiments that—

(1.) Ordinary coast batteries should be of masonry, with a shield of iron at each gun, affording protection against fire, equal at least to that of its own gun.

(2.) The most advanced and important sea forts should be protected by walls entirely of iron.

Coast forts. (1.) Fig. 5 shows a specimen of an ordinary **coast fort**: at Picklecombe, Plymouth Sound, the armour is in three thicknesses of only 5 inches each; in other cases it is more—as much as three

Fig. 5.—Picklecombe and Garrison Point Forts.



plates of 10 inches in the fort at Garrison Point, Sheerness, which is of similar construction. Care is taken to build the armour into the masonry to prevent it from being driven in when struck by an enemy's projectiles.

At St. Helen's, Spithead, and also at lower positions in Malta and Gibraltar, guns are provided with turntables, and they can fire out of

* *Vide* Lecture by Major-General Inglis, C.B., R.E., Pro. R.A.I., January 1881, from which several of the diagrams in this Chapter are taken.

either of two embrasures as desired, thus obtaining an extended lateral fire of 120°. The front protection is similar to that previously mentioned, but it is partly circular in plan.

Fig. 6.—Curve Fronted Casemate.

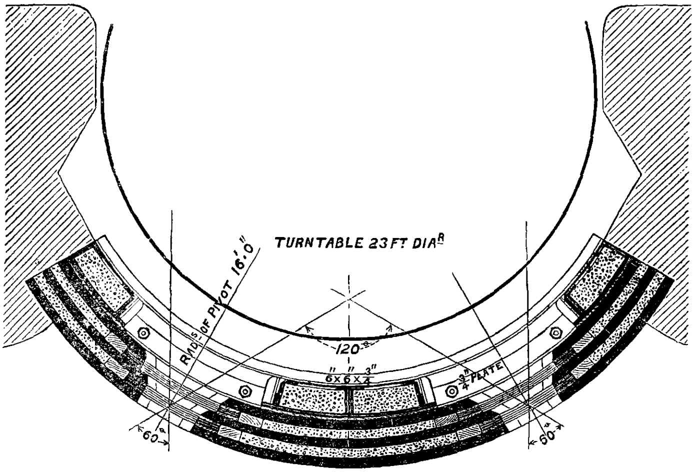
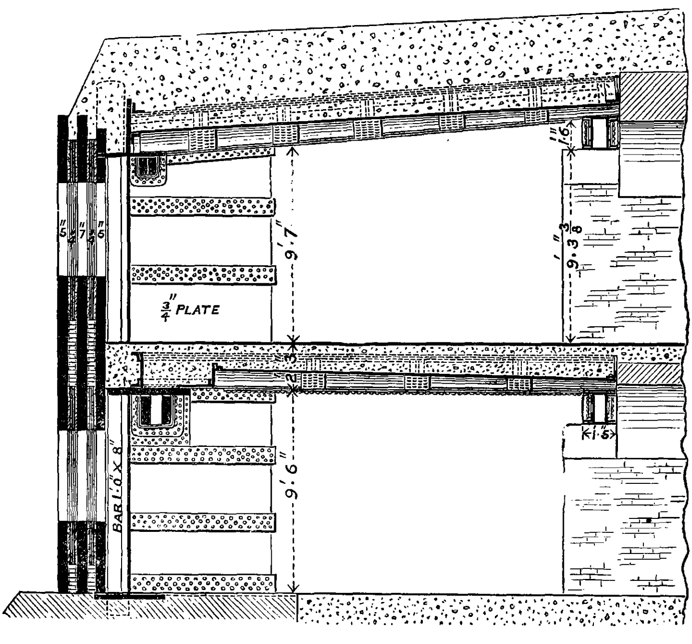


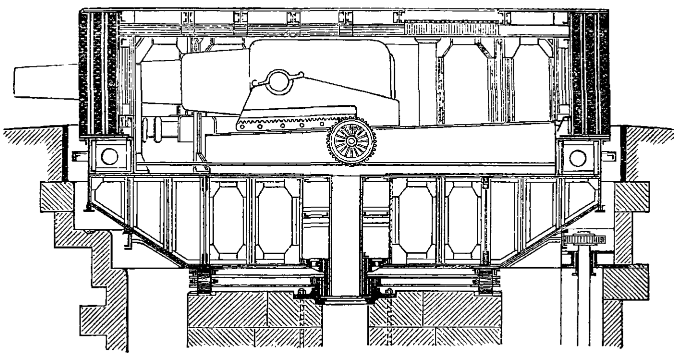
Fig. 7.—Horse Sand and No Man's Land Forts, Spithead.



Sea forts. (2.) Forts entirely surrounded by iron walls, called **Sea Forts** (Fig. 7), are built on foundations raised above high water mark, to defend important harbours. The armour rests upon base plates, and it is strengthened by thick vertical armour bars at close intervals. Care is taken to separate the armour from the flooring, so that when the fort is struck by an enemy's projectile, the racers may not be bent, for if they were the guns could not be traversed. The armour rests against and covers a series of masonry piers between each gun.

Turrets. Although the sea forts were prepared for the addition of **turrets**, the only English one actually constructed at present (Fig. 8) is that at

Fig. 8.—Turret for two 80-ton guns at end of Dover Pier.



the end of the Admiralty Pier, Dover, which contains two 80-ton R.M.L. guns. The total weight of the turret complete with guns, &c., is estimated to be no less that 895 tons. The whole turns round a central steel pivot, and revolves on a live ring on steel rollers by suitable machinery, worked by steam power, which is placed in safety below, as are also the cartridge and shell stores. The turret is capable of all-round fire, and the guns are loaded from under the glacis, by running them back and depressing the muzzles. The armour is wrought-iron, in three thicknesses of 7 inches each, with intermediate 2-inch plates.

It will be noticed, that in all the above armour, the plate-upon-plate system has been adopted. This has the advantage of permitting an addition of an extra thickness, should it be deemed necessary at any future time; this could hardly be done with the continental cast-iron armour.

Difficulty has been experienced in carrying out all the necessary arrangements for guns in forts, from the fact of the increase in size of the more recently constructed guns; frequently, works designed for certain pieces were armed with others of a heavier calibre, thus necessitating many alterations in the arrangements of the forts and armour.

Considerable progress has lately been made with turrets or cupolas on land, generally armed with two heavy guns: these may be for two separate services, either for coast defence, as those lately designed by Italy, to resist the fire of the heaviest guns, which however cannot be very accurate and may probably only last for a short time; or for the defence of important points subject to the attack of much lighter siege guns, which however will fire accurately and

continuously. For the former chilled cast iron seems valuable, as it readily resists heavy blows which do not fall close together, and probably the muzzle of the gun in the cupola may be allowed to project without much risk of its being struck and disabled by the enemy's projectiles, if the structure can be readily rotated. For positions subject to land attack, wrought iron, mild steel, or compound armour is more suitable, as the blows will fall close together; plates liable to cracking should be avoided, and but little of the muzzle should project outside the armour.

The Italians have determined to erect two massive cupolas for *coast defence* of Grüson's chilled cast iron armour, each to contain two guns of 120 tons. The shield is to be built up in 15 segments, each of which will weigh 85 tons, except those near the muzzles of the guns, which will be somewhat less; the total weight to be rotated will be 1378 tons. In April, 1886, one segment of such a shield (suitably set in masonry and iron to represent the support which, would be received from the other segments in the complete structure) received three blows from the 100-ton gun at a short range with forged steel Krupp projectiles; although it was a good deal cracked it had afforded excellent protection, there were no bolts or fastenings to fly about inside, and the resistance was considered most satisfactory (Figs. 9, 10, and 11).

Fig. 9.—Italian cast-iron target* (after the first round).

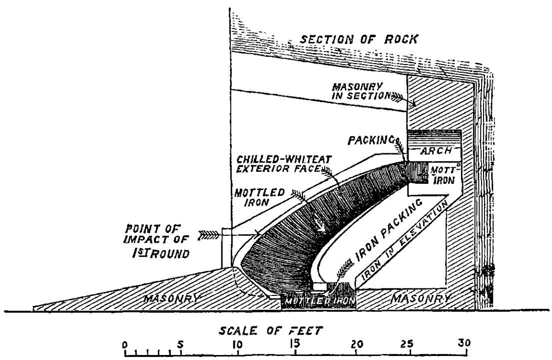
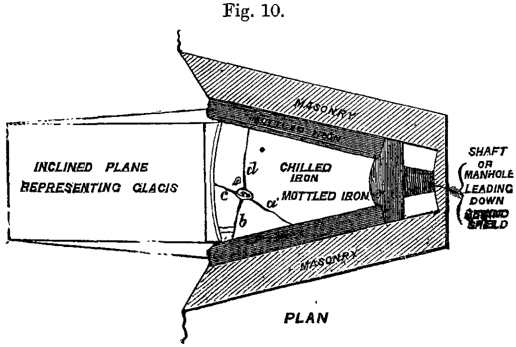
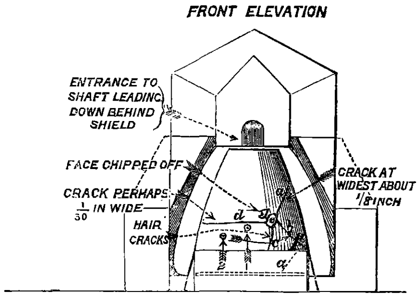


Fig. 10.



* Diagrams 9 to 13 are from the "Engineer."

Fig. 11.—Front elevation.



A couple of months before the Italian experiment, two rival cupolas for *land defence* were subjected to a severe attack by siege guns at Bucharest: Grüson's dome of compound armour contained two muzzle pivoting 5·9-inch guns: this was found difficult to injure by horizontal fire, but the guns in it were somewhat small, and it seemed doubtful if heavier ordnance could be efficiently mounted and worked without injury to the structure as no recoil is allowed (Fig. 12).

Fig. 12.—Schumann-Grüson cupola.

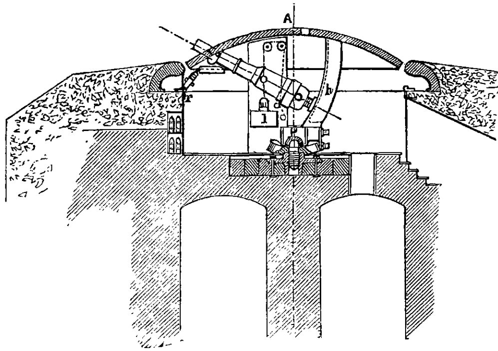
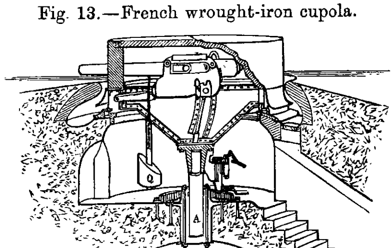


Fig. 13.—French wrought-iron cupola.



A French turret with vertical sides of wrought iron rising some 3 feet above the glacis was also fired at: the thick armour of 17·7 inches offered good resistance to the continued fire of the siege guns; the guns inside were allowed a slight recoil, and the front of the slides being pivoted under the muzzles allowed the use of small ports.

Both cupolas were readily rotated, and no case of jamming occurred, although they were struck by many projectiles.

Experiments have also lately been made in England by firing at an experimental conical cupola at Eastbourne.

Application of Armour to Ships.

Our first ironclad, the "Warrior," had armour for about two-thirds of its length, affording protection about $5\frac{1}{2}$ feet below water, and to the long battery of broadside guns. Wrought-iron plates of $4\frac{1}{2}$ inches, with backing and inner skin, were employed, but this thickness of wrought iron can now be penetrated at short ranges with a field gun (the 13-pr.) with specially hard pointed projectiles.

As guns became more powerful more armour was added, by making it thicker, and also by extending it over the entire length of the ship, as in the "Minotaur."

But soon it became evident that ships could not carry much additional weight; so in the "Hercules" and others in which the armour is 9 inches and more in thickness, the guns are collected in a central battery, which is well protected, and only a narrow band of armour covers the waterline of the remainder of the ship.

As still heavier and more powerful guns came into use, the broadside principle and the central battery gave place to turrets capable of fire in nearly all directions; these are placed and revolve in a central breastwork or citadel, which covers the vital parts of the vessel. Both turrets and breastworks are heavily armoured, and only the waterline of the rest of the ship is protected.

In the "Inflexible" (with 24 inches of iron plates in the citadel, and 17 inches compound armour on the turrets) a part only of the waterline is defended by side armour; the remainder is protected by a 3-inch armoured water-tight deck below water; on this rests an unarmoured superstructure, which may easily be shot away in action, but the loss is calculated not to injure the fighting qualities of the ship, or its seaworthiness to any important extent.* Many modern armour-clad ships such as the "Benbow," have guns mounted *en barbette*, instead of in turrets: weight is thus saved, which allows the use of rather heavier ordnance.

Sir W. Armstrong, in his Presidential Address, Inst. C.E., 1/10/82, thinks fast unarmoured cruisers with heavy guns preferable to ironclads, as they are so much cheaper, three cruisers costing about the price of one armour-clad. Their speed would be most useful to protect our commerce, and they would be available for ramming and torpedo work. In our Navy, however, both types of ships are necessary.

In the attack of an armour-clad ship by the guns of a fort, it is important to know what the vessel's armour consists of; probably this information could generally be obtained beforehand, as the details of armoured ships possessed by all nations are generally known. If the

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* For details of armoured ships, *see* Manual of Gunnery for Her Majesty's Fleet 1886, and Naval Annual, 1886, by Lord Brassey, K.C.B.

armour of the turrets or breastwork of the ship is far beyond the power of the guns in the fort, it would be best to attack the unamoured ends with common shell, or even with shrapnel.*

As ships are now provided with auxiliary armaments of machine guns and quick firing guns, most of which would be in the unarmoured portions, it becomes an important matter to subdue their fire and drive the men working them into the armoured parts of the vessel.

If the armour of the ship is far below the power of the guns, common shell might be used, even against the armour, as they will as a rough rule penetrate a depth of iron equal to half their diameters at short ranges.

If the fort is at a considerable elevation, a depressed fire may be brought to bear on the deck of an opposing ship with very destructive effect, from even moderately powerful guns, against the strongest ironclads, if the angle of descent is sufficiently large to prevent glancing.

The fire of heavy rifled howitzers is now so accurate that the decks of an enemy's ships at anchor (and perhaps even in motion) may be struck by a fair percentage of shells with destructive effects even at long ranges. If ships do not anchor they may encounter great risks from submarine mines, and the fire of a moving ship is generally greatly wanting in accuracy. Designs have been made for utilising the 9-inch R.M.L. gun (as a species of howitzer) for curved fire at very long ranges to fall on ships' decks at a considerable angle of descent; experimental fire of this description has given good results.

Experience of the attack of armour gained from actual battle was furnished in the late Chili and Peru war, by the capture of the "Huascar," which had weak armour of only $5\frac{1}{2}$ and $4\frac{1}{2}$ inches; this was easily penetrated by the 9-inch Palliser shell, which burst inside with terrible effect. Had the armour been thicker the broken shells might still have got through, but the fire would have been kept out, as the burst takes place very rapidly on impact.

Although the turret was repeatedly struck in this action, it revolved freely, and no jam occurred. This was also found to be the case in previous experiments made on the English coast.

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* *Vide* Lecture by Captain C. O. Browne, Pro. R.A.I., Feb., 1882.

CHAPTER XVII.—THE PENETRATION OF ARMOUR.

THE exact laws which govern the penetration of armour by projectiles are not known, because the conditions are so complicated; our knowledge on this subject is of a purely practical character, deduced directly from the results of actual experiments. **Exact laws unknown.**

The question to be determined is, how shall the projectile be constructed, that the energy which it contains shall cause the maximum of destructive effect on the target?

The energy in the projectile may assume several different forms on impact with the target; it may

1. Cause change of shape and fracture in the projectile.
2. Heat the projectile.
3. Heat the plate.
4. Cause change of shape and fracture in the target.

All these effects are more or less always produced; but what is sought for, is that the greater portion of the **energy** shall be **expended** in **changing the shape of the target,**—piercing or fracturing it, and as little as possible in breaking or bending the projectile, or in heating it or the target. Wrought iron has been found to be quite unsuitable for projectiles, because a considerable proportion of the energy is employed in bending it out of shape, and also much heat is developed on impact; this, of course, means so much less work in a useful form.

The effect on the plate may be either local or distributed; this depends partly on the projectile and partly on the target. The Americans formerly advocated the use of large spherical projectiles, which distributed their blows over large surfaces, and tended to shake the whole structure to pieces; this was termed *racking*, but it is not now considered a good plan.

On the other hand, an elongated projectile moving at a high velocity tends to *punch* a nearly clean round hole, if fired against a wrought iron plate, which is little injured except quite near the point struck (Figs. 1 and 2). In a steel or steel-faced target, however, the blow is generally distributed, and cracks extend to some distance, though the actual resistance at the point struck is much more than with wrought iron: (Plate IV, Fig. *A*, p. 209).

Very good estimates can be formed beforehand, by calculation, of the probable penetration on wrought-iron armour by projectiles moving at given velocities, if the plate is of ordinary quality; but the resisting power of compound armour is difficult to foretell.

In a general way the amount of damage appears to depend upon the total amount of energy in the projectile, almost irrespective of calibre, and the resisting power of the plate seems to be more nearly proportional to its weight than to its thickness, as the blow is more or less distributed over the whole mass, and is not localized as with wrought-iron: the backing has a very great influence on its resistance, a rigid unyielding one being best, as it has been found that solid

Fig. 1.—Wrought-iron plate perforated (front face).

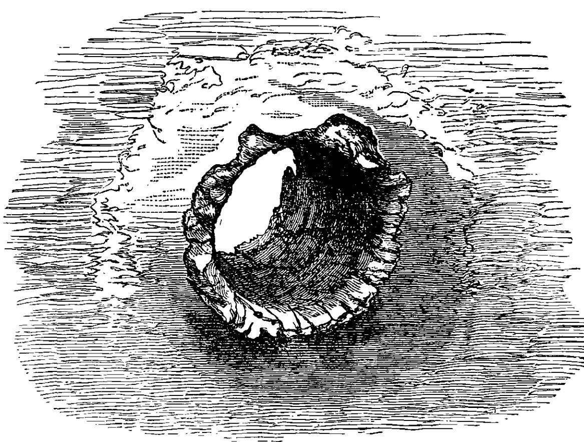
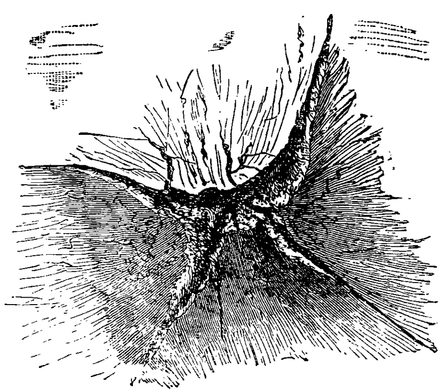


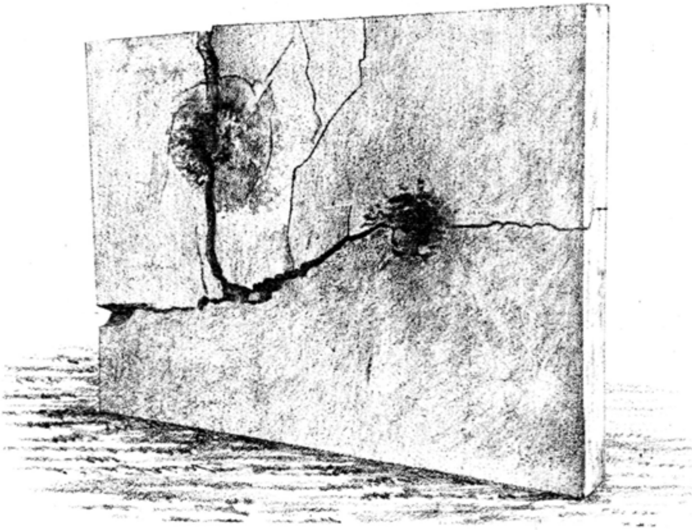
Fig. 2.—Wrought-iron plate nearly perforated (rear face), showing how the metal gives way; the cracks do not greatly exceed the diameter of the projectile.



granite has given most excellent results, greatly preventing local deformation at the point of impact; on the other hand, a soft backing, such as wood, or even iron girders which are liable to bend unless very massive, allows the plate to bulge where struck, when cracks rapidly appear, extend over a great portion of the plate and quickly lead to its destruction; it would thus appear that this form of armour can be more advantageously employed on land defences than on ships, though it must be stated that it is difficult to construct an experimental target representing part of a ship's armour with exactly the rigidity and strength derived from the neighbouring plates and from the whole framework of the vessel on which it is built. The method of attachment of the armour to the structure to be defended is of im-

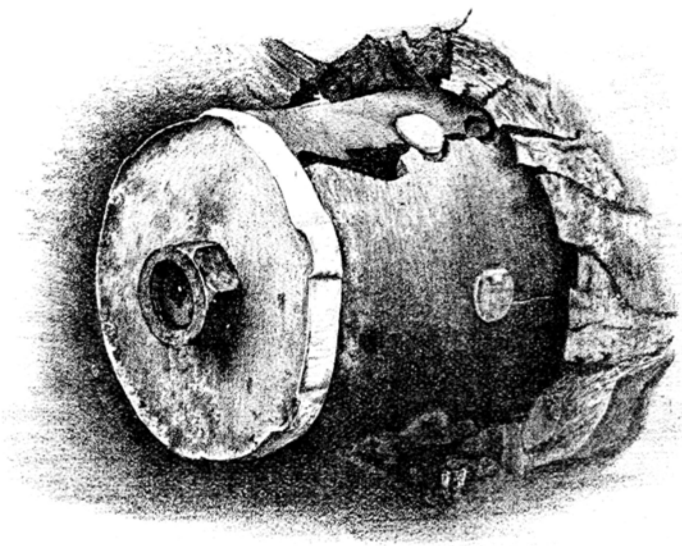
Plate IV.

FIG. A.



COMPOUND PLATE 9 INCHES THICK (6 FEET BY 9 FEET.)
AFTER TWO BLOWS.
(FROM A PHOTOGRAPH.)

FIG. B.



9" PROJECTILE IN A WROUGHT IRON PLATE SHEWING THE FRACTURE THROUGH STUD HOLES.
(FROM A PHOTOGRAPH.)

DANGERFIELD, LITH. 22, BEDFORD ST COVENT GARDEN 12/85. 10982

To face p. 209.

portance, as it has been found in several experiments that large masses fell down when plates held on by only a few bolts were cracked through; they would have given good protection had they been held up in their places, even though detached from the rest of the armour; on the other hand, each bolt-hole weakens the armour to a certain extent. The union of the two materials, steel and wrought-iron, probably leads to complication and uncertainty in results.

Experiments with armour-piercing projectiles.

Experiments have within the past few years been carried out to determine the best **material** and **form** for **armour-piercing projectiles.**

With these objects in view, different makers were invited some years ago to send 9-in. projectiles of a uniform weight of 280 lbs. to a competitive trial: these were front fired at similar 12-in. wrought-iron plates 4 feet square, each shell having a striking velocity of about 1500 f.s.

The projectiles tried were of the following materials:—

(1.) Chilled cast-iron (the most numerous).
(2.) Chilled cast-iron heads and cast-steel bodies.
(3.) Cast steel.
(4.) Forged steel.

The prices ranged from £1 4*s.* 9*d.* for Service chilled, to as much as £15 for one of the forged steel shells.

The best results were obtained by Whitworth's forged steel shells, which were so little altered in shape, that some were fired twice, and one even three times through the 12-in. *wrought-iron* plate; this is, however, not such a severe test as firing against hard *steel-faced* armour with a high velocity.

The results with cast steel were not satisfactory, but the best of the first two classes, viz., chilled cast-iron and chilled cast-iron heads and steel bodies gave good results.

Form of armour-piercing projectile.

To determine, in addition, the best *form* for armour-piercing projectiles, the experimental Committee began by using rotating gas checks instead of studs, which formed lines of weakness along which splits developed on impact (Plate IV, fig. B). The heads varied from an ogival of 1·5 to 3 diameters; a flat-headed projectile was also tried, and some of special forms.

The trials were—

(*a.*) Front fire against 14 inches unbacked wrought iron.
(*b.*) Obliquely against a 12-inch plate at angles varying from 60° to 53° with the face.
(*c.*) Front fire against 12-inch compound plate (4 inches of steel welded to 8 inches of wrought iron).
(*d.*) Oblique fire against a 10-inch compound plate (4 inches of steel, 6 inches of wrought iron) at angles varying from 65° to 63° with the face of the plate.*

In (*a*) experiment the shells with sharpest heads did best.

In (*b*) an ogival of two diameters was found most suitable; although the R.L. chilled projectiles broke up, they produced rather more effect than the steel shells.

In (*c*) steel shells only were tried; Cammell's with cast steel chilled heads were rather the best. None remained unbroken after impact. The forged steel projectiles broke up into larger fragments

* *Vide* "Report on Experiments with Armour-Piercing Projectiles up to 9-inch Calibre." 22nd June, 1880.

than the others. Only one chilled iron shell was fired on this occasion, and that produced only a trifling effect.

In (*d*) the Cammell's steel projectiles were again a very little superior; the break up of all the projectiles was more complete than with front fire.

Conclusions. Taking all the experiments into consideration the Whitworth forged steel shells were considered to be of the best material, as they alone could have carried a bursting charge through the 12-inch plate.

Though chilled cast-iron shells do well against wrought iron, steel projectiles were considered necessary for the attack of steel-faced armour: since that time, however, chilled cast-iron projectiles have been made whose effect is almost as good as *ordinary* steel shells.

An ogival head, with a radius of two diameters was, on the whole, considered the best form of head, as, though a sharper point gave greater penetration with front fire, the reverse was the case with oblique fire. Flat-headed projectiles were not then found to be successful: but they penetrate water without deflecting, and also bite a plate when fired obliquely, at a smaller angle than an ogival-headed shell. At a trial of Grüson curved cast-iron shields in 1885, at Buckau, flat-headed shells bit the plate, and did far more damage than those with pointed heads.

Armour-piercing shells have been much improved of late years; in England a good chilled projectile is still employed: but France and Germany have both produced excellent steel shells; Holtzer's and Firminy's steel shells have lately (1886) proved very destructive to compound armour at Shoeburyness.

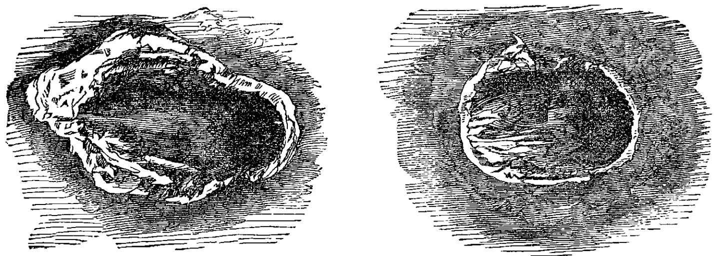
Value of $\frac{w}{d^3}$. The best weight for a projectile of given diameter was considered. To express a proportion between the weight and diameter, the latter must be cubed, since w is a quantity connected with capacity, which is of three dimensions, and the problem thus comes to finding the ratio of w to d^3 suitable for all armour-piercing projectiles; the Committee in 1880 considered that 0·37 was a value of $\frac{w}{d^3}$, which gave good results; but they were unable to pursue the subject fully.

Latterly, this matter has attracted a good deal of attention, and at present a value of $\frac{w}{d^3} = 0{\cdot}45$ to $0{\cdot}5$ is considered best; thus for the 10-inch projectile $\frac{w}{(10)^3}$ should be $= 0{\cdot}45$ to $0{\cdot}5$, whence $w = 450$ lbs. to 500 lbs.

Speaking in general terms, the advantage of a light projectile, with an equal charge, is that its muzzle velocity is higher, the trajectory is flatter, and the ease of hitting at short ranges is increased; with heavy projectiles, on the other hand, though the muzzle velocity is comparatively low, the resistance of the air produces less effect, and thus projectiles, heavy in proportion to the diameter, are advantageous at long ranges.

Oblique fire. With **oblique fire** (Fig. 3) it has been found that when wrought-iron armour cannot be perforated by frontal fire, that glancing takes place when shells strike at about 60° with the surface of the plate, but, when the plate has been below the power of the gun, the angle at which the shells pierced is about 50°. Penetration can doubtless be effected at smaller angles still, if the power of the gun is much more than that of the plate, as this is found to be the case with Nordenfelt bullets, which have penetrated thin plates at angles of 20°, and even less.

Fig. 3.—Effect of oblique fire against wrought-iron plates: projectile has glanced off.



On the other hand, steel-faced armour caused the projectile to glance more readily. When this armour is beyond the power of the gun, the projectile does not bite the plate at a less angle than about 65°, but when the armour is not a match for the gun firing frontally, penetration takes place at this angle.

Gun-cotton bursting charges.

It has been found that a gunpowder bursting charge cannot be restrained from exploding when the projectile strikes armour, and it cannot burst a tough steel shell. Experiments have therefore been made with *gun-cotton bursters*, which will not explode on impact; no accidents took place by premature bursts at the gun in the few rounds fired, and the shells exploded after penetrating thick armour, thus carrying fire through; this cannot be done by ordinary means. At present, however, our armour-piercing projectiles have no bursting charges.

Void space between plates.

When a *void space* has been left *between plates*, chilled projectiles have broken up on penetrating the first plate, and the fragments fell harmlessly on the inner one. As steel shells will, however, go through both plates without breaking, it is not proposed to make armour on this plan.

Calculation of Penetration.

Calculations.

Various formulæ, more or less empirical, have been given from time to time for **calculating the penetration** of solid wrought-iron plates. It was early noticed that the thickness (t) perforated in inches varied approximately with the energy of the projectile in foot-tons *per inch of circumference* (e). And thus the perforation of a projectile of one calibre could be compared with any other, provided the weights and velocities were known.

One of the first formulæ employed was

$$t = \sqrt[1\cdot 6]{\frac{e}{2\cdot 53}} \quad \text{(i)}$$

in which $e =$ energy in foot-tons per inch of circumference

$$= \frac{wv^2}{2g\pi d \times 2240}.$$

$w =$ the weight of the projectiles in lbs.,

$d =$ the diameter of the projectile in inches,

$v =$ striking velocity in feet per second.

The factor 2240 is introduced into the denominator to reduce the foot-pounds to foot-tons, and πd is employed to find the energy per linear inch of the circumference of the projectile.

It was found, however, with more extended experiments with high velocities, when plates a good deal thicker than the diameter of the projectile were perforated, that this relation is not very accurate. General Inglis, R.E., has prepared another formula, which has been reduced to one of the same form as the last, except that the square root of e is taken, and the coefficient is not constant, but varies slowly with variations of the proportion of the diameter of the projectile to the thickness of the plate or with the striking velocity—

$$t = \sqrt{\frac{e}{c}} \quad \text{.......................} \quad \text{(ii)}$$

where t and e have the same meaning as before; c must be found from the table given below :—

TABLE A.

Proportion of thickness of plate (t) to diameter of projectile (d). $\frac{t}{d}$.	*Approximate* striking velocity. f.s.	Corresponding value of c.	Proportion of thickness of plate (t) to diameter of projectile (d). $\frac{t}{d}$.	*Approximate* striking velocity. f.s.	Corresponding value of c.
0·5	500	1·300	1·3	1300	0·981
0·6	600	1·250	1·4	1400	0·964
0·7	700	1·178	1·5	1500	0·950
0·8	800	1·125	1·6	1600	0·938
0·9	900	1·083	1·7	1700	0·946
1·0	1000	1·050	1·8	1800	0·917
1·1	1100	1·022	1·9	1900	0·908
1·2	1200	1·000	2·0	2000	0·910

Example 1. Find the velocity with which an 8-inch projectile ($d =$ 7·97 inches), weight 182½ lbs., will just perforate a 12-inch wrought-iron plate.

$$\frac{t}{d} = \frac{12}{7{\cdot}97} = 1{\cdot}556.$$

Opposite 1·5 in Table A we find $c = 0{\cdot}95$ taking differences for 1·556, $c = 0{\cdot}9433$. Substituting in (ii) the values of t, e, and c, we have—

$$12 = \sqrt{\frac{182{\cdot}5v^2}{2 \times 32{\cdot}19 \times 2240 \times \pi \times 7{\cdot}97 \times 0{\cdot}9433}},$$

$$\text{whence } v = \sqrt{\frac{2 \times 32{\cdot}19 \times 2240 \times 3{\cdot}14159 \times 7{\cdot}97 \times 0{\cdot}9433 \times 144}{182{\cdot}5}}$$

This calculation is troublesome to do by arithmetic, but it is readily done by 4-figure logarithms (Table XIX (A), p. 318—319.

$$
\begin{array}{lll}
\log\ 2 & = & 0{\cdot}3010 \\
\log\ 32{\cdot}19 & = & 1{\cdot}5077 \\
\log\ 2240 & = & 3{\cdot}3502 \\
\log\ 3{\cdot}1459 & = & 0{\cdot}4971 \\
\log\ 7{\cdot}97 & = & 0{\cdot}9015 \\
\log\ 0{\cdot}9433 & = & \bar{1}{\cdot}9746 \\
\log\ 144 & = & 2{\cdot}1584 \\
\hline
 & & 8{\cdot}6905 \\
\log\ 182{\cdot}5 & = & 2{\cdot}2613 \\
\hline
 & & 2)6{\cdot}4292 \\
\hline
\log\ 1639 & = & 3{\cdot}2146
\end{array}
$$

whence $V = 1639$ f.s.

Formula (i) gives a result

$$v = 1633 \text{ f.s.}$$

If the striking velocity of the same projectile is increased to 1800 f.s., find the perforation to be expected. **Example 2.**

From Table A we find that the value of c for a velocity of 1800 f.s. may be approximately taken as 0·917.

Substituting values we have—

$$t = \sqrt{\frac{182{\cdot}5 \times 1800^2}{2 \times 32{\cdot}19 \times 2240 \times 3{\cdot}14159 \times 7{\cdot}97 \times 0{\cdot}917}}$$

$$
\begin{array}{lll}
\log\ 182{\cdot}5 & = & 2{\cdot}2613 \\
2 \log 1800 & = & \begin{cases} 3{\cdot}2553 \\ 3{\cdot}2553 \end{cases} \\
\hline
 & & 8{\cdot}7719
\end{array}
$$

$$
\begin{array}{lll}
\log\ 2 & = & 0{\cdot}3010 \\
\log\ 32{\cdot}19 & = & 1{\cdot}5077 \\
\log\ 2240 & = & 3{\cdot}3502 \\
\log\ 3{\cdot}14159 & = & 0{\cdot}4971 \\
\log\ 7{\cdot}97 & = & 0{\cdot}9015 \\
\log\ 0{\cdot}917 & = & \bar{1}{\cdot}9624 \\
\hline
 & & 6{\cdot}5199 \ldots\ldots 6{\cdot}5199 \\
\hline
 & & 2)2{\cdot}2520 \\
\hline
\log\ 13{\cdot}37 & = & 1{\cdot}1260
\end{array}
$$

whence $t = 13{\cdot}37$ inches.

When working out a number of penetrations, the calculations may be shortened, but they have been shown at full length in both the preceding examples. Sometimes it may also be required to find (E) the total energy in foot-tons, and (e) the energy per inch of circumference; these can readily be found at the same time by a slightly different arrangement, and by looking out one or two extra logarithms.

Formula (i) gives a result

$$t = 13{\cdot}57 \text{ inches.}$$

The method of calculation at present adopted is as follows:—it has been noticed that for all projectiles having the same proportion of weight to calibre $\left(\frac{w}{d^3} \text{ constant}\right)$, the penetration measured *in their own calibres* is practically the same. **Maitland's formula.**

The value of $\frac{w}{d^3}$ in the most reliable experiments was 0·37, and this is accordingly taken for the proportions of the standard projectile striking with velocity v_1.

To find the velocity of a projectile of other proportions with velocity v to perforate the same thickness of wrought iron ; this relation exists from the equation of energies,—

$$v_1 : v :: \sqrt{\frac{w}{d^3}} : \sqrt{0{\cdot}37}, \quad \ldots\ldots\ldots\ldots\ldots\ldots \text{(iii)}$$

and if k is the thickness *in calibres* perforated by the *standard projectile*, it was found by experiment that—

$$k = 0{\cdot}001 v_1 - 0{\cdot}14 \ldots\ldots\ldots\ldots\ldots\ldots \text{(iv)}$$

when the striking velocity is more than 700 f.s., and

$$k = 0{\cdot}0008 v_1 \ldots\ldots\ldots\ldots\ldots\ldots \text{(v)}$$

when the striking velocity is less than 700 f.s. (an unfrequent case).
from (iii) we have—

$$v_1 = \frac{v}{\sqrt{0{\cdot}37}}\sqrt{\frac{w}{d^3}},$$

and substituting this value in (iv) and (vi) they become

$$k = \frac{v}{608{\cdot}3}\sqrt{\frac{w}{d^3}} - 0{\cdot}14 \ldots\ldots\ldots\ldots \text{(vi)}$$

for velocities over 700 f.s.

$$\text{and } k = \frac{v}{760{\cdot}4}\sqrt{\frac{w}{d^3}} \quad \ldots\ldots\ldots\ldots\ldots\ldots \text{(vii)}$$

for velocities under 700 f.s.

The penetration in inches $t = kd$, if d is the diameter in inches, hence (vi) becomes

$$t = \frac{v}{608{\cdot}3}\sqrt{\frac{w}{d}} - 0{\cdot}14\,d \ldots\ldots\ldots\ldots \text{(viii)}$$

This is known as Maitland's formula of 1880, and it is now generally employed.

Example 3. Working out Example 1 by this method, assuming that v is more than 700 f.s. (if it should be less it will soon be discovered, as even the wrong formula of the two will give an approximately correct result, which will indicate that the other should be employed).

Substituting the values in (viii) we have—

$$12 = \frac{v}{608{\cdot}3}\sqrt{\frac{182{\cdot}5}{7{\cdot}97}} - 0{\cdot}14 \times 7{\cdot}97$$

whence $v = 1667$ f.s.

Example 4. The inverse problem to find the penetration when the striking velocity is known can also be determined by formula (viii) thus :—

Find the penetration of 10 in. M.L. Palliser projectile in wrought iron, striking velocity 1400 f.s., $w = 412{\cdot}4$ lbs., $d = 10$ inches.

Substituting the values in (viii) we have—

$$t = \frac{1400}{608{\cdot}3}\sqrt{\frac{412{\cdot}4}{10}} - 0{\cdot}14 \times 10$$

whence $t = 13{\cdot}38$ inches.

What must be the value of $\frac{w}{d^3}$ of a projectile if it will effect the same penetration in wrought iron as the standard projectile, but with 15 per cent. less velocity? Example 5.

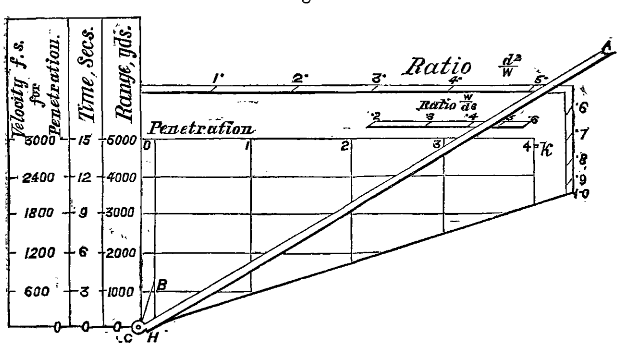
$$\text{Here } v = 0{\cdot}85 v_1$$

$$\therefore\ v_1 : 0{\cdot}85 v_1 :: \sqrt{\frac{w}{d^3}} : \sqrt{0{\cdot}37}$$

$$\text{whence } \frac{w}{d^3} = 0{\cdot}5122$$

A graphic diagram based on formulas (vi) and (vii) has been devised by Colonel Maitland, R.A. This saves calculation, and the results can be read off on a scale. A piece of strong cardboard, about ten times the scale shown on Fig. 4, is ruled over with a number of

Fig. 4.



horizontal and vertical lines at equal distances apart (of which only a few are shown in the sketch): towards the left bottom corner a flat metallic bar CA is pivoted, and at the top and right is a scale of Ratio $\frac{w}{d^3}$, and a scale of Ratio $\frac{d^2}{w}$. On the left side are three vertical scales of equal parts—one of velocity f.s. for penetration, another of times of flight from 0 to 15 seconds, and the third of ranges from 0 to 5000 *yards* (not *feet* as in Bashforth's tables) A scale of values of k is provided. In the rare case of the velocity being less than 700 f.s., an addition must be made to the value of k equal to the distance (measured on the k scale) intercepted between the fixed lines CB and BH. A detached flat wood scale is also employed, one side of which is graduated for distances and velocities, the other side for times of flight and velocities, the graduations in each case being simply derived from Bashforth's Tables of $\frac{d^2}{w}s$ and $\frac{d^2}{w}t$ respectively.

Fig. 5.

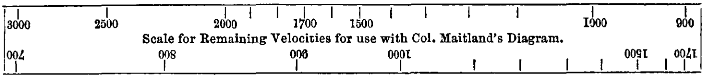


Fig. 5 represents the wood scale $\frac{1}{6}$th of its real size on the side referring to distance (or space in yards) and velocity. It is graduated more fully than can be denoted in a small sketch, and on it are the words as shown. On the other side are the words "Scale for Times of Flight for use, &c."

Both edges on each side are graduated, and thus a rather greater length of scale can be obtained than if one edge only is used; but as the graduations are all unequal, a considerable part has to be repeated, for if the graduations on the lower edge began at 900 f.s. where the upper one leaves off, it would be difficult to apply the scale when the velocity fell from any above 900 f.s. to any below it; consequently the upper edge in Fig. 5 is graduated for velocities from 3000 f.s. down to 900 f.s., and the lower edge from 1700 f.s. down to 700 f.s., and thus the part from 1700 down to 900 f.s. is on both edges; this gives ample margin, as it would never practically be required to find so long a range as that over which the velocity of a projectile falls from more than 1700 to less than 900 f.s.

To make use of the diagram for finding penetration, the radius rod must be swung round to the value on the scale "Ratio of $\frac{w}{d^3}$" which belongs to the projectile employed, this must be *calculated* in the ordinary way, or it may be found on reference to a previously prepared table.

If the velocity is given, carry the eye along a line horizontally from the scale "Velocity f.s. for Penetration," till it comes to the edge of the radius rod: then follow a vertical line till the k scale is reached; the value of k indicated gives the penetration *in calibres.*

If the penetration is given the velocity required can be found by beginning on the k scale, following a vertical till the radius rod is reached, and then along a horizontal till the velocity is indicated on the "Velocity f.s. for Penetration" scale.

This device is also constructed to work out the remaining velocities at the end of given ranges and times of flight (or the converse), in fact the problems connected with Tables VI and VII may be solved by its aid almost without any calculation (Figs. 6 and 7).

Fig. 6.

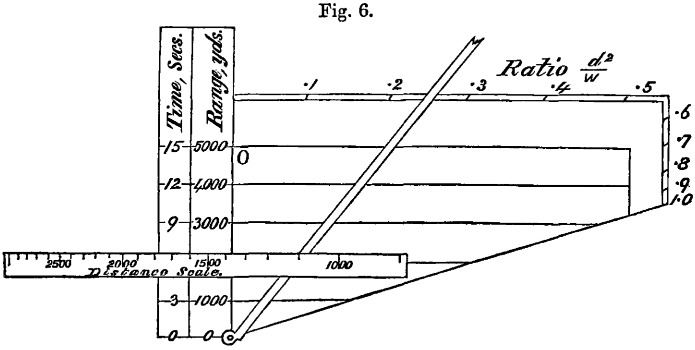
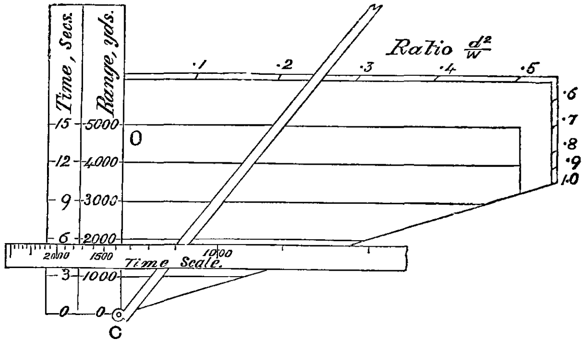


Fig. 7.



Note.—Figs. 6 and 7 represent the same diagram as Fig. 4, but for simplicity the scales "Velocity f.s. for Penetration," Ratio $\frac{w}{d^3}$ and Penetration k are omitted.

To make use of the diagrams for this purpose turn the radius rod to the calculated "ratio of $\frac{d^2}{w}$" (which should include the factors σ and τ if the projectile is one of new type, and if the atmospheric conditions differ from the normal), apply the wooden distance scale, and move it parallel to the horizontal lines till the velocity marked on it corresponding to the M.V. of the projectile employed cuts the vertical line CO, and the velocity on it corresponding to the striking velocity just touches the edge of the radius rod: the range can then be read off by noting the graduations on the vertical scale of "Range Yards" intersected by the wooden scale. If the muzzle velocity and range are given, it is evident that the final velocity can be found by this means, or if the range and final velocity are known the muzzle velocity can be determined.

The relation between time of flight and velocity is treated similarly (Fig. 7)—the wooden time scale being used with the vertical scale "Time secs.," in the same way as the distance scale was employed with the vertical scale of "Range Yards" (Fig. 6).

At what range will the 10-in. R.M.L. Palliser shell penetrate 10·2 inches of wrought iron if the M.V. is 1364 f.s. Example 6.

$$\text{Take } \frac{w}{d^3} = \cdot 414 \text{ and } \frac{d^2}{w} = 0\cdot 242.$$

The first thing to be done is to find the velocity to give the requisite penetration, and then, knowing the final velocity and the M.V., to determine the range.

As the calibre is 10 inches and the penetration 10·2 *inches*, the penetration k *in calibres* must be $10\cdot 2 \div 10 = 1\cdot 02$.

Turn the radius rod to 0·414 on $\frac{w}{d^3}$ scale (Fig. 4), carry the eye down vertically from 1·02 on the k scale till the edge of the radius rod is reached, and then look horizontally until the requisite velocity of 1100 f.s. is indicated on the "Velocity f.s. for penetration" scale.

Turn the radius rod to 0·242 on the $\frac{d^2}{w}$ scale (Fig. 6), and apply the wooden distance scale as shown, so that 1364 f.s. on it cuts the vertical line CO, and 1100 f.s. on it cuts the edge of the radius rod: a range of 2230 yards is then indicated on the vertical "Range Yards" scale.

Fig. 9 is adjusted to show the time of flight under the same conditions.

The results obtained agree very closely with those furnished by calculation, and time is saved. Mr. Niven's table of inclination in degrees and velocities (Table VIII, p. 293) has not been adopted for use in this diagram.

Orde Browne's rule.

Captain Orde Browne has furnished a quick rough rule of thumb, that the penetration (t) in inches is equal to the velocity (v) in *thousands* of feet per second, multiplied by the diameter (d) of the projectile in inches

$$\text{or } t = vd \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(ix)}$$

This is perhaps more correctly stated as follows:—With all projectiles (except those which are abnormally heavy for their calibre), the extreme penetration to be expected in wrought iron, is one cálibre for every thousand feet of velocity.

Example 7.

Working out Examples 1 and 2 respectively by this rule,—

$$12 = 7{\cdot}97\, v, \qquad t = 18 \times 7{\cdot}97$$
$$\text{whence } v = 1506 \text{ f.s.} \qquad \text{whence } t = 14{\cdot}35 \text{ inches.}$$

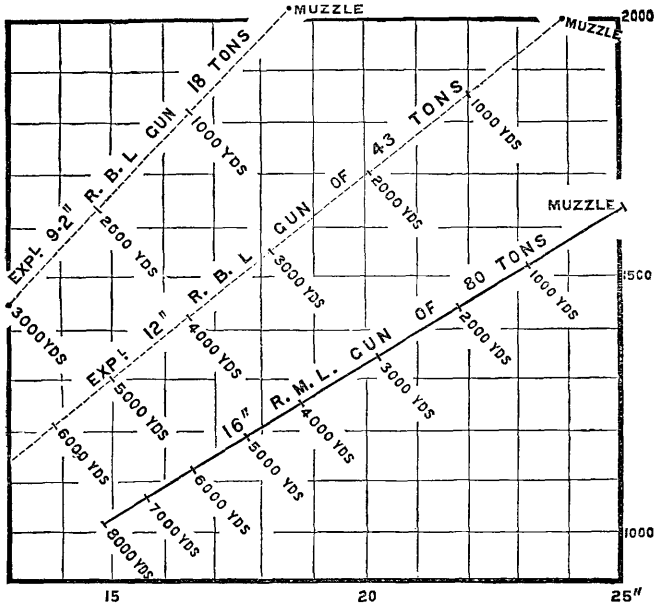
Comparing this with Colonel Maitland's plan, it will be seen that no account is taken of variation in the value of $\frac{w}{d^3}$, which is assumed to be constant; the newer armour-piercing projectiles are to have a value of $\frac{w}{d^3}$ of over 0·45, when this approximate rule will give more accurate results than at present. Making a rough allowance for the value of $\frac{w}{d^3}$, a rapid and fairly accurate estimate of probable penetration can be readily formed by this means.

By the use of Bashforth's tables of $\frac{d^2}{w}s$, the remaining velocity at any range may be calculated, and hence the probable penetration may be estimated by any of the above methods. General Inglis has prepared a diagram (Fig. 8) which shows at a glance the penetration of solid unbacked wrought-iron plates of average quality at different ranges, when front fired at by steel or chilled cast-iron shells, with ogival heads of $1\frac{1}{2}$ diameters. With heads of 2 diameters' radius, the penetration is from 5 to 10 per cent. more. If partial penetration only is effected, only about nine-tenths of the indicated depth is reached, as the unpenetrated metal gives great resistance. The word perforation is often used to express complete penetration.

Oblique fire.

When a target is penetrated by *oblique fire*, it is generally found that the projectile turns and penetrates nearly at right angles. It is supposed that this is caused by the head being suddenly stopped on striking the target, when the base tending to go on partly revolves

Fig. 8.—Part of General Inglis's Diagram of Penetration on a reduced scale.

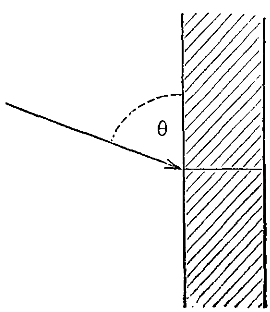


Thickness of wrought-iron plates in inches.

round it, as it is found that when projectiles glance, they frequently quit the plate base first; the normal velocity required for penetration becomes $\frac{v}{\sin\theta}$, if θ is the angle made with the face of the plate (Fig. 9), and the energy per inch of circumference required for perforation in (ii), p. 212 becomes

$$\frac{wv^2}{2g \times 2240 \times \pi d \sin^2\theta} \text{ ft. tons.} \ldots\ldots\ldots\ldots\ldots \text{(x)}$$

Fig. 9.



Inclined armour is now coming into use with Grüson's shields, and the barbettes of our ships have inclined compound plates.

CHAPTER XVIII.—THE PENETRATION OF EARTH AND MASONRY.

THE successful use of indirect fire by the Germans at the siege of Strasburg to breach an unseen and distant escarp has led to the development of this method of fire, which is rendered necessary by modern improvements. Since that time various experiments have been carried out with siege pieces by different nations, and those which we have undertaken will now be considered.

In 1880 trials were made at Lydd* to determine if practicable breaches could be formed at ranges of 1600 and 2500 yards without the expenditure of an excessive amount of ammunition. Preliminary experiments were conducted to find out the lowest effective limit for **striking velocity**, and also the **amount of obliquity** (horizontally) of the line of fire with the face of the work attacked, as it will often be necessary to fire obliquely at a wall which is well covered by a glacis in order to hit it sufficiently low down to make a breach without the employment of a very large angle of descent, when accuracy and velocity are both diminished.

An idea of the necessary conditions can be formed from the fact that the lowest effective striking velocity of the 70-lb. shell of the 6·3-inch howitzer with frontal fire was found to be about 300 f.s., but an excessive amount of ammunition was then required: when the velocity was increased to 400 f.s. the line of fire could be made as oblique as 55° with the face of the work, as although the first few rounds did little more than roughen the wall as they glanced, the way was prepared for the action of subsequent shells. With the 180-pr. shell of the 8-in. howitzer, and striking velocity 600 f.s., the fire was effective when this angle was as low as 40°, the greater obliquity allowable being due to the increased weight as well as to the increased velocity of the projectile.

Breaching Hidden Escarps.

Owing to the impossibility of very great accuracy of fire when attacking a distant covered escarp with a deep narrow ditch in front of it, the breaching experiments were conducted on the **demolition** plan, which consists in distributing the fire over the portion to be breached: this system presents the advantage that the masonry is well broken up, and it thus readily combines with the earth of the parapet to form a practicable slope.

The escarp wall attacked was made partly of brick and partly of concrete, and each breach was made half in one and half in the other material in order to compare their resisting power: the height of the

* See Reports of Experiments at Lydd; also "The Effect of Projectiles on Masonry and Earthworks," by Colonel Bayley, R.A., P.P.R.E., Vol. X, Paper II, from which the diagrams of this chapter are taken. Lecture by Captain Main, R.E., at Chatham, November, 1885, and "Lydd Experiments," by Captain Clarke, R.E., P.P.R.E., Vol. X, Paper VII.

wall was 18 feet, and the top of it was 4 feet below the crest of the glacis which was 43 feet distant.

Two breaches were made by the 8-in. howitzer of 70 cwt., and two others by the 6·6-in. howitzer of 36 cwt. with the following results :—

TABLE A.

Nature of piece.	Weight of shell.	Weight of bursting charge.	Range.	Horizontal angle of line of fire with face of wall.	Angle of descent.	Striking velocity.	No. of rounds fired.	No. of effective hits. On wall.	No. of effective hits. On parapet.	Breached.	Remarks.
	lbs.	lbs.	yards.	° ′	° ′	f.s.		p.c.	p.c.		
8″ Howr. of 70 cwt.	180	14	1600	54 30	14 18	579	112	34·8	29·5	Yes. 36 feet wide at neck.	
			2500	48 0	12 12	742	139	33·1	30·2	Yes. 30 feet wide at neck.	
6·3″ Howr of 36 cwt.	100	5½	1600	60 0	14 12	547	300	24·0	33·0	Yes. 30 ft. wide at neck.	
			2300	54 30	14 8	666	175	·15·0		No.	No more shells available; shooting inaccurate.

The rate at which firing could practically be carried on was found to be about 20 rounds an hour from each piece, consequently the **heavier howitzer** has the great **superiority** that it can form a breach in less than half the time taken by the other, and it requires a less total weight of ammunition than the lighter one : these advantages of the heavier howitzer are due to greater accuracy, increased shell power, and harder hitting. Such importance was attached to the employment of large capacity shells by the Committee carrying on the experiments that they recommended the use of jointed pieces, in cases where difficulties of transport might otherwise prevent such powerful ordnance from being placed in position.

The shingle of which the parapets were composed slid down more readily than earth would do, and consequently the breaches were doubtless made rather sooner than if earth had been employed: brickwork was found to give more resistance than concrete, and it was considered that the latter material should only be employed for foundations or other parts not exposed to fire.

The practice was carried on as nearly as possible under Service conditions, and the shooting was corrected by observing the effects of fire from the battery : it was noticed that when the glacis was struck the flash of the bursting shell was plainly visible, but it was not so clearly seen when the parapet was hit, though more material was moved; when the wall was struck no flash was seen. Towards dark, however, the strongly illumined smoke resembled the flash itself, but fragments of masonry rising up could generally be seen indicating a

hit on the hidden wall, and if the hit was low down only a flattened cloud of smoke appeared with fragments of masonry; the width of the breach could fairly be estimated from the amount of material which disappeared from the parapet.

Breaching detached hidden wall.

Another experiment was made in breaching a strong **detached wall** 13 feet 6 inches high, the top being 2 feet 8 inches below the crest of the glacis, which was 38 feet 3 inches distant, with results as under:—

TABLE B.

Nature of piece.	Weight of shell.	Weight of bursting charge.	Range.	Horizontal angle of line of fire with face of wall.	Angle of descent.	Striking velocity.	No. of rounds fired.	No. of effective hits on wall.	Breached.	Remarks.
	lbs.	lbs. oz.	yards.	°	°	f.s.		p. c.		
6·3″ Howr. of 18 cwt.	70	7 2	1000	60	20	370	123	28	A gap made 6 ft. wide, 2 ft. 9 in. high.	Wall also much shaken.

The shells were unsteady in flight from the small charge used in order to obtain the necessary angle of descent at the range of 1000 yards; and probably many rounds went through old gaps, as it was difficult to correct the shooting from observing the effect of fire.

The result of these experiments showed that curved fire can be employed in breaching hidden escarps and covered detached walls at a considerable range without the employment of an excessive amount of ammunition.

Fire at visible masonry.

With regard to fire at masonry, it had long been known that even granite can be gradually destroyed by the fire of comparatively light guns with suitable hard-headed projectiles. Ordinary cast-iron common shells break up on impact; the line of fire should not make a less angle than 60° with the face of the work attacked; and it has been found that a few well-directed blows from the heaviest guns rapidly do great damage. An experimental casemate thickly faced with granite, and with an iron shield at the gun port, in 1865, at Shoeburyness, received as many as 54 hits, the great majority on the granite, from 7, 8, 9 and 10-inch guns, at ranges of from 600 to 1000 yards, before it was considered that its fire would have been silenced.

Curved Fire from Guns.

Curved fire from guns.

In 1884 a few rounds with common shell for accuracy were fired from the 6-in. **gun** on a naval slide with a **reduced charge** of 6 lbs. R.L.G.[2] to obtain an angle of descent of 14° at a range of 1600 yards, as it was thought that cases might occur in the land attack of maritime fortresses when such pieces might be landed from ships of the Royal Navy; the shells were steady in flight, but their vertical dispersion was considerable on a wooden target 18 feet high: on an earthwork the effect of these shells was small as, though the line was good, the vertical dispersion was again considerable, and the striking

velocity being only 580 f.s., did not allow of sufficient penetration with the direct action fuze to attain the best effects from the bursting charges, but no shells scooped and rose out of the parapet. From the few rounds fired it appeared that fairly good results might be expected.

Fire at screened batteries.

The results of firing at a gun portion in a screened battery with high angle fire and common shells were not very great; as the effect of even the best howitzers is small, on account of want of sufficient accuracy; the conclusion arrived at was that it is best under these circumstances to fire howitzers with large charges and low angles of elevation, in order to obtain the maximum of accuracy.

Penetration in Concrete.

The penetration of unburst shells into concrete at short ranges to ensure hitting the desired spots was found to be as under in 1881:—

TABLE C.

Nature of gun.	Weight of shell.	Weight of bursting charge in common shell.	Range.	Striking velocity.	Maximum penetration when unburst. Palliser shell.	Maximum penetration when unburst. Common shell.	Mean penetration of common shells to point of burst, with direct action fuzes.	Remarks.
	lbs.	lbs.	yds.	f.s.	ft. in.	ft. in.	ft. in.	
16″ R.M.L., 80 tons.	1700	..	200	1586	34 0	..	..	This experiment was made previously at Shoeburyness. Concrete of broken granite.
10″ R.M.L., 18 tons.	408	20¼	145	1424	17 0	13 10	8 9	Concrete made of rounded pebbles.
6·6″ R.M.L., 70 cwt.	100	5½	41	1497	8 2	8 5	4 7	Concrete made of rounded pebbles.
6″ B.L., 80 cwt.	80	5	145	1893	12 7	10 9	5 9	Concrete made of rounded pebbles.

The tunnel formed by each shell was much more than the diameter of the projectile, especially as the velocity was increased. The 6-inch common shells of cast iron with high velocity did not break up, but, strange to say, they did so when fired against a strong earth parapet. Most of the shells deviated to the right, but a few went in straight, and one or two turned a little to the left in the butt. The sharp point of the Palliser projectile was supposed to account for its greater penetration. In no instances were the bursting charges of the Palliser shell ignited by the shock of impact, as is the case when armour is struck, but filled unfuzed 6-inch common shells burst in the butt from the heat generated by striking with very high velocity. With shells fuzed with direct action fuzes, the small time taken to burst the shell allowed of a very considerable penetration, and when delay action

fuzes were used with two shells from the 6·6-inch gun, which gave time for more penetration, the effect produced was not so good as in the other case, the small bursting charge of 5 lb. 8 oz. probably acting as an undercharged mine. The powerful shells of the 10-inch gun not only made craters 10 to 14 feet in diameter, but also shook the whole structure, and threw down the concrete in large masses; common shell were better than Palliser, on account of their larger bursting charges.

Penetration in Earth and Sand.

The **earth** was a loamy clay with lumps of chalk and old bricks, but the **sand** was remarkably pure. Shells were first fired unfuzed to determine the penetration, with the following comparative results in 1881:—

In Earth.

(1.) The penetration was variable, depending on the quality of the soil and its degree of dampness.
(2.) No shells turned more than half round, except one, which entered the shingle part of the butt.
(3.) All 6-inch shell with high velocity broke up on impact.
(4.) The displacement of material by those shells which did not break up was greater in earth than in sand.

In Sand.

(1.) The penetration was fairly constant, only from two-thirds to seven-eighths of what it is in earth.
(2.) The projectiles all turned completely round in the butt, and this probably lessened penetration.
(3.) 6-inch common shell with high velocity broke up on impact, but Palliser shell did not do so.

In agreement with the above (except perhaps in the penetration of sand being fairly constant) are the results of Italian experiments carried out in 1884, some of which are given below :—

TABLE D.

Projectile.	No. of rounds.		Striking velocity.	Material fired into.	Penetration.		
	Fired.	Effective.			Min.	Max.	Mean.
			f.s.		feet.	feet.	feet.
21 cm., or 8·27″	14	10	804	clay	16·0	20·2	18·1
	13	8	807	sand	10·5	13·1	11·8
15 cm., or 5·91″.	15	10	1565	clay	14·1	22·6	20·0
	12	10	1604	sand	8·2	13·1	10·6
12 cm., or 4·72″	6	4	1378	clay	12·0	15·4	13·7
	8	4	1414	sand	8·2	11·5	9·7

Extracted from translation by Major A. W. Anstruther, R.A., Pro. R.A.I., No. 4, Vol. XIV, from "Rivista di Artiglieria e Genio, 4/85.

The results of fire at a parapet 30 feet thick, exclusive of slopes, were as follows, common shell being used in every case mentioned in Table E (*see* also Plate V, Fig. *A*, p. 234) :—

TABLE E.

Nature of piece.	Charge.		Weight of shell.	Weight of bursting charge.	Range.	Striking velocity.	Fuze.	Number of rounds.	Nature of parapet.	Material displaced.	Remarks.
	Weight.	Description.									
	lbs.		lbs.	lbs.	yds	f.s.				cub.yds.	
10″ R.M.L. Gun ..	95	P².	408	20¼	195	1416	direct action	3	Earth	63	Through in 2 rounds.
							direct action	3	Sand	24	
′ R.M.L. Howr.	11½	R.L.G².	180	14	195	921	direct action	10	Earth	54	Through in 8 rounds.
							delay action	10	Earth	27	Through in 7 rounds.
							direct action	10	Sand	18	
6·6″ R.M.L. Gun..	25	P.	100	5½	195	1452	direct action	12	Earth	17	
6·6″ R.M.L. Howr.	5	R.L.G².	100	5½	195	841	direct action	10	Earth	12	
6″ B.L. Gun.....	42	P².	80	Nil	195	1875	Nil	10	Earth	12	
				5	195	1875	direct action	10	Earth	16	Shells broke up on impact.

Under these circumstances high velocity is *not* generally an advantage, as common shells of cast iron break up on impact. In firing at an earthwork with a high velocity projectile, the first round or two generally penetrates well into the exterior slope if it is fairly steep, but its course in the earth is very erratic, and it *may* turn upwards towards a line of least resistance, and get clear out before bursting, unless a quickly acting fuze is employed. As the firing goes on and the breach grows larger, with sides of loose gently sloping earth, gun shells tend to ricochet at nearly every round, and it is only by the use of the quickly acting fuze that their small bursting charges can displace any appreciable quantity of earth. For work of this kind it appears that with a given weight of piece, a howitzer with large bore, and consequently having a shell of large capacity though with low muzzle velocity and inferior accuracy, is preferable to the smaller bore with smaller capacity shell, though with higher velocity; in other words, it is better to have a large proportion of gunpowder in the bursting charge of the shell, but not so much in the cartridge in the piece. The high velocity shell, however, possesses superior accuracy, this is specially the case when the wind is gusty; and if a high velocity shell does not burst it displaces more earth than one with a low velocity. Although delay action fuzes *occasionally* produced very good results with howitzer shells, the Committee recommended that only the quickly acting direct action fuze should be employed. The great

effect produced by the 10-inch shell in breaching a 30-foot parapet in two rounds is worthy of notice, and demonstrates the advantage of the employment of heavy ordnance when possible.

The superior resisting power of sand over earth was confirmed in this experiment, when it was also found that bursting shells scatter less sand than earth.

Breaching Visible Earthworks.

In 1882 the various siege pieces were tried against each other, in **breaching visible earthworks** 30 feet thick, from the first and second artillery positions, 2400 and 1200 yards. At the longer range several pieces produced only insignificant results, but at the shorter range all did well, the 8-inch howitzer of 70 cwt. being much the best.

The arrangement for reverse laying with the howitzer was found conducive to good shooting when the target was indistinct and distant; on the other hand, the coarse sighting arrangement of the 6-inch gun was difficult to use with accuracy.

The following are the results of 20 rounds from those pieces which did best:—

TABLE F.

Nature of piece.	Charge.		Weight of shell.	Weight of bursting charge.	Range.	Angle of descent.	Striking velocity.	No. of effective hits.	Earth displaced.	Remarks.
	Weight.	Description.								
	lbs.		lbs.	lbs.	yds.	°	f.s.	p.c.	cub. yds.	
8″ R.M.L. Howr. 70 cwt.	11½	R.L.G.[2]	180	14	1200	4	876	85	46·0	Through in 7 rounds.
					2400	9	809	25	17·6	
6″ B.L. Gun	42	P[2]	100	6½	1200	1½	1552	80	26·0	Shells all broke up.
					2400	3¼	1286	30	14·13	It is believed many shells broke up.

The 6·6-inch howitzer produced about the same effect as the 6-inch gun at 1200 yards, but was inferior to it at the longer range.

Experimental firing was carried on in 1884 against earthworks having **gradual exterior slopes** of 15° and 8°, as it was thought that they would greatly tend to deflect projectiles on striking. The result with the 8-inch howitzer was that the effect of the earlier rounds was less than when the slope was steeper, as not only were several of the first shells liable to be deflected, but those which did penetrate to a good distance had only a slight depth of earth above them, and consequently when the large bursting charges exploded only small craters were formed. It was considered from the rather scanty data obtained that almost double as many rounds are required

Gradual exterior slopes.

by the howitzer at 1200 yards to breach an earthwork having an exterior slope at 15° as another which has the ordinary slope of 45°.

The advantage of the gentle slope to resist the fire of common shell from guns was not so apparent. The 6-inch gun was fired with a charge which gave a striking velocity of 1375 f.s., and the shells did not break up on striking the gentle exterior slope, but they did not experience a very great resistance, as they ploughed along and raised a considerable quantity of earth in doing so; on the other hand, with the steeper exterior slope, when a shell penetrates at the beginning of the firing the effect of the small bursting charge is smothered. As the practice went on against the parapet with the gentle exterior slope, the projectile penetrated into the heart of the mass, when, notwithstanding the reduced striking velocity, a proportion of the shells broke up from the increased resistance. The condition of dampness of the clay parapet being different at different times prevented a very accurate comparison from being made, as damp clay or earth is more readily breached than that which is dry.

In 1885 it was determined to discontinue the use of the alluvial clay which is found at Lydd, as it is most easily penetrable by projectiles, and it suffers great disruptive effects.

An artificial mixture of two parts sandy loam and one part clay loam was then employed for the parapets, with the result that much greater resistance was obtained: this was probably partly due to the unusual dryness of the season. The 8-inch howitzer and the 6-inch gun were fired at a 30-feet parapet with an exterior slope of 15°, and the 9·2-inch gun at one with a slope of $\frac{1}{2}$: the resistance was considerable, and it was considered that this thickness of sandy loam gives efficient protection for both coast and land defence.

A layer of 1 foot of sandy loam over a clay parapet was found to increase the resistance considerably; but layers of rails, which were tried as a protection, were hurled about in a destructive manner when struck.

Fire of field and medium guns at parallels.

In 1884 the **first parallel** as at present constructed was cut through by the 6-inch gun in one round at a range of 800 yards. The shells did not break up on impact, presumably because the parapet did not offer sufficient resistance; one round was required from the 5-inch gun for the same purpose, unless the shell struck very near the ground line, when a second was necessary, and the 12-pr. B.L. gun cut through the parapet in from four to six rounds, when a steel shell having a bursting charge of P. and F.G. powder mixed together was employed. To form a breach 10 yards wide, it was roughly estimated that about 9 rounds from the 6-inch gun, or 18 from the 5-inch, or 40 from the 12-pr. would be required.

In 1885 it was found that sandy loam offered more resistance than clay: with the 6-inch gun the difference was slight, but with the smaller natures of ordnance the expenditure of ammunition would be considerably increased when breaching sandy parallels.

The 5-inch gun cut through the second parallel (of clay) at a range of 400 yards in one round, and the 12-pr. also did so in 3 rounds. The shell of the 6-pr. quick firing Hotchkiss gun has but a small bursting charge; nevertheless owing to its accuracy and high velocity it cut through the parallel in some 15 rounds.

As the parallels are very long, can easily be repaired, and afford concealment from view, it would not probably be often worth while to fire against them with medium or field guns, except perhaps when the guard of the trenches discover their position by opening fire. Certain parts might perhaps be made stronger than others to afford protec-

tion to masses of men behind them; the present forms of parallels hide men behind them from view, but they give little protection against artillery fire from even light guns.

Field guns, and the heavier quick firing guns firing 6-pr. projectiles, were directed against saps with such destructive effects that it seems doubtful if sapping can in future be carried on except at night: it must be remembered, however, that the besieger will do all in his power to silence such pieces by converging on them the fire of several accurate guns. Fire at saps.

Saps may perhaps be made deeper in future, or false saps may be started with the object of diverting the enemy's fire from the real ones.

Silencing the Fire of Guns.

It was considered from the experience gained in 1883 that the best way to silence guns mounted in permanent or in siege works by frontal fire is by the employment of shells with the largest possible bursting charges to destroy the protecting earthwork, as a gun showing above a parapet, especially at the longer range, is a target extremely difficult to hit. The 8-inch howitzer of 70 cwt. showed itself the most suitable of all the siege pieces for this purpose at the longer range of 2400 yards. At the shorter range (1200 yards) when the 8-inch howitzer was not used, the 6·6-inch gun and the 6·6-inch howitzer each dismounted a gun in a sunken battery with a covering screen in front in 20 rounds; the former had also just cut through the parapet. A 6·3-inch jointed wire gun was also fired at this range, but though its shells displaced a large amount of earth the shooting was inaccurate, the parapet was not breached, and the gun fired at was not dismounted. Silencing guns by frontal fire.

In 1884 further trials were carried on at the 2400 yards range against two dummy wooden guns on overbank carriages in an ordinary screened sunken battery, the effects of fire being corrected by observation from the firing battery and flanks only as follows:—The officer on range duty held up a plank occasionally for a few seconds to give an indication of the position of the target battery resembling that afforded by the smoke of its guns at the moment of firing.

The *line* was obtained at the firing battery by pointing the sights of a straight edge on this plank when it was held up; an auxiliary mark was then laid out in front, and the gun laid on it, with suitable deflection, as afterwards found to be necessary by looking over the sights of the straight edge which had not been moved, and observing the position of the burst of the shell to be to the right or to the left. The officers at the flanks (whose positions should be to the front, clear of smoke and on an elevation if possible), aligned their straight edges on the plank when held up, and as the shells burst right or left of the fixed line thus obtained, they signalled over or under to the battery. Excellent results were obtained; some such plan seems an absolute necessity when firing at guns in a screened battery. Common shell with percussion fuzes were employed, and the results of fire were as follows:—

TABLE G.

Nature of Piece.	Charge.		Angle of descent.	Striking velocity.	No. of rounds fired.	Remarks.
	Weight.	Description.				
	lbs.		° ′	f.s.		
8″ R.M.L. Howr..	11½	R.L.G.[2]	9 12	809	23	22nd round; shell burst on superior slope and blew off chase of a dummy gun; probably would not have silenced a real gun. Proper straight edges not provided.
6·6″ R.M.L. Gun .	25	P.	5 38	892	23	20th round dismounted a gun by a ricochet.
6″ B.L. Gun......	34	P[2].	3 50	1150	23	12th round dismounted No. 1 gun by a direct hit. 22nd round dismounted No. 2 gun by a shell bursting in the scoop: uncertain if a real gun would have been silenced.
5″ B.L. Gun......	16	P.	4 15	1064	25	17th round dismounted No. 1 gun by a direct hit. 20th round burst on superior slope; a splinter knocked off left trunnion of No. 2 gun; uncertain if a real gun would have been silenced.

It appeared from this experiment that guns can often be dismounted by frontal fire before the parapet in front of them is cut through.

In 1885, 16 rounds were fired from the 8-inch howitzer at a dummy gun, mounted on a hydro-pneumatic disappearing carriage; although the parapet was a good deal damaged the results were not considered conclusive.

Silencing guns by enfilade fire.

In 1883 experiments were made in silencing guns by **enfilade fire.** Although a traverse 20 feet thick and 9 feet high was employed, it appeared that guns could be readily dismounted, and again the advantage of the large bursting charge of the 8-inch howitzer was apparent. At very long ranges, however, when the trajectory of guns is considerably curved, it was expected that their superior accuracy would render them more suitable than howitzers; but in 1885 the 6·6-inch gun fired at a range of 2400 yards produced only a slight effect when enfilading 2 guns. The 8-inch howitzer would probably have been more efficient. It appeared that the protection afforded by the present arrangement of traverses against enfilade fire of modern siege pieces is inadequate.

High Angle Fire at Field Magazines from the First Artillery Position (range 2400 yards).

Fire at field magazines.

In 1882 three howitzers were employed, viz., 8-inch of 70 cwt., 8-inch of 46 cwt., and the 6·6-inch of 36 cwt., each firing 30 rounds. The target consisted of two empty **field magazines** (to double the chances of hitting), one roofed with one layer of oak timbers 12′ × 12″ × 12″, the other with two layers of fir 12′ × 10″ × 10″, and

both covered with 5 feet of earth. The projectiles fired against them were common shells with quick (direct action) fuzes: the results were that the penetration before burst was only 3 feet to 3 feet 6 inches on account of the quick action of the fuze, and although a good deal of earth was displaced and a few cap sills broken, no material harm was done, and the magazine would not have been blown up unless one shell had fallen into an old crater (an unlikely event). Rounds were fired at blindages with similar results.

The experiments were continued in 1883 when the 8-inch howitzer of 46 cwt. was not employed, as it had failed in accuracy; the magazine was strengthened by an additional $1\frac{1}{2}$ feet of earth on the top; instead of the direct action percussion fuze, unbored 30 seconds time fuzes with detonators having thin (0·012-inch) suspending wires, to ensure action with the small charge in the howitzers, were employed to imitate the action of delay action percussion fuzes, which were not then available: the results were much better than before, the heavier shells burst right down on the baulks, having penetrated the superincumbent earth: a few of these wooden fuzes, however, were driven in on impact and burst their shells at once.

In 1884 only the 8-inch howitzer of 70 cwt. was employed, as although the accuracy of the 6·6-inch was good, its bursting charge was not so effective as that of the heavier piece; suitable experimental delay action fuzes had been constructed to act with small charges: some were fixed in the nose and another pattern in the base of the shell: the target was a single magazine, but larger than the Service one to ensure several hits (*see* Plate V, Fig. *B*, page 234) and it was further strengthened, having 7 feet thickness of earth above it and a double row of light rails above the baulks; nevertheless, out of 26 rounds, one (No. 20) which hit the top near the exterior slope penetrated 18 to 20 feet and burst as it struck the sheeting; this would certainly have blown up the magazine; two other rounds would very possibly have blown it up, the last one injured the structure so much by penetrating and bursting on the rails of the overhead cover that the broken *débris* which fell down prevented the magazine from being entered.

Further experiments in the same year were made with the 8-inch howitzer against a magazine still more strengthened by the addition of an extra layer of fir baulks, with a double layer of iron rails over them: this was intended to intercept any shell which might take a line towards the front side of the magazines (Plate V, Fig. *C*). Most of the (16) rounds were with experimental shells having sharp pointed heads (ogivals of 6 diameters) and with delay fuzes in their bases; they were steady in flight, and very good practice was made. Three shells were of steel containing bursting charges (21 lbs. 11 oz.) more than half as much again as those in the Service pattern of cast iron (13 lbs. 10 ozs.); two of these rounds were effective: one struck the berm and formed a very large crater, the next round (No. 6) struck 2 feet higher up in the crater just formed, and burst after penetrating about 16 feet to a point 5 or 6 feet from the front frames of the magazine (Plate V, Fig. *D*); hardly any earth was disturbed externally, but the whole of the front sheeting of the magazine was blown in, and it was about one-third filled with earth; a chamber 7 feet deep, about 10 feet in diameter, and extending up to the baulks of the extra layer, had suddenly been formed in the middle of the earth mass by the explosion of this shell. Other rounds also produced destructive results.

From this experiment it was concluded, although the magazine was

undoubtedly stronger than before, that the front side was still liable to be blown in by shells penetrating a certain distance into the earth and then bursting.

In 1885 the results of firing were inferior; the practice was inaccurate with the long shells employed, and but few effective hits were obtained, only one of which produced a large crater, and even that did little damage to the magazine itself.

The following table gives the results of the firing with the 8-inch howitzer of 70 cwt. from 1882 to 1885:—

TABLE H.

Nature of Piece.	Range.	Striking velocity.	Angle of descent.	Year of firing.	Fuze.	Number of rounds fired.	Effective hits.	Remarks.
	yds.	f.s.	°				p.c.	
8″ Howr. 70 cwt.	2400	486	32 to 35	1882	Direct action.	10	0	Weather unfavourable.
				1882	Direct action.	20	30	Penetration slight before burst, consequently small craters.
				1883	Wood, time, unbored.	10	20	Craters larger than in previous year.
				1884	Experimental (nose) delay percussion.	26	15·4	Weather favourable: other hits partly effective; magazine would have been blown up.
				1884	3 with nose delay action, 13 with base delay action.	16	62·5	Practice very good.
				1885	Special delay action.	24	..	Practice inaccurate.
				1885	Special delay action.	20	..	

An enemy's magazine is generally hidden from view; unless its position can be *seen* from some accessible spot so that the shooting may be corrected, it appears to be almost hopeless to attempt to blow it up by high angle fire; on the other hand, these experiments demonstrated that in order to *ensure* the protection of a magazine from high angle shell fire with large bursting charges, whether from a chance round or from careful firing, a very enormous amount of heavy baulks and iron rails are required. An alternative plan was suggested of placing powder in metal-lined cases in numerous traversed recesses below the ground, with covering masses of earth in front; these would necessitate the employment of a very small amount of material, and the blowing up of one of them containing only a limited quantity of powder, would not be so serious as that of a magazine of present construction, which holds a large amount.

Steel Common Shells.

Strong **steel common shells** were tried for the double purpose of not breaking up on impact at high velocity and to contain larger bursting charges, which were still further augmented by the use of compressed powder. The cast-iron common shell of the 6-inch gun contains 5 lbs. of powder, but the longest steel shell will hold $17\frac{1}{2}$ lbs. of compressed powder. As the walls of steel shells are thin, they are longer than cast-iron ones of equal weight, and this probably accounted for the unsteadiness of some of them in flight, the ordinary twist of rifling not being sufficient.

It was found that a quickly acting percussion fuze fitted in the base of a steel shell, and which was in immediate contact with the charge, burst the shell much sooner than a direct action fuze in the nose of a cast-iron common shell in which there is a space caused by the setting back and caking of the bursting charge on firing the piece. So quick was the base percussion fuze in its action that the shell burst almost immediately on entering the earth, and a mere flake only was blown off, and thus the powerful bursting charge was of little effect: one round, however, which for some reason was delayed in bursting, produced a large crater. The best results at 1200 yards were produced by light 6-inch shells containing $12\frac{1}{2}$ lbs. of compressed powder and base delay fuzes; the clay parapet 30 feet thick was breached in 15 rounds. The same number of rounds with a heavier 6-inch shell containing $17\frac{1}{2}$ lbs. of compressed powder produced only a small effect, as the long shells were unsteady and inaccurate in flight.

The steel shells had the advantage over cast iron ones that they did not break up (generally) on impact, but, on the other hand, a cast-iron shell breaking up on impact moves more earth than a steel shell bursting on the surface, but the latter when burst properly in the work is by far the most effective.

It is thus seen how very much depends on the correct action of the fuze: it appears that a quickly acting one is best for common shells with high velocity from guns, as they are likely to scoop and rise out of the parapet, yet not a *very* quick one, which bursts a shell at the surface, blowing away only a mere flake of earth; with howitzer shells at low angles of descent, a comparatively quickly acting fuze may also be employed, but at considerable angles of descent when projectiles have no tendency to rise, and when it is desirable for the destruction of magazines and overhead cover that shells should burst at extreme penetration, a long delay action fuze is most suitable.

Steel shells with various natures of bursting

Experiments were conducted in 1884 to compare the effects of steel shells filled with ordinary shell L.G. powder, with others containing P and pistol powder mixed: the 6-inch shells employed held from 8 to 9 lbs. of the former, but 10 to $12\frac{1}{4}$ lbs. of the mixture could be put into them. The shells were not fired filled and fuzed into earth, as there were no means of ensuring that they would be equally tamped on penetration; they were first fired filled with powder but unfuzed, in order to obtain the setting back and caking action on the shock of discharge, and also the supposed setting forward movement on striking. Some of those fired were recovered, carefully buried, and exploded by electricity. It was found that a shell with a bursting charge of 9 lbs. L.G. powder loses in effect when burst at a greater depth than about 5 feet below the surface; the other shells, however, produced excellent results when exploded at a depth of 6 feet, when each displaced about $12\frac{1}{2}$ cubic yards; this greater effect is believed

to be due not only to the greater charge of P and pistol powder, but also to less loss from caking on the shock of discharge than with the shell L.G. powder.

Shrapnel Shell Fire.

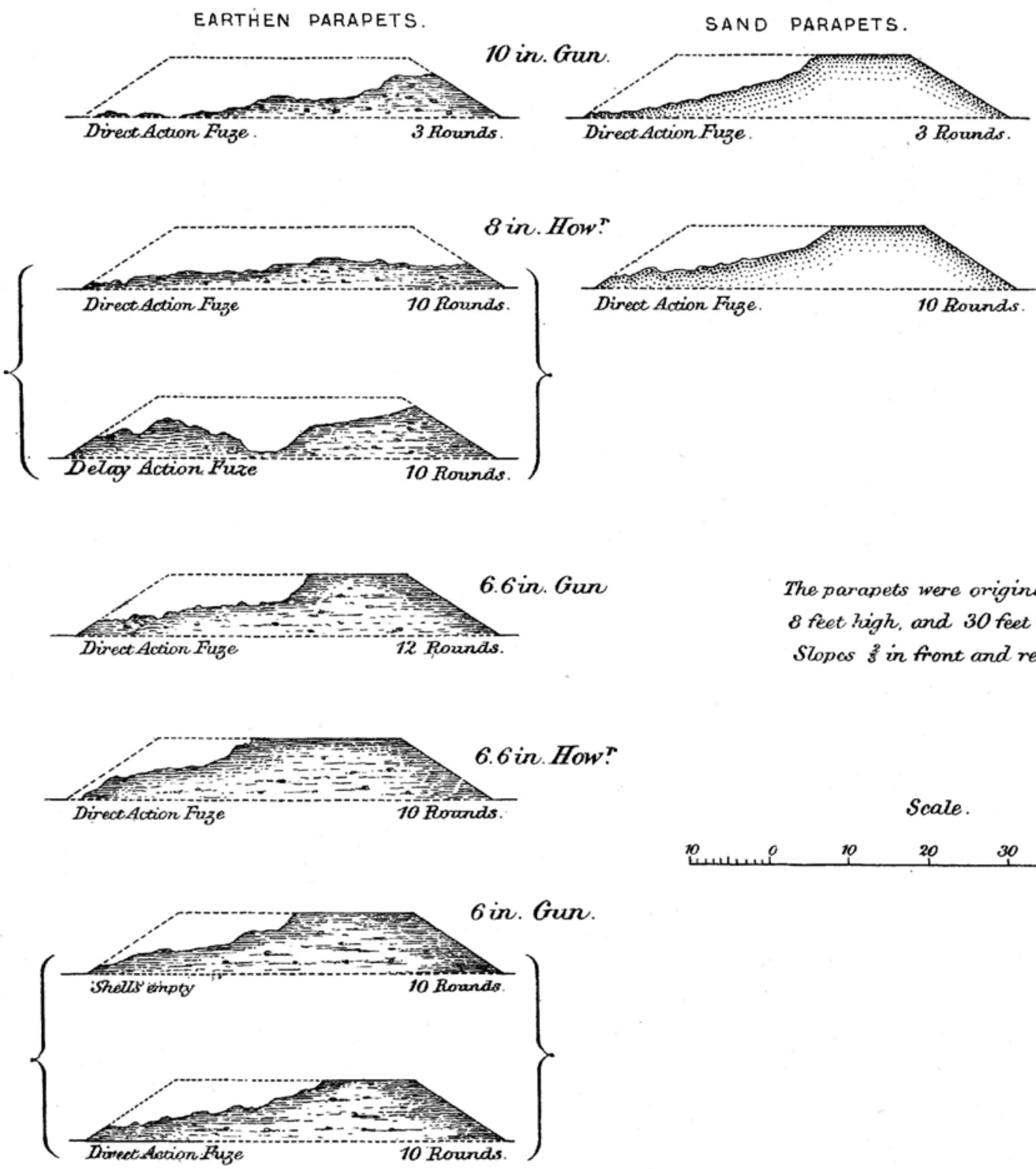
In 1882 the effect of **shrapnel**, *under favourable circumstances*, was found to be considerable, the 8-inch howitzer of 70 cwt. giving 234 hits (lodged or through; splinters and balls which did not lodge not being counted) on three rows of wooden targets at 2400 yards in 18 rounds; the results obtained tend to show that an enemy's working parties can be prevented from repairing a breach; and that the interior of works which have no overhead cover can be searched out. In the following year a comparison was made between service cast-iron shrapnel and steel shells at a range of 1600 yards, which resulted in the success of the latter, although they contain a less number of bullets than the others. A steel shrapnel has a body so strong that it does not break up when the large bursting charge acts, but the bullets are projected from it like case-shot from a small cannon, and thus their striking velocity is increased; the steel shells, however, had the advantage of the more accurate Armstrong, metal time fuze, while the cast-iron shells had only the wooden 15 secs. fuze. Although the results appeared to show the decided superiority of the steel shell, the experiments were too limited to settle the matter: as a single round bursting in a good position produces a better effect than the combined action of many others, the accuracy of the time fuze being of the first importance. Experimental firing at a range of 1200 yards at a battery concealed from view by a screen in front, seemed to show that the direction could be fairly obtained from the battery by observing the position of the flash of the enemy's gun, and if flanking parties are pushed well forward, provided with pivoted straight edges or some similar arrangement, and with the means of signalling the results of their observations to the battery, a close approximation could generally be obtained of the distance under or over of the burst for each round, though doubtless the expenditure of ammunition would be considerable to produce any great effect. In 1885 it was found that steel plates $\frac{1}{4}$-inch and $\frac{1}{2}$-inch thick gave good protection against shrapnel shell fire from the 6·6-inch and 8-inch howitzers respectively at a range of 1600 yards.

In 1884 rounds of 10-inch shrapnel shell were fired from H.M.S. "Sultan" at a gun mounted *en barbette* at Inchkeith, but several causes prevented the fire from being accurate, and the results were hardly conclusive.

Although peace experiments are valuable in showing the comparative value of various natures of fire employed at sieges, it should be remembered that much less effect must be expected on active service, when the conditions cannot be so favourable as on the practice ground.

FIRE AT PARAPETS AND MAGAZINES.

FIG. (A)



The parapets were originally
8 feet high, and 30 feet thick,
Slopes $\frac{2}{3}$ in front and rear

Scale.

MAGAZINES

FIG. (B)

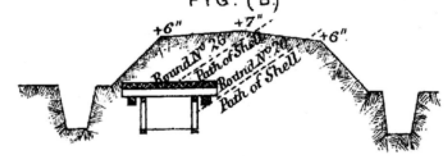


FIG. (C)

Penetration of Shell (Round No. 6) before bursting.

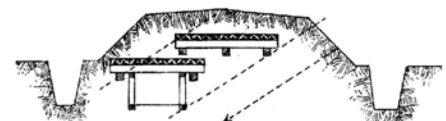
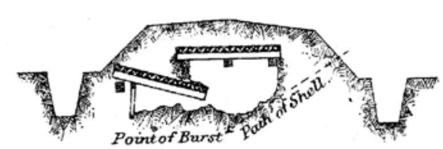


FIG. (D)

Crater formed in the interior of the Earth, by the bursting of Shell (Round No. 6.)



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PART II.

CONTENTS.

CHAPTER I.—GUNNERY INSTRUMENTS.

We shall now proceed to some of the more difficult problems in gunnery, for a full consideration of which, however, reference will be made to the standard works which are devoted to each subject. Part II is arranged in the same general order as Part I, with trifling exceptions in Chapter I, which is somewhat miscellaneous.

Internal Ballistics.

(*See* also Part I, Chaper II).

We shall commence, as in Part I, with the subject of **Internal Ballistics**. Noble and Abel obtained their results (quoted in Part I, Chapter II) from calculations on the data furnished by experiment* as follows:—v is the volume of the interior of the explosion chamber, and δ the fraction of it occupied by the charge (gravimetric density reduced to unity), whose volume is then $v\delta$; suppose α the fraction of the volume of the charge occupied by the residue formed on explosion, this must be a volume $\alpha v\delta$. Consequently the space to be occupied by the exploded gas is $v - \alpha v\delta$ or $v(1 - \alpha\delta)$; and since $v\delta$ (the volume of the powder charge) is proportional to the volume of gas produced, its density must be proportional to $\frac{v\delta}{v(1 - \alpha\delta)}$.

According to Boyle's law the pressure p exerted by a gas is proportional to its density—

$$\therefore p = R\frac{\delta}{1 - \alpha\delta}, \quad \ldots\ldots\ldots\ldots\ldots\ldots \text{(i)}$$

in which R is some constant.

Values of p had been obtained for corresponding values of δ by pressure gauges.

For instance, when

$\delta = 0{\cdot}85$, p was found $= 4350$ atmospheres, or $28{\cdot}5$ tons on sq. in.

$\delta = 1{\cdot}00$, p " $= 6400$ " " $42{\cdot}0$ " "

Substituting successively these values for p and δ in equation (i), R is eliminated, and we obtain $\alpha = 0{\cdot}6$ nearly.

Had other experimentally determined values of p for different values of δ been taken, approximately the same value for α would be obtained, but generally a little higher. The experimenters had reason, however, to believe that this was the most trustworthy result, and consequently they adopted it as the mean value for α.

This shows that the residue, which is liquid at the moment of explosion, thus occupies about $0{\cdot}6$ of the volume of the original charge; when cold and solid, it is found to be only about half that bulk: to verify the correctness of the value of α, some residues were heated in crucibles and melted; at about the supposed temperature of explosion

* *See* "Phil. Trans., R.S.," "Fired Gunpowder," 1875 and 1880.

it was found that the volumes were nearly double what they had been when cold and solid.

They found the **temperature of explosion** thus—

If t is the temperature reckoned in degrees Centigrade from absolute zero ($-273°$ C.), this relation holds good (Boyle's and Charles' law combined).

$$pv = Qt, \quad \ldots\ldots\ldots\ldots\ldots\ldots \text{(ii)}$$

where Q is a constant, v is the volume occupied by the exploded gas, and p is the pressure exerted by it.

Suppose the gravimetric density is unity; taking the value $\alpha = 0{\cdot}6$, it follows that the gases must be confined at the moment of explosion in only 0·4 of the interior space. It was found by direct measurement of the gases drawn off from the explosion chamber that their volume at 0° C. and 760 mm. is 280 times that of the original charge. If therefore they are compressed $\frac{10}{4}$ of $280 = 700$ times, they exert a pressure of 700 atmospheres at 0° C.

Hence by substitution in (ii) we can obtain the value of the constant Q when $t = 0°$ Centigrade $= 273°$ reckoned from absolute zero.

$$\therefore 700 \times 0{\cdot}4 = Q\,273,$$
$$\text{whence } Q = 1{\cdot}0256.$$

But at the moment of explosion we know from the crusher gauge that the pressure is some 6400 atmospheres; v and Q remain the same; hence in this case—

$$6400 \times 0{\cdot}4 = 1{\cdot}0256t,$$
$$\text{whence } t = 2496°.$$

But this is the temperature of explosion reckoned from absolute zero: subtracting from it 273°,—to bring the scale to the ordinary zero Centigrade (that of melting ice), the temperature of explosion becomes

$$2496° - 273° = 2223° \text{ Centigrade.}$$

Velocity Instruments.

(*See* also Part I, Chapter X).

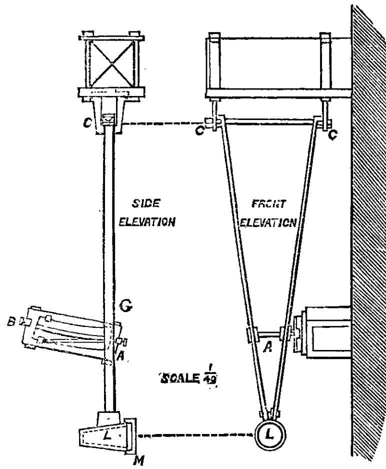
The **ballistic pendulum**,* invented by Robins, and described by him in 1765, was the instrument employed for finding velocities until about 30 years ago; although it has been entirely superseded by electrical chronographs, which are more convenient, it is well to consider the important dynamical principles involved in its employment. Ballistic pendulum.

It consists essentially of a strong large pendulum CL (Fig. 1), which has its axis of suspension at C, and a core or block ML at its lower part. The projectile is fired into this core and remains there, causing the pendulum at the same time to swing through a certain angle θ with the vertical; this is measured automatically on the fixed arc AB by a needle attached to the pendulum.

Before using the pendulum certain adjustments as to symmetry and level are necessary, and it is important to arrange that it may be

* For detailed drawings and full descriptions *see* Boxer's "Treatise on Artillery," Sec. 1, Part I, 1860.

Fig. 1.—Pendulum for Small Arms.



struck (if possible) at such a point that no impulse shall be given to the axis of suspension C; in other words, that the centre of percussion shall be hit.

If struck above the point sought for, an impulse will be given to the axis of suspension in the direction of the motion of the projectile: if struck below, an impulse will be given in a contrary direction. Officers' swords have the centre of percussion marked at the back of the blade at about two-thirds of the length from the handle; on striking an object with this part, little or no jar is given to the hand in delivering the cut.

It is an established fact that the centres of percussion and oscillation are coincident; and the centre of oscillation is readily found by causing the pendulum to vibrate through a small arc, and observing the number of vibrations (n) in a time (T); then $\frac{T}{n}$ gives the time (t) of a single vibration with considerable accuracy. The length of the corresponding simple pendulum, *i.e.*, the distance of the centre of oscillation, or percussion from the axis of suspension, is then known from the elementary formula

$$t = \pi \sqrt{\frac{l}{g}}.$$

If the distance of the centre of the core L from the axis of suspension C is exactly equal to l, the instrument is in adjustment; but if this is not the case, weights must be pushed up or down the pendulum by trial and error, till the time of oscillation is that of a simple pendulum whose length is CL.

The weight of the pendulum being known, the distance h of its centre of gravity G from the axis of suspension is then found experimentally (if not already determined). A cord, with a known heavy weight at the end of it, is passed over a pulley (in the plane of oscillation), and attached to the lower part of the pendulum, which it pulls through a certain angle α, which is measured. The distance of the point of attachment of the cord from the axis of suspension is also measured; and from these data the position of the centre of gravity of the pendulum is easily found.

Let W = weight of pendulum in pounds,
w = ,, projectile ,,
v = striking velocity of projectile in f.s.,
l = distance of centre of percussion from axis in feet,
h = ,, centre of gravity ,, ,,
ω = angular velocity of pendulum at instant of impact,

The projectile is fired into the middle of the core, and the pendulum swings through the angle 2θ.

Suppose the pendulum is struck at a distance l_1 from the axis; taking moments about this axis—

$$wvl_1 = Whl\omega + wl_1^2\omega,$$

$$\therefore\ v = \frac{\omega}{w}\left(Wh\frac{l}{l_1} + wl_1\right) \ldots\ldots\ldots\ldots\ldots \text{(iii)}$$

—the general expression.

If the centre of percussion is struck $l_1 = l$, and (iii) becomes—

$$v = \frac{\omega}{w}\left(Wh + wl\right) \ldots\ldots\ldots\ldots\ldots\ldots \text{(iv)}$$

$$\text{now } \omega = 2\sqrt{\frac{g}{l}}\ \sin\frac{\theta}{2}.$$

$\therefore$ (iv) becomes

$$v = 2\sin\frac{\theta}{2}\,\frac{Wh + wl}{wl}\sqrt{gl}\ \ldots\ldots\ldots \text{(v)}$$

An endeavour was generally made to hit the centre of percussion, as not only were the calculations simplified, but jar on the axis was avoided.

Gun pendulum.

The **gun pendulum** contains a gun instead of a core; the axis of the gun is made to coincide with the centre of percussion, and the velocity of recoil on firing is measured; from these data the velocity of the projectile can be deduced, but not with great exactness, as we have previously noticed that the relation between the velocity of the projectile and that of the piece is a little uncertain (*see* p. 106).

The gun pendulum has been occasionally used in experiments to find the recoil of small arms.

Whirling machine.

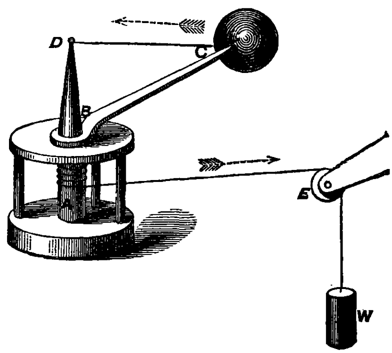
When firing projectiles at very low velocities, the ballistic pendulum was not found to be satisfactory, as rebound occurred, and thus the impact was not that of inelastic bodies; with the electrical instruments at present employed, difficulties are also experienced at low velocities, as wires and currents cannot readily be broken by the projectile in flight, and thus records cannot be obtained. The **whirling machine** designed by Robins, however, offers a means of finding the resistance of the air to bodies moving at low velocities.

It consists of a cylinder A (Fig. 2), and a cone B, carrying a long feather-edged arm BC, to the end of which is attached the body C, to which the resistance of the air is to be found. A light wire CD acts as a brace to prevent bending.

The whole is caused to revolve round a vertical axis by means of a silk thread wound round the cylinder, and passing over a pulley E with a weight W at the end of it.

When set in action, the velocity is found to become uniform in a short time; the period of performing several revolutions is then

Fig. 2.—Robins' Whirling Machine.



noted, and hence the time of one revolution is found with accuracy. The body C is then replaced by an equal flat weight of lead, which is put edgeways and exposes very little surface to the air in direction of its motion. By trial and error it is found that a smaller weight w will now give the same uniform velocity of rotation to the system as before. Hence the difference of weight $W - w$ in the two instances must be due to the resistance of the air on the surface C, and the true amount (R) of it is obtained by the proportion

$$R : W - w :: r : l,$$

where r is the radius of the cylinder, and l is the distance of the centre of C from the axis of rotation.

The velocity of the body C is known from the dimensions of the machine, and from the time to perform one revolution.

Example 1. Thus, when the object was about the size and shape of a 12-lb. spherical shot, a weight of $3\frac{1}{4}$ lbs. caused each revolution to be performed in one second, corresponding (from the length of the arm AC) to a velocity of 25 f.s. for the object C. But when a thin plate of lead of equal weight was put in its place, a weight of 1 lb. caused the same uniform rotation; hence the difference of weight, $2\frac{1}{4}$ lbs., was due to the resistance of the air on C, and the proportions of the radius of the cylinder A to the length of the arm AC being as 1 : 50, it follows that the resistance R of the air to the ball C is found from the relation

$$R : 2\tfrac{1}{4} \text{ lbs.} :: 1 : 50,$$
$$\text{whence } R = \tfrac{3}{4} \text{ oz. nearly.}$$

Twist of Rifling.

(*See* also Part I, Chapter XI.)

The **twist of rifling** at any point in the bore of a gun with an increasing twist is found as follows:—

Let n_1 = No. of cals. in which the rifling makes one turn at breech.

Let n = No. of cals. in which the rifling makes one turn at a distance l inches from the first part of greatest twist.

n_2 = No. of cals. in which the rifling makes one turn at the part of greatest twist.

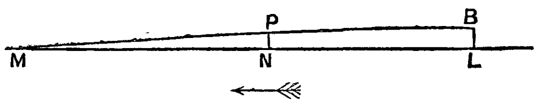
L = length of part of the bore rifled with increasing twist, in inches.

l_1 = length of part at muzzle (recent guns only) where the twist is uniform, in inches.

l = distance of the point, in inches, where the twist is 1 in n calibres from the point where it first is 1 in n_2 calibres.

Then $l + l_1$ = distance of the point, in inches, from the muzzle where the twist is 1 in n calibres.

Fig. 3.



to muzzle — direction of axis.

Develope that part of the groove which has an increasing twist on to a flat surface so that B corresponds to the breech, M to the point of greatest twist towards the muzzle, and LM to the direction of the axis of the gun. The groove is then represented by the parabola MB, having the tangents of the angles which the tangent lines make at B and M with an ordinate of x $\frac{\pi}{n_1}$ and $\frac{\pi}{n_2}$ respectively. We are required to find the horizontal ordinate $l = MN$ of some intermediate point P where the tangent of the corresponding angle is $\frac{\pi}{n}$.

The equation to the parabola, origin at M, is—

$$y^2 = bx - cx^2$$

$$\therefore \frac{dy}{dx} = b - 2cx \quad \ldots\ldots\ldots\ldots \text{(vi)}$$

In (vi) let $x = 0$ $\qquad \therefore b = \frac{\pi}{n_2}$

and $x = L$ $\qquad \therefore 2c = \frac{\pi(n_1 - n_2)}{Ln_1 n_2}$

Substituting the values of these constants in (vi) and putting $\frac{\pi}{n}$ for $\frac{dy}{dx}$ and l for x we find—

$$\text{Distance } l = L\frac{n_1}{n} \cdot \frac{n - n_2}{n_1 - n_2} \quad \ldots\ldots\ldots\ldots \text{(vii)}$$

With the more recent rifled guns a distance l_1 at the muzzle is rifled with a uniform twist; in that case the distance of the part where the twist is one turn in n calibres measured from the muzzle is—

$$= L\frac{n_1}{n} \cdot \frac{n - n_2}{n_1 - n_2} + l_1 \quad \ldots\ldots\ldots\ldots \text{(viii)}$$

Example 2. In the 10-in. R.M.L. gun of 18 tons the length of the rifled part of the bore is 118 inches, and the twist increases from one turn in 100 calibres at the breech, to one in 40 at the muzzle. At what point is it one turn in 60 calibres?

Substitute the values in equation (vii), and we find that the required point is 65·56 inches distant from the muzzle.

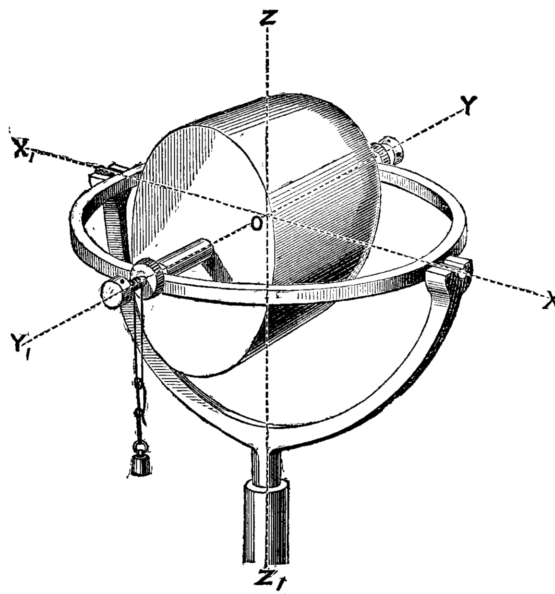
Drift.

(*See* also Part I, Chapter XI, p. 131.)

The **gyroscope** illustrates the stability of the direction of the axis of a spinning body and the tendency of a rifled elongated projectile to travel nearly point first in flight, and it also gives a reason for the slight turning movement of the axis of the projectile caused by the resistance of the air.

A gyroscope (Fig. 4) for such a purpose consists of a carefully centred heavy model of an elongated projectile, some $2\frac{1}{2}$ inches long, free to revolve on pivots inside a brass ring, the axis of revolution being a diameter of the ring, which has externally two arms in its

Fig. 4.



own plane, in prolongation of a diameter at right angles to the axis of the projectile. On these the ring can revolve in bearings in a vertical brass half ring, which has a stem under its middle fitting into a socket in a heavy stand. The half ring can revolve round its vertical axis.

The centre of gravity of the model projectile is adjusted to be over the vertical stem by screwing the bearings of the pivots in or out together, and it will then remain in any position. All the three axes

of rotation intersect in the centre of gravity O of the projectile, and movement of the axis in any direction is easy.

If the gyroscope's projectile (not spun) is placed horizontally as in the figure, and a small weight is suspended from the brass ring behind it, the point of the projectile will immediately rise; and thus the effect of the couple caused by the resistance of the air in raising the point of a *non-rotating* elongated projectile in flight is imitated.

But if the model projectile is caused to spin round its longer axis with a right-handed rotation, the addition of the small weight causes the axis to move its point round to the right (looking from behind it) with a slow movement in a *horizontal* plane, and thus the couple caused by the resistance of the air on a *rotating* projectile in flight is imitated.

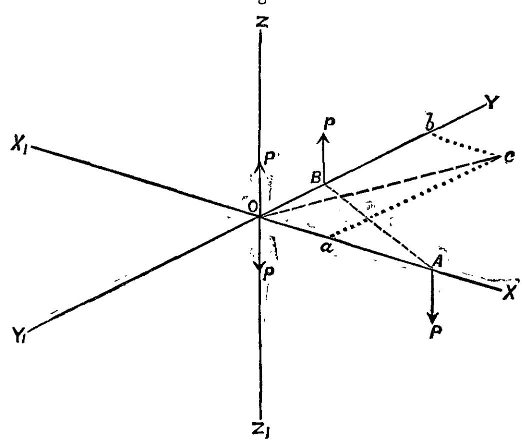
If one of the bearings on the brass ring is now pushed laterally in the direction of this motion, it does not increase the movement, but the axis turns in a plane at right angles and the point dips.

In these last two cases a turning motion of the axis ensues in a plane at right angles to the couple, similar to the lateral movement of the axis of an elongated projectile in flight.

This can be explained as follows:—

Place the brass ring horizontal, and through O, the centre of gravity of the model projectile (Fig. 4), draw the three axes of revolution at right angles to each other, and call them OX, OY, OZ.

Fig. 5.



Reproduce these lines in Fig. 5, which (for simplicity) is divested of the rings, projectile, &c., of Fig. 4.

The original rotation round the axis YY_1 (Fig. 5), may be represented by a couple P.OA; with a certain length of arm OA: the couple caused by the weight, or by the resistance of the air in the case of a real projectile in flight, may be shown by P.BO, the force P being taken equal in both couples, but the arms different in length, since the couples are not equal to each other.

Mark off on OY a distance Ob = OA, on OX make Oa = OB, and complete the parallelogram ab with its diagonal Oc.

The forces P and P at O neutralise each other as they are equal, and act along the same line in opposite directions, and there remain

P at B, and P at A, constituting only one couple, causing rotation about a fresh axis O*c*, which is the diagonal of the parallelogram *ab*, and at right angles to AB. This indicates that the axis of revolution slowly moves from the direction Y_1 Y to O*c*; and this turning movement will continue as long as both couples exist.

The very slow continuous change of the inclination of the earth's axis, called the precession of the equinoxes, is a movement of a similar nature, which can also be illustrated by means of the gyroscope.

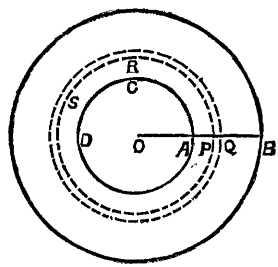
CHAPTER II.—THE STRENGTH OF GUNS.

(*See* also Part I, Chapter V.)

In this chapter the formulæ referred to in Chapter V, Part I, will be investigated; the problem is to determine the condition of a thick metallic cylinder under internal and external stresses.

The following is one of the simplest of the many proofs which have been given of the fundamental equations (**viii**) and (**ix**), pp. 246 and 247:—

Fig. 1.



See Fig. 1. Let

$r_0 \quad$ = (OA) *internal* radius of cylinder,
$r_1 \quad$ = (OB) *external* „ „
$r \quad$ = (OP) *internal* radius of *any* thin cylindrical film,
$r + dr$ = (OQ) *exterior* „ „ „

$p_0 \quad$ = intensity of normal or radial pressure acting uniformly over *interior* surface of radius r_0, tending to compress the thickness of the wall AB,
$p_1 \quad$ = similar normal pressure at *exterior* surface radius r_1,
$p \quad$ = similar normal pressure on *interior* of *any* thin cylindrical film, radius r, tending to compress PQ,
$p + dp$ = similar normal pressure on *exterior* of this film,

$t_0 \quad$ = intensity of hoop tension acting uniformly at *interior* surface radius r_0, tending to lengthen the circumference ACD,
$t_1 \quad$ = similar hoop tension at *exterior* surface radius r_1,
$t \quad$ = similar hoop tension on *interior* of *any* thin cylindrical film and radius r, tending to lengthen the circumference PRS,
$t + dt$ = similar hoop tension on *exterior* of cylindrical film.

Considering the **hoop tension** of the **thin cylindrical film** when strained to the elastic limit, the two halves tend to be burst apart by

the interior pressure p acting over 2r or by 2pr; while the exterior pressure p + dp acting over 2(r + dr) or 2(p + dp)(r + dr) tends to keep it together. The elastic strength exerted by the hoop itself is its thickness on each side 2dr multiplied by its mean tension $\frac{t + (t + dt)}{2}$ or $2dr\left(t + \frac{dt}{2}\right)$ and it is equal in amount to the difference of the interior and exterior pressures exerted over their respective diameters.

$$\therefore\ 2dr\left(t + \frac{dt}{2}\right) = 2pr - 2(p + dp)(r + dr),$$

$$\therefore\ t\times dr = -r\times dp - p\times dr \ldots\ldots\ldots\ldots\ldots\ldots \text{(i)}$$

$dr \times dt$ and $dp \times dr$ being so small that they may be neglected.

The stresses applied slightly alter the dimensions of the cylinder according to the elasticity of the metal: thus the *interior* circumference of the cylindrical film PRS is elongated by the hoop tension t, call this elongation $\delta(2\pi r)$, its original length being $2\pi r$.

Then, according to Hooke's law, if E is the modulus of elasticity,

$$\frac{\delta(2\pi r)}{2\pi r} = \frac{1}{E}t,$$

$$\therefore\ \delta r = \frac{1}{E}rt \ \ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(ii)}$$

Similarly, considering the elongation of the *exterior* of the cylindrical film, we have—

$$\delta(r + dr) = \frac{1}{E}(r + dr)(t + dt) \ \ldots\ldots \text{(iii)}$$

Hence, subtracting (ii) from (iii), we have for the elongation of PQ or dr—

$$\delta dr = \frac{1}{E}(r\times dt + t\times dr) \ldots\ldots\ldots \text{(iv)}$$

But PQ is *compressed* by a pressure p.

$$\therefore\ -\delta dr = \frac{1}{E}p\times dr \ldots\ldots\ldots\ldots\ldots\ldots \text{(v)}$$

$\therefore$ from (iv) and (v)

$$p\times dr = -r\times dt - t\times dr \ \ldots\ldots\ldots\ldots \text{(vi)}$$

And from (i) and (vi)

$$dt = dp$$

Integrating
$$t = p + 2m \ \ldots\ldots\ldots\ldots\ldots\ldots \text{(vii)}$$

which tells that the hoop tension exceeds the normal pressure by some constant 2m.

Hence by (i) and (vii)

$$(p + 2m)dr = -r\times dp - p\times dr,$$
$$\therefore\ (2p + 2m)dr = -r\times dp.$$

Integrating

$$-\tfrac{1}{2}\log(2p + 2m) = \log r + c,$$

$$\therefore\ 2p + 2m = \frac{2a}{r^2},$$

$$\therefore\ p = \frac{a}{r^2} - m \ldots\ldots\ldots\ldots\ldots \quad \textbf{(viii)}$$

Substituting this value of p in (vii)

$$t = \frac{a}{r^2} + m \text{ ....(ix)}$$

The two last are fundamental general equations, from which if two definite values can be given for p and t, the pressures and tensions at other points can be determined.

Subtracting (viii) from (ix) we obtain

$$t - p = 2m \text{ (a constant) ....(x)}$$

Adding (viii) and (ix)

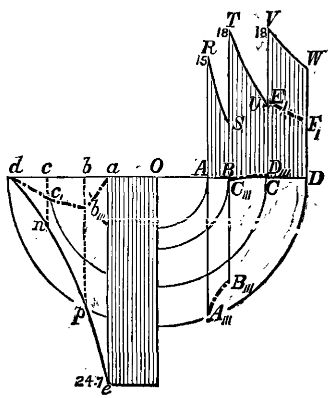
$$t + p = \frac{2a}{r^2} \text{ ....(xi)}$$

Two formulæ useful for reference.

We are now in a position to investigate the formulæ in Chapter V, Part I; for facility of reference several diagrams will be repeated.

Suppose it is known that on firing the gun the maximum hoop tensions on the interiors of the various cylinders are to be $t_0, t_1 \ldots t_n$, which will be a little less than the elastic limits of the steel. If there are n + 1 cylinders, the pressure p_{n+1} on the exterior surface of the outer one may be considered = 0, as it is only that of the atmosphere, which may be neglected, as it is small in comparison to powder pressures.

Fig. 2.



Substituting t_n, p_n, &c., for t, p, &c., in (viii) and (ix) (Fig. 2, in which t_n = CV, p_n = cn),

$$t_n = \frac{a_n}{r_n^2} + m_n \text{ .... (xii)}$$

$$p_n = \frac{a_n}{r_n^2} - m_n \text{ .... (xiii)}$$

for the *interior* of the last or outer cylinder. For the *exterior* of the same, p_n and r_n become p_{n+1} and r_{n+1} respectively, but the constants a_n m_n remain the same throughout the thickness of the outer cylinder—

$$\therefore\ p_{n+1} = \frac{a_n}{r^2_{n+1}} - m_n \text{ .... (xiv)}$$

but $p_{n+1} = 0$, since it is only that of the atmosphere which may be neglected.

$$\therefore a_n = r_{n+1}^2 m_n \ldots\ldots\ldots\ldots\ldots\ldots \quad \text{(xv)}$$

Eliminating a_m and m_n from equations (xii), (xiii), and (xv), we obtain—

$$p_n = \frac{r_{n+1}^2 - r_n^2}{r_{n+1}^2 + r_n^2} t_n \ldots\ldots\ldots\ldots\ldots \quad \textbf{(xvi)}$$

whence p_n can be found since t_n is known.

With regard to the next cylinder or the second from the outside, $t_{n-1} = BT$ and $p_{n-1} = bp$, we obtain in like manner

$$t_{n-1} = \frac{a_{n-1}}{r_{n-1}^2} + m_{n-1} \ldots\ldots\ldots\ldots\ldots \quad \text{(xvii)}$$

$$p_{n-1} = \frac{a_{n-1}}{r_{n-1}^2} - m_{n-1} \ldots\ldots\ldots\ldots\ldots \quad \text{(xviii)}$$

$$p_n = \frac{a_{n-1}}{r_n^2} - m_{n-1} \ldots\ldots\ldots\ldots\ldots\ldots \quad \text{(xix)}$$

the radial pressure being the *same* on the interior of the outer cylinder and on the exterior of the next one, p_n is employed in both equations (xiii) and (xix); the constants, however, differ, as (xiii) refers to the outer cylinder, and (xix) refers to the next.

Eliminating a_{n-1} and m_{n-1} from (xvii), (xviii), and (xix), we obtain

$$p_{n-1} = \frac{r_n^2 - r_{n-1}^2}{r_n^2 + r_{n-1}^2}(t_{n-1} + p_n) + p_n \ldots \quad \textbf{(xx)}$$

Hence p_{n-1} can be found as t_{n-1} is known, and p_n can be found from (xix).

In a similar way the firing pressure on the interior of each cylinder can be found. Hence if p_0 is the maximum allowable powder pressure in the bore on firing—

$$p_0 = \frac{r_1^2 - r_0^2}{r_1^2 + r_0^2}(t_0 + p_1) + p_1 \ldots\ldots \quad \textbf{(xxi)}$$

(in which $t_0 = AR$, $p_0 = ae$, Fig. 2).

whence p_0 can be found as t_0 is given, and p_1 can be found from successive solutions of formulæ (**xvi**) and (**xx**).

The values of a and m for each cylinder can be found when two definite values of hoop stress and radial pressure can be assigned, as in (xii), (xiii), and (xiv), &c. This is a somewhat laborious operation, all the curves drawn in the diagrams have been readily constructed from these calculations for a 6-inch gun; in all the diagrams in this Chapter and in Chapter V, Part I, the curves on the right are of hoop stress, and have the equation $t = \frac{a}{r^2} + m$; while the curves of radial pressure are drawn for distinction on the left sides of the diagrams, and have the equation $p = \frac{a}{r^2} - m$.

In Fig. 3, of a homogeneous gun under fire, the curves are drawn on the supposition that the internal powder pressure p_0 is the same as in the composite gun of the same thickness in Fig. 2, the external pressure being 0; sufficient data are afforded by use of equations (**viii**) and (**ix**) pp. 246–247, to determine the curves EH and do by finding the values of a and m.

Fig. 4 is the difference of Fig. 3, which represents a homogeneous gun under fire, and Fig. 2, which is a gun made with initial stress, under fire; for instance $\mathrm{AA}_{\prime\prime\prime}$ of Fig. 4 = AE of Fig. 3 −AR of Fig. 2. It shows the initial stresses on the completed composite gun before firing.

In order to find the stresses as each part is put on in succession in manufacture, imagine that the outer cylinder is stripped off the completed gun.

Fig. 3.

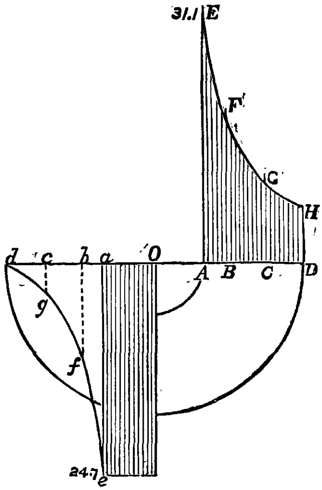
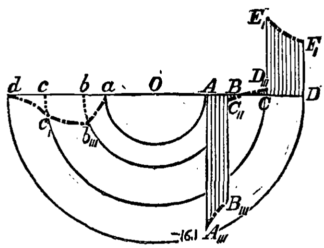


Fig. 4.



This will cause alteration in all the remaining initial stresses, as the pressure $cc_{\prime}$ at c in Fig. 4 sinks to 0 at c in Fig. 5. To find the effect of the removal of this radial pressure $cc_{\prime}$ of Fig. 4 on the interior cylinders of the gun, imagine them to be homogeneous and subject to an exterior radial pressure $cc_{\prime}$ at c and interior pressure 0 at a (*see* Fig. 6). The hoop pressures can then be found from (**viii**) and (**ix**), pp. 246–247; the values of a and m can be determined and the curves drawn. Fig. 5, then shows the effect of removing the exterior

Fig. 5.

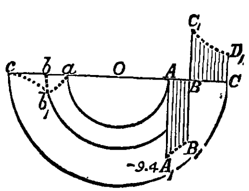
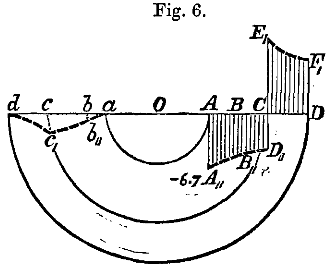


Fig. 6.



cylinder; the hoop stresses and radial pressures are the differences between those in Fig. 4 and Fig. 6; for instance $\mathrm{AA}_{\prime}$ in Fig. 5 = $\mathrm{AA}_{\prime\prime\prime}$ in Fig. 4 $-\mathrm{AA}_{\prime\prime}$ in Fig. 6. If the interior part is composed of more than two cylinders, a similar process will show the effect of taking

off another covering; if as in Fig. 5, there are only two cylinders left, the removal of the outer one will cause the other to return to its normal state.

Shrinkage.

The simplest case is that in which each part of the gun has the same modulus of elasticity.

If $\mathrm{t_n}$ as before is the maximum allowable hoop tension at the inner surface of the (n + 1) hoop (*see* CV, Fig. 2, p. 247); if t_n (*in italics*) is the hoop tension on firing in a homogeneous gun at the same distance from the centre (*see* $CG_{,,}$ Fig. 3, p. 249), then $\mathrm{t_n} - t_n$ (*see* $CE_{,}$ Fig. 4, p. 249) is the hoop tension of the interior of the n + 1 hoop before firing. This is caused by an initial radial pressure $\mathrm{p_n} - p_n$ due to shrinkage (cn of Fig. 2 —cg of Fig. 3 which = $cc_{,}$ of Fig. 2 and of Figs. 4 and 6).

Suppose the n inner tubes to be homogeneous (*see* Fig. 6), they are compressed before firing by an external radial pressure $\mathrm{p_n} - p_n$ ($cc_{,}$), and there is no internal radial pressure. Call the hoop pressures thus produced $-\tau_n$ and $-\tau_0$. (*See* $CD_{,,}$ and $AA_{,,}$, Fig. 6.)

At the surface of contact of the exterior of the n hoop and the interior of the n + 1 hoop, we have the latter extended circumferentially by $\mathrm{t_n} - t_n$ ($CE_{,}$, Figs. 4 and 6), and the former compressed by $-\tau_n$ ($CD_{,,}$, Fig. 6). The whole shrinkage to be given must then be proportional to the sum of these or

$$\mathrm{t_n} - t_n - \tau_n \qquad \text{(xxii)}$$

Since $-\tau_n$ is negative tension.*

As the inner n cylinders (*see* Fig. 6) not under fire, are compressed by $\mathrm{p_n} - p_n$ radially, we have from (x) and (xi)—

$$\tau_n - (\mathrm{p_n} - p_n) = \tau_0 - 0$$

$$\{\tau_n + (\mathrm{p_n} - p_n)\}r_n^2 = (\tau_0 + 0)r_0^2.$$

Eliminating τ_0 we obtain

$$-\tau_n = (\mathrm{p_n} - p_n)\frac{r_n^2 + r_0^2}{r_n^2 - r_0^2}.$$

Hence (xxii) becomes

$$\mathrm{t_n} - t_n + (\mathrm{p_n} - p_n)\frac{r_n^2 + r_0^2}{r_n^2 - r_0^2} \qquad \text{(xxiii)}$$

Next to eliminate t_n and p_n—

$$(t_n - p_n) = t_0 - p_0,$$

$$(t_n + p_n)r_n^2 = (t_0 + p_0)r_0^2,$$

$$\text{and } p_0 = \mathrm{p_0},$$

$$\therefore t_n = \frac{t_0(r_n^2 + r_0^2) - \mathrm{p_0}(r_n^2 - r_0^2)}{2r_n^2}$$

$$p_n = \frac{\mathrm{p_0}(r_n^2 + r_0^2) - t_0(r_n^2 - r_0^2)}{2r_n^2}.$$

Substitute these values in (xxiii) and divide by E, the modulus of

---

* In previous pages hoop stress has always been represented by t, and radial stress by p; following this arrangement, we now use $-\tau_n$ in preference to p for negative hoop tension, which is of course hoop pressure.

elasticity, to obtain the shrinkage per linear inch; t_0 disappears, and the expression reduces to

$$_{n}S_{n+1} = \frac{t_n + p_0 - (p_0 - p_n)\dfrac{r_n^2 + r_0^2}{r_n^2 - r_0^2}}{E} \quad \ldots\ldots\ldots\ldots \quad \textbf{(xxiv)}$$

If $_{n}S_{n+1}$ denotes the shrinkage *per inch* between the n and the n + 1 hoop; since $2r_n$ is the diameter in inches of this surface of contact, the *actual* shrinkage $_{n}\mathrm{S}_{n+1} = {_{n}S_{n+1}} \times 2r_n$.

Equation **(xxiv)** holds good for the shrinkage between any two surfaces when the modulus of elasticity is the same throughout the gun. In the particular case of the surface between the exterior of the inner tube and the interior of the next cylinder, the equation may be simplified by reduction to

$$_{1}S_2 = \frac{p_0 - p_1 + t_1 - t_0}{E} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad \textbf{(xxv)}$$

Summary.

Supposing as a definite case a gun composed of three cylinders, the equations used to determine the maximum allowable powder pressure p_0 are from **(xvi)**, **(xx)**, and **(xxi)**—

$$\left.\begin{aligned} p_2 &= \frac{r_3^2 - r_2^2}{r_3^2 + r_2^2} \cdot t_2 \\ p_1 &= \frac{r_2^2 - r_1^2}{r_2^2 + r_1^2}(t_1 + p_2) + p_2 \\ p_0 &= \frac{r_1^2 - r_0^2}{r_1^2 + r_0^2}(t_0 + p_1) + p_1 \end{aligned}\right\} \ldots\ldots\ldots\ldots \quad \textbf{(xxvi)}$$

The actual or absolute shrinkages in inches from **(xxiv)** and **(xxv)** are

$$\left.\begin{aligned} {_2}\mathrm{S}_3 &= \frac{t_2 + p_0 - (p_0 - p_2)\dfrac{r_2^2 + r_0^2}{r_2^2 - r_0^2}}{E} \times 2r_2 \\ {_1}\mathrm{S}_2 &= \frac{p_0 - p_1 + t_1 - t_0}{E} \times 2r_1 \end{aligned}\right\} \ldots \quad \textbf{(xxvii)}$$

If four or more cylinders are used, the actual formulæ to employ can readily be written out from an inspection of the general equations **(xxvi)** and **(xxvii)**.

Lamé and Rankine have investigated the general mathematical question of the strength of cylinders subject to great internal pressure, and the application of these calculations to ordnance has been fully entered into by Longridge in England, by Virgile of the French Artillery, by Clavarino in Italy, and by Birnie in the United States "Notes on the Construction of Ordnance."

Further account has been taken of the laws of elasticity. When a bar of metal is subjected to tension t, elongation takes place according to Hooke's law in the direction of the tension, but the bar becomes thinner at the sides, although a certain actual increase in volume takes place in the whole bar.

Wertheim finds with iron and steel as an approximation that the

lateral compression is the same as would be produced by two independent pressures or negative tensions acting at right angles to each other and to the original tension each equal to magnitude to $\frac{1}{3}$t. (For a more accurate conclusion, *see* "Natural Philosophy," by Thomson and Tait, 2nd Edition, Part II, pp. 220 and 221.)

When a gun is fired the stresses produced in three directions at right angles to each other are hoop *tension* t; radial *pressure* p; and longitudinal *tension* q, in the direction of the length l, the first and last being regarded as positive, and the other as negative in sign. If E is the modulus of elasticity, we have in each direction according to Wertheim—

$$\left.\begin{aligned} &+\frac{1}{E}(t + \tfrac{1}{3}p - \tfrac{1}{3}q) \\ &-\frac{1}{E}(p + \tfrac{1}{3}t + \tfrac{1}{3}q) \\ &+\frac{1}{E}(q - \tfrac{1}{3}t + \tfrac{1}{3}p) \end{aligned}\right\} \qquad \text{(xxviii)}$$

We also have

$$t = \frac{a}{r^2} + m \qquad \text{(xxix)}$$

$$p = \frac{a}{r^2} - m \qquad \text{(xxx)}$$

Putting q = 0, the elongations in the direction of the circumference, of the radial thickness of the wall, and of the length of the cylinder become respectively—

$$\left.\begin{aligned} &+\frac{1}{E}\left(\frac{2}{3}m + \frac{4}{3}\frac{a}{r^2}\right) \\ &+\frac{1}{E}\left(\frac{2}{3}m - \frac{4}{3}\frac{a}{r^2}\right) \\ &-\frac{1}{E}\left(\frac{2m}{3}\right) \end{aligned}\right\} \qquad \text{(xxxi)}$$

If the internal and external pressures are known, a and m can be determined from (xxix) and (xxx), and their values substituted in (**xxxi**). This method leads to somewhat different results from the simpler assumption made before, but it cannot be further followed in these pages on account of want of space.

It may happen that the modulus of elasticity differs in the various cylinders of which the gun is composed, as it appears to be advantageous to have the inner tube softer than the rest. In this case the calculations become longer and more involved, and reference should be made to the works of the writers above quoted.

CHAPTER III.—BASHFORTH'S TABLES AND THE COEFFICIENT K.

(*See* also Part I, Chapter XII, and Tables VI and VII, pp. 283–292.)

THE difficult problem of the effect of the resistance of the air to projectiles in flight, and the calculation of trajectories has been undertaken by many, chief among whom may be mentioned Robins, Hutton, Didion, Piobert, Hélie, Mayevski, Bashforth, Niven, Greenhill, Siacci, and Ingalls. We shall confine our attention to the modern English methods originated by Bashforth and Niven.

The calculation of Bashforth's tables of $\frac{d^2}{w}t$ and $\frac{d^2}{w}s$ (*see* pages 283—292) depends upon the following considerations:—

Relation between velocity and *time.*

The formula $v = ft$ always holds good when the acceleration is uniform; but when the acceleration is variable, it is only true for an indefinitely short interval of time, dt, or

$$dv = fdt,$$

which gives the small increment of velocity dv, under acceleration f in the small space of time dt;

$$\text{or } \frac{dt}{dv} = \frac{1}{f} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(i)}$$

Substituting from (iv), p. 142, the value $f = -\frac{d^2}{w}K\left(\frac{v}{1000}\right)^3$, with sign reversed,

$$\frac{dt}{dv} = +\frac{w}{d^2}\frac{(1000)^3}{K}\frac{1}{v^3}.$$

The plus sign being used because Mr. Bashforth has calculated his table on the supposition that the motion is reversed, and therefore the retardation caused by the resistance of the air becomes an acceleration in the table. This makes the differences in the tables always positive, and thus mistakes in calculation are not likely to occur.

If we now integrate this between the limits V and v, the velocities at the beginning and end of time t respectively, we obtain

$$\frac{d^2}{w}t = \frac{(1000)^3}{2K}\left(\frac{1}{V^2} - \frac{1}{v^2}\right), \ldots\ldots\ldots\ldots \text{(ii)}$$

an equation connecting *time* and velocity, from which, if any four of the five quantities t, V, v, d, or w are known, the fifth may be found. A certain mean value for K must be selected, since, as we know, K varies with the velocity: this has been done by Mr. Bashforth who has calculated Table VI, p. 283, which gives the times taken by a *certain unit projectile*, and thence for any projectile,

to attain any velocity (between 100 and 2900 f.s.) from a certain time of starting, under the influence of an *acceleration*, equal in magnitude to the *retardation* caused by the resistance of the air.

Relation between velocity and *space.*

The relation between the velocity at the beginning and end of any *distance* is found thus :—

As before in (i)

$$f = \frac{dv}{dt}$$

$$= \frac{dv}{ds} \cdot \frac{ds}{dt} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(iii)}$$

Now $v = \frac{s}{t}$ generally, when the velocity is uniform; when the space and time are indefinitely diminished, $v = \frac{ds}{dt}$, even when the velocity is not uniform; substituting this value for v in (iii) we have

$$\frac{ds}{dv} = \frac{v}{f}.$$

Substituting the value of f, as before, we have

$$\frac{ds}{dv} = \frac{w}{d^2} \cdot \frac{(1000)^3}{K} \cdot \frac{1}{v^2},$$

and integrating this between the limits V and v, the velocities at the beginning and end of the distance s, we obtain

$$\frac{d^2}{w}s = \frac{(1000)^3}{K}\left(\frac{1}{V} - \frac{1}{v}\right), \ldots\ldots\ldots\ldots \text{(iv)},$$

an equation connecting *distance* and velocity. From this, if any four of the five quantities s, V, v, d, or w, are known, the fifth may be found.

From this formula (iv) Mr. Bashforth has calculated a Table VII of $\frac{d^2}{w}s$, p. 288, on the same principle as Table VI of $\frac{d^2}{w}t$, on p. 283.

Time and Space (or Distance) known, but Velocity unknown.

Relation between time and space.

As we have seen before, p. 149, there is no direct relation between time and distance, they can only be linked together by the relation of each to velocity, but the assumption that the mean velocity or

$$\frac{\text{distance between the screens in feet}}{\text{time in seconds over the distance between the screens}}$$

is the *actual* velocity in f.s. at the *middle point* between the screens of the Boulengé chronograph is strictly true when the cubic law of resistance holds good, and this is very approximately the case in the short distance considered ; it may be proved as follows :—

In the equations (ii) and (iv)

$$\frac{d^2}{w}t = \frac{(1000)^3}{2K}\left(\frac{1}{V^2} - \frac{1}{v^2}\right)$$

$$\text{and} \quad \frac{d^2}{w}s = \frac{(1000)^3}{K}\left(\frac{1}{V} - \frac{1}{v}\right).$$

write $2b$ for $\frac{d^2}{w}\frac{K}{(1000)^3}$, and they become respectively

$$\frac{1}{v^2} = \frac{1}{V^2} + 4bt \ldots\ldots \quad \text{(v)}$$

$$\frac{1}{v} = \frac{1}{V} + 2bs \ldots\ldots \quad \text{(vi)}$$

Substitute the value of $\frac{1}{v^2}$ from (vi) in (v), and we obtain—

$$t = \frac{s}{V} + bs^2 \ldots\ldots \quad \text{(vii)}$$

Now if v_2 be the velocity at the end of the distance $\frac{1}{2}s$, we have (writing v_2 for v and $\frac{1}{2}s$ for s in (vi),

$$\frac{1}{v_2} = \frac{1}{V} + bs \ldots\ldots \quad \text{(viii)}$$

And the *mean* velocity in f.s. over the distance $s = \frac{\text{distance in feet}}{\text{time in seconds}}$

$$= \frac{s}{t}$$

Substitute the value of t from (vii),

$$= \frac{s}{\frac{s}{V} + bs^2}$$

$$= \frac{1}{\frac{1}{V} + bs}$$

$$= v_2 \text{ from equation (viii).}$$

Hence v_2 the velocity at the middle point $\frac{1}{2}s$ is the mean velocity over the whole distance, if the cubic law of resistance assumed in (v) and (vi) above holds good.

THE COEFFICIENT K.*

(*See* Tables IV and V, pp. 281–282).

Mr. Bashforth has deduced the values of K, previously referred to, for Service elongated projectiles for a range of velocities between 100 and 2900 f.s., by numerous experiments with his chronograph, which registers the time taken by a projectile in flight to pass between 10 screens at 150 feet interval. (With low velocities the screens were 75 feet apart, and the number in the series was reduced.)

To understand his method the following considerations are necessary:—

* *See* Bashforth's "Motion of Projectiles."

If A, B, C, D, &c., be a series of numbers increasing or decreasing in magnitude, and

$$\begin{array}{lllll} B - A = \alpha & & & & \\ & \text{also } \beta - \alpha = l & & & \\ C - B = \beta & & \text{and } m - l = a & & \text{\&c.} \\ & \gamma - \beta = m & & & \\ D - C = \gamma & & \text{\&c.} = \text{\&c.} & & \\ & \text{\&c.} = \text{\&c.} & & & \\ \text{\&c.} = \text{\&c.} & & & & \end{array}$$

Then $\alpha.\beta.\gamma.$, &c., are called first differences (Δ),
$l.m.n.$, &c., „ second „ (Δ^2),
$a.b.c.$, &., „ third „ (Δ^3),

of the original numbers, A, B, C, D, &c.

Evidently there *may* be 4th, 5th, and higher differences.

This is expressed in symbols thus :—

$$\begin{array}{llllll} & \Delta & \Delta^2 & \Delta^3 & \Delta^4 & \text{\&c.} \\ A & & & & & \\ & \alpha & & & & \\ B & & l & & & \\ & \beta & & a & & \\ C & & m & & \text{\&c.} & \\ & \gamma & & b & & \text{\&c} \\ D & & n & & \text{\&c.} & \\ & \delta & & \text{\&c.} & & \\ E & & \text{\&c.} & & & \\ & \text{\&c.} & & & & \\ \text{\&c.} & & & & & \end{array}$$

We have $B = A + \alpha$,

$$\begin{aligned} C = B + \beta &= (A + \alpha) + (\alpha + l) = A + 2\alpha + l, \\ D = C + \gamma &= (A + 2\alpha + l) + (\beta + m), \\ &= (A + 2\alpha + l) + (\alpha + l) + (l + a), \\ &= A + 3\alpha + 3l + a, \quad \ldots\ldots\ldots\ldots\ldots (\text{ix}) \end{aligned}$$

And so generally we find that the coefficients are those of the binomial expansion.

Now, suppose A replaced by t_s (time to pass over distance s), which is some function, $f(s)$ of s. Let B be replaced by $f(s + l)$, where l is a certain increment of distance (the interval between the screens, in the particular case we are considering). Replace C by $f(s + 2l)$, when the distance is increased by another equal increment. Let the next terms be $f(s + 3l)$, $f(s + 4l)$, &c.

Let the first differences

$$\begin{array}{lll} f(s + l) - f(s) & \text{be represented by} & \Delta t_s, \\ f(s + 2l) - f(s + l) & \text{„} & \Delta t_{s+l}, \\ \text{\&c.,} & & \text{\&c.} \end{array}$$

Let the second differences

$$\begin{array}{lcl} \Delta t_{s+l} - \Delta t_s & \text{be represented by} & \Delta^2 t_s, \\ \Delta t_{s+2l} - \Delta t_{s+l} & ,, & \Delta^2 t_{s+l}, \\ \&c., & & \&c., \end{array}$$

and the 3rd and 4th, &c., differences in like manner.

The following scheme will explain the notation :—

	Δ.	Δ^2.	Δ^3.	Δ^4.	&c.
By definition—					
$t_s \;\;= f(s)$					
	Δt_s				
$t_{s+l} \;= f(s + l)$		$\Delta^2 t_s$			
	Δt_{s+l}		$\Delta^3 t_s$		
$t_{s+2l} = f(s + 2l)$		$\Delta^2 t_{s+l}$		$\Delta^4 t_s$	
	Δt_{s+2l}		$\Delta^3 t_{s+l}$		&c.
$t_{s+3l} = f(s + 3l)$		$\Delta^2 t_{s+2l}$		$\Delta^4 t_{s+l}$	
	Δt_{s+3l}		$\Delta^3 t_{s+2l}$		
$t_{s+4l} = f(s + 4l)$		$\Delta^2 t_{s+3l}$		&c.	
	Δt_{s+4l}		&c.		
$t_{s+5l} = f(s + 5l)$		&c.			
	&c.				
&c. = &c.					

Adopting the notation found to hold good in (ix), we have

$$\begin{aligned} f(s + nl) &= t_s + n\Delta t_s + \frac{n \,.\, n - 1}{1 \,.\, 2}\Delta^2 t_s + \frac{n \,.\, n - 1 \,.\, n - 2}{1 \,.\, 2 \,.\, 3}\Delta^3 t_s, + \&c. \\ &= t_s + n(\Delta t_s - \tfrac{1}{2}\Delta^2 t_s + \tfrac{1}{3}\Delta^3 t_s - \&c.) \\ &\quad + n^2(\tfrac{1}{2}\Delta^2 t_s - \tfrac{1}{2}\Delta^3 t_s + \tfrac{11}{24}\Delta^4 t_s - \&c.) \\ &\quad + \&c. \quad \ldots\ldots\ldots\ldots \quad \text{(x)} \end{aligned}$$

Now $f(s + nl)$ can also be expanded by Taylor's theorem.

$$= t_s + \frac{dt_s}{ds}nl + \frac{d^2 t_s}{ds^2}\frac{n^2 l^2}{1 \,.\, 2} + \&c. \quad \ldots\ldots\ldots \quad \text{(xi)}$$

Equating the coefficients of the first and second powers of n in the two expansions of $f(s + nl)$ (x) and (xi) we have—

$$l\frac{dt_s}{ds} = \Delta t_s - \tfrac{1}{2}\Delta^2 t_s + \tfrac{1}{3}\Delta^3 t_s - \&c. \ldots\ldots\ldots \quad \text{(xii)}$$

$$l^2\frac{d^2 t_s}{ds^2} = \Delta^2 t_s - \Delta^3 t_s + \tfrac{11}{12}\Delta^4 t_s - \&c. \quad \ldots\ldots\ldots \quad \text{(xiii)}$$

Again, following the same notation of finite differences, and writing $s - l$ for s, we have

$$\begin{aligned} t_{s-l} &= f(s - l), \\ f(s) &= f(s - l) + \Delta t_{s-l}, \\ \text{or } t_s &= t_{s-l} + \Delta t_{s-l}; \end{aligned}$$

$$\begin{aligned}
\text{Hence} \quad \Delta^2 t_s &= \Delta^2 t_{s-l} + \Delta^3 t_{s-l}, \\
\text{and} \quad - \Delta^3 t_s &= - \Delta^3 t_{s-l} - \Delta^4 t_{s-l} \\
\text{also} + \tfrac{11}{12} \Delta^4 t_s &= \tfrac{11}{12} \Delta^4 t_{s-l} + \tfrac{11}{12} \Delta^5 t_{s-l}, \\
\&c. \qquad & \qquad \&c.
\end{aligned}$$

Now, taking the sum of these, and remembering the value of the left side, from (xiii) we have—

$$l^2 \frac{d^2 t_s}{ds^2} = \Delta^2 t_{s-l} - \text{some fourth difference, \&c.} \quad \text{(xiv)}$$

the fourth differences with the succeeding terms may generally be neglected, as they are so small.

We are now in a position to find the *velocity* (v) and retardation (f) at each screen; we have generally—

$$v = \frac{ds}{dt}$$

$$\therefore \text{ from (xii) } v = \frac{l}{\Delta t_s - \frac{1}{2}\Delta^2 t_s + \frac{1}{3}\Delta^3 t_s - \&c.} \quad \ldots\ldots \text{(xv)}$$

To obtain the *retardation* (f) at each screen—

$$f = \frac{dv}{dt} \text{ and } v = \frac{ds}{dt}$$

$$\begin{aligned}
\therefore \quad f &= \frac{d^2 s}{dt^2} \\
&= \frac{d}{ds}\left(\frac{1}{\frac{dt}{ds}}\right)\frac{ds}{dt} \\
&= -\frac{d^2 t}{ds^2}\left(\frac{ds}{dt}\right)^3.
\end{aligned}$$

Substituting the value of $\frac{d^2 t}{ds^2}$ from (xiv), and putting v for $\frac{ds}{dt}$,

$$= -\frac{v^3}{l^2}(\Delta^2 t_{s-l} - \&c.) \quad \ldots\ldots\ldots\ldots \text{(xvi)}$$

Example 1. With an elongated shot weighing 23·84 lbs., diameter 4·92 inches, the marks made on the revolving cylinder as the projectile passed each screen were at the following screen readings :—

Screen.	Readings of scale.
1	74·89
2	77·48
3	80·14
4	82·85
5	85·61
6	88·44
&c.	

The following were the readings of the same scale for each second marked by the clock:—

Screen.	Readings of scale.
1	25·90
2	49·79
3	73·64
4	97·43
5	121·17
&c.	

Find the velocity, retardation, and value of K at the fifth screen.

Before practical use can be made of the above readings, it is necessary to apply certain small corrections, due to accidental irregularities in the motion of the chronograph, &c. These depend on the following facts:—In a consecutive series of numbers following *any* law; the first differences are more nearly equal to each other than the original numbers, and the second differences are more nearly alike than the first, and so on. For instance, if we take the square or cubes of numbers, we find the second and third differences respectively actually equal, in which case there are of course none higher, thus:—

Number.	Squares.	Δ.	Δ^2.
1	1		
		+ 3	
2	4		+ 2
		+ 5	
3	9		+ 2
		+ 7	
4	16		+ 2
		+ 9	
5	25		+ 2
		+ 11	
&c.	&c.	&c.	&c.

Logarithms of numbers are not related to each other in such a simple way, but here the second differences are *nearly* equal to each other, and vary but slowly, as seen below—

Number.	Corresponding logarithm.	Δ.	Δ^2.
1280	107210		
		+ 3380	
1290	110590		− 27
		+ 3353	
1300	113943		− 25
		+ 3328	
1310	117271		− 25
		+ 3303	
1320	120574		− 25
		+ 3278	
1330	123852		− 25
		+ 3253	
1340	127105		− 24
		+ 3229	
1350	130334		

Suppose now that an error had been made in the calculation of the logarithm of the number 1310, and that by mistake, it is put down as 1172*61*, the differences become—

Δ.	Δ^2.	Correction to apply to second differences.
	− *35*	+ 10
+ 33*18*		
	− *05*	− 20
+ 33*13*		
	− *35*	+ 10

the different type showing the erroneous figures.

The errors thus caused in the second differences are seen at once, and we can correct by making them correspond with the others. The first differences are then altered, and then the original error in the logarithm itself.

In using the chronograph (*see* p. 118) a smoothly revolving cylinder with a heavy fly wheel attached, is set in motion by the hand, and then left to itself: its angular velocity gradually decreases owing to friction. A clock, by electrical means, makes a mark on the surface of the revolving cylinder once in a second: the distances apart of these marks represent a scale of time; but the distances are not equal to each other, but gradually becoming less as the angular velocity decreases.

As this decrease follows *some* law, there must be some set of differences for this time scale, which is regular or varies but slowly.

It is found that the third differences are nearly regular: if they are not in any particular case, corrections are applied to make them so, as slight irregularities in the motion of the instrument are thus eliminated. In the experiment quoted above, the time scale of the instrument read as follows:—

Seconds.	Reading of scale.	Correction applied.	Corrected reading.	Δ.	Δ^2.	Δ^3.
1....	25·90	− ·001	= 25·899			
				+ 23·893		
2....	49·79	+ ·002	= 49·792		−0·048	
				+ 23·845		−0·003
3....	73·64	− ·003	= 73·637		−0·051	
				+ 23·794		−0·003
4....	97·43	+ ·001	= 97·431		−0·054	
				+ 23·740		
5....	121·17	+ ·001	= 121·171			

The corrections applied may be said to reduce the motion of the instrument to smoothness.

To find where the half seconds should be marked, use Equation (x), p. 257, making $n = \frac{1}{2}$. Suppose we wish to find what division on the scale corresponds to $3\frac{1}{2}$ seconds, it is—

$$= t_3 + \tfrac{1}{2}\Delta t_3 + \frac{\frac{1}{2}\,.\,(\frac{1}{2}-1)}{1\,.\,2}\Delta^2 t_3 + \&c.$$

Substitute from above—

$$= 73{\cdot}637 + 11{\cdot}897 + {\cdot}007 + \text{very small terms.}$$
$$= 85{\cdot}541 \text{ scale divisions on the cylinder.}$$

By a further interpolation making $n = \frac{1}{5}$, the division of the scale corresponding to the tenths of seconds can be found; they are—

Seconds.	Divisions.	Δ.
3·0......................	73·637	
		+ 2·382
3·1......................	76·019	
		+ 2·381
3·2......................	78·400	
		+ 2·381
3·3......................	80·781	
&c.	&c.	&c.

Here it is plain that smaller divisions of time may be found by the ordinary rule of proportional parts, as the first differences are so nearly alike.

The chronograph is so arranged that as the projectile cuts each screen and breaks an electric current, a record is made on the surface of glazed paper covering the revolving cylinder: these marks come among the time scale. The screen readings were as under in the above experiment:—

Screen.	Reading of scale.	Correction applied.		Corrected reading.	Δ.	Δ².
1....	74·89	− ·001	=	74·889		
					+ 2·596	
2....	77·48	+ ·005	=	77·485		+0·057
					+ 2·653	
3....	80·14	− ·002	=	80·138		+0·057
					+ 2·710	
4....	82·85	− ·002	=	82·848		+0·058
					+ 2·768	
5....	85·61	+ ·006	=	85·616		+0·058
					+ 2·826	
6....	88·44	+ ·002	=	88·442		

The corrections being applied on the same principle as before; it is seen that the second differences are nearly regular.

The corrected reading for the first screen was .. 74·889 scale divisions.
The corrected reading for 3·0 seconds was .... 73·637 „ „

Difference 1·252

At that time, a tenth of a second is seen to correspond to 2·382 scale divisions, so that the shot passed the first screen at $3{\cdot}0 + \frac{1}{10} \times \frac{1{\cdot}252}{2{\cdot}382} = 3{\cdot}0526$ seconds: in the same manner the other corrected screen readings are converted into seconds, as follows:—

Screen.	Passed at seconds.	Δ.	Δ².
1.........................	3·0526		
		+ 0·1009	
2.........................	3·1616		+0·0024
		+ 0·1114	
3.........................	3·2730		+0·0024
		+ 0·1138	
4.........................	3·3868		+0·0025
		+ 0·1163	
5.........................	3·5031		+0·0025
		+ 0·1188	
6.........................	3·6219		+0·0025

&c., the second differences being nearly equal.

Since the second differences of the times of passing the screens are nearly equal, it must follow from what was found to hold true on p. 259 about the second differences of square numbers, that t and s are (nearly) related together, as expressed by the equation—

$$t = as + bs^2,$$

differentiating and inverting

$$\frac{ds}{dt} \text{ or } v = \frac{1}{a + 2bs},$$

and differentiating again—

$$\frac{d^2s}{dt^2} \text{ or } f = -\frac{2b}{(a + 2bs)^2}\frac{ds}{dt}$$

$$= -2bv^3;$$

$$\text{or } f \propto v^3.$$

This, however, is only true for velocities nearly the same as that just found, and does not hold good universally.

In agreement with this, independent experiments give nearly equal values for K when the velocity is only a little greater or less than 1276 f.s., or the cubic law holds good at about this velocity, since K is nearly constant (*see* Table IV) for velocities from about 1100 to 1300 f.s.

Having obtained the corrected readings we can now apply the formulæ to find the velocity, retardation, and value of K at the fifth screen.

The screens were 150 feet apart $= l$.
From (**xv**) the velocity v_5, at the fifth screen,

$$= \frac{150}{\Delta t_5 - \frac{1}{2}\Delta^2 t_5}$$

The third difference is almost insensible, and is neglected ; Δt_5 is the time from the fifth screen to the next.

$$= \frac{150}{0{\cdot}1188 - \frac{1}{2}{\cdot}0025}$$

$$= 1276{\cdot}1 \text{ feet per second.}$$

The retardation f_5 at the same screen from (**xvi**)

$$= -\frac{v_5^{\,3}}{l^2}(\Delta^2 t_4 - \text{some fourth difference}).$$

Substituting the value of v_5, just obtained—

$$= -\frac{(1276{\cdot}1)^3}{(150)^2}(0{\cdot}0025)$$

$$= -\ 230{\cdot}9 \text{ feet per second.}$$

Now from (iv) page 142—

$$f = -\frac{d^2}{w}K\left(\frac{v}{1000}\right)^3.$$

Therefore, substituting the value of f, just obtained in this experiment, and subsequently the values of w and d, we have—

$$K = \frac{w}{d^2}\left(\frac{1000}{v_5}\right)^3\frac{(v_5)^3(0{\cdot}0025)}{(150)^2}$$

$$= \frac{(23{\cdot}84)}{(4{\cdot}92)^2}\,\frac{(0{\cdot}0025)(1000)^3}{(150)^2}$$

$$= 109{\cdot}4.$$

A slight correction must be applied on account of the density of the air at the time of the experiment, varying a little from the standard. In a similar way the value of K was found for other velocities.

The annexed curve (Fig. 1) shows the variation of K in a graphic form : the horizontal ordinates representing velocities, and the vertical ones the corresponding value of K at each velocity.

It is simply plotted from the values given in Table IV, page 281.

It will be noticed that a very marked maximum summit occurs at velocities near or just exceeding that of sound ; if it were not for this, the curve would be much more regular in its appearance, and there would probably be more likelihood of the discovery of some general law.

Up to about 800 f.s. the product Kv is found to always nearly equal 57800, hence the first part of the curve is regular, but it afterwards varies considerably.

In a similar way a diagram (Fig. 2) may be made of the resistance of the air (p) per circular inch (a circular inch is the area of a circle one inch in diameter) to a projectile moving at various velocities, in which another bend in the curve is distinctly visible in the neighbour-

Fig. 1.—Diagram of K.

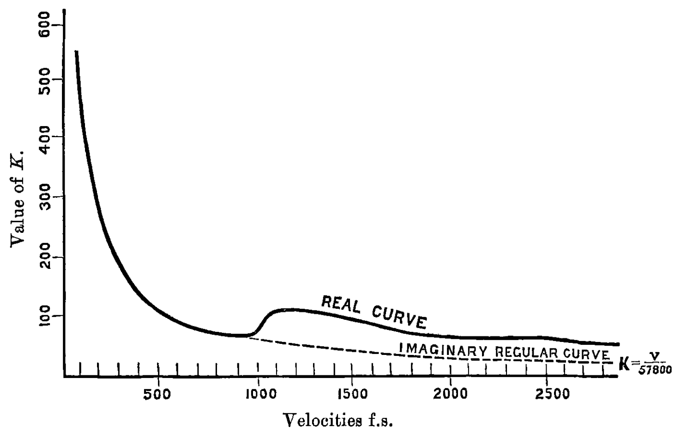
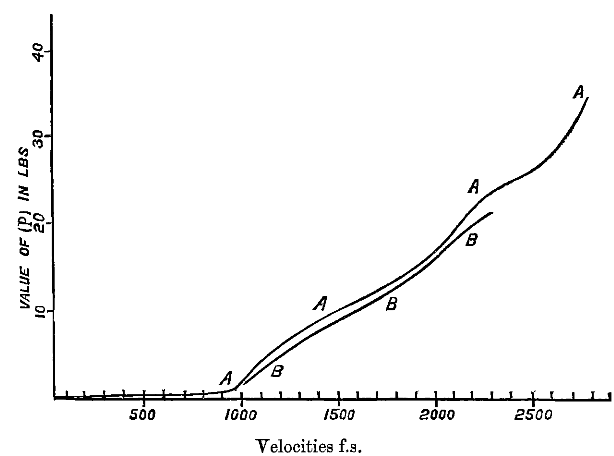


Fig. 2.



hood of $v = 2413$ f.s.,—the velocity at which air rushes into a vacuum. The curve AAAA is deduced from the tables founded on Bashforth's data: the curve BBB corresponds with the results of Krupp's later experiments when sharper headed and smoother projectiles were employed, which experience less resistance from the air.

Terminal Velocity.

If a heavy body falls downwards under the uniform acceleration of gravity, its velocity continually increases; but when it falls in a resisting medium there is a limit to the velocity, which can be attained, and the body then falls down at a uniform speed, called the **terminal velocity**; this limit is reached when the acceleration of gravity acting on the mass of the body, that is the weight of the body in pounds, is equal to the resistance of the air in pounds; this can be illustrated in a child's air balloon, which very soon attains a uniform velocity when falling in still air.

The tendency of the resistance of the air to reduce a high velocity to the terminal velocity is well shown in the case of meteorites.

"Meteorites enter the atmosphere with planetary velocities ranging from 10 to 45 miles per second. . . . . A very speedy change must take place. . . . . It is clear that the speed of the meteorite after the whole of the atmosphere has been traversed will be extremely small, and comparable with that of an ordinary falling body."

"From experiments on the penetration of a similar body into earth, made by Professor A. S. Herschel, it has been calculated that the striking velocity of the meteorite which fell at Middlesbrough in Yorkshire on March 14, 1881, was only 412 feet per second." ("Guide to Collection of Meteorites, South Kensington Museum." Dr. L. Fletcher.)

Example 1. Find the terminal velocity of a 64-pr. projectile, dropped point downwards; atmospheric conditions normal.

$$\text{Here } d = 6{\cdot}3 \text{ inches,}$$
$$w = 64 \text{ lbs.}$$
$$g = 32{\cdot}19$$
$$\text{Take } K = 75.$$

Assuming a value for K looks like assuming the answer to the whole question; as we see from Table IV, that when K is equal to 75 the velocity must be somewhere between 840 f.s. and 1000 f.s.

A guess of the correct value of K to take must however be made; if not correct it will not affect the resulting velocity very much, since K is involved in the equation only as a first power, while v appears as a cubic quantity.

Thus, suppose we had erroneously assumed $K = 100$ (corresponding as we observe from Table IV, to velocities of 602·6, 1073·5, or 1771·4 f.s.); if the equation is worked out with that value of K, we obtain $v = 803{\cdot}7$ f.s., which is a fairly good approximation. If we wish for a more accurate result, the example may be gone through again, taking the value of K (76), corresponding to a velocity of 803·7 f.s. Working out the problem with the new value of K we obtain $v = 880{\cdot}7$, a closer approximation than the last. If a more accurate result still is required we can take the value of K (75), corresponding to a velocity of 880·7 f.s., and this time we arrive at an answer as below, in which the resulting velocity agrees with the value of K assumed.

By definition, the resistance of the air (R) must be equal to the weight of the projectile (w).

Now from (i) p. 141, $R = d^2\dfrac{K}{g}\left(\dfrac{v}{1000}\right)^3$

Substituting the values of R. d. K. and g. we obtain—

$$v = 884{\cdot}5 \text{ f.s.}$$

CHAPTER IV.—TRAJECTORIES AT ANY ANGLE OF ELEVATION.

(See also Part I, Chapter XIII.)

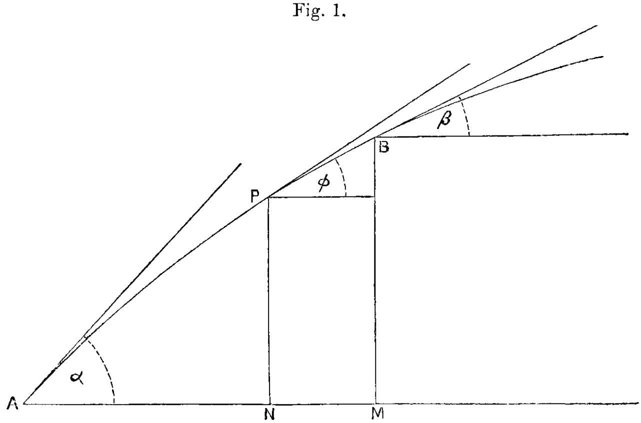
In mathematical calculations of the range, time of flight, greatest height and angle of descent, it is usual, for the sake of simplicity, to neglect the deviation of the axis of the projectile from the trajectory, and also the drift, partly because these circumstances have but little effect (especially at the lower angles of elevation), but chiefly because of the very great difficulty of their calculation.

Bashforth's Method.

It is assumed that the retardation varies as the cube of the velocity (the coefficient changing), or $f = -2bv^3$, in which

$$2b = \frac{d^2}{w} \cdot K \cdot \frac{1}{(1000)^3} \ldots\ldots\ldots\ldots\ldots \text{(i)}.$$

Fig. 1.



Suppose A the point of projection, and APB part of a trajectory in a resisting medium (Fig. 1).

Let α = the angle of departure or inclination of the tangent to the trajectory to the horizontal at initial point A.

β = the inclination of the tangent to the trajectory to the horizontal at some later definite point B.

ϕ = the inclination of the tangent to the trajectory of the horizontal at any point P.

V = the velocity in f.s. at muzzle.

v = the velocity in f.s. at any point P.

p = horizontal component of velocity in f.s. at A.

Let q = horizontal component of velocity in f.s. at B.

u = horizontal component of velocity in f.s. at any point P.

${}_\alpha t_\beta$, or for brevity, t = the time in seconds taken to pass from A to B while ϕ changes from α to β.

${}_\alpha x_\beta$,, ,, x = the hor. distance in feet from A to M while ϕ changes from α to β.

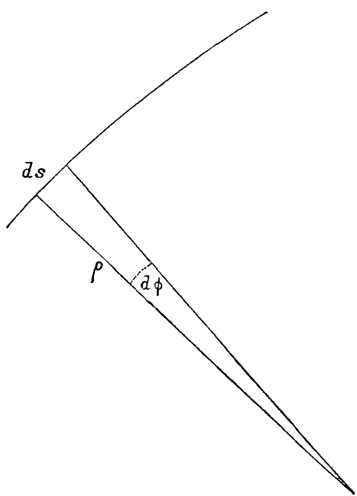
${}_\alpha y_\beta$,, ,, y = the vert. height in feet of B above A while ϕ changes from α to β.

Since the retardation at any point is $-2bv^3$, the horizontal component of it or the rate of diminution of the horizontal velocity is $-2bv^3 \cos\phi$, and since $f = \frac{dv}{dt}$.

$$\therefore \frac{du}{dt} = -2bv^3 \cos\phi \dots\dots\dots\dots\dots\dots \text{(ii)}$$

The equation (ii) has first been found, as it is the simplest equation of motion; since by taking the rate of change of the horizontal component of velocity, the acceleration of gravity is avoided. We will now find the rate of change of velocity in the direction of the normal to the trajectory, thereby avoiding the retarding effect of the resistance of the air, which acts tangentially.

Fig. 2.



If ρ be the radius of curvature of any small part ds of the trajectory inclined at an angle ϕ to a horizontal (Fig. 2); then ds subtends an angle $d\phi$ at the centre of curvature; the normal acceleration is $\frac{v^2}{\rho}$, and this must equal the normal component of the acceleration of gravity—

$$\text{or } \frac{v^2}{\rho} = g\cos\phi \dots\dots\dots\dots\dots\dots \text{(iii)}$$

$$\text{Now } \rho = -\frac{ds}{d\phi}$$

(the negative sign being taken, since ϕ diminishes as s increases),

$$\text{and } \frac{ds}{dt} = v,$$

$$\text{hence } \rho = -v\frac{dt}{d\phi}.$$

Substituting this value of ρ in (iii) we have—

$$v\frac{d\phi}{dt} = -g\cos\phi, \ldots\ldots\ldots\ldots \text{(iv)}$$

Dividing (ii) by (iv) we obtain—

$$\frac{du}{d\phi} = \frac{2b}{g}v^4. \ldots\ldots\ldots\ldots\ldots\ldots \text{(v)}$$

$$\text{Since } v = u\sec\phi$$

$$\therefore \frac{du}{d\phi} = \frac{2b}{g}u^4\sec^4\phi.$$

Integrate each side between the limits $u = p$, and $u = q$, when the limits of ϕ must be α and β

$$\int_p^q \frac{du}{u^4} = \frac{2b}{g}\int_\alpha^\beta \sec^4\phi d\phi,$$

whence

$$\left(\frac{1000}{q}\right)^3 - \left(\frac{1000}{p}\right)^3 = \frac{d^2}{w} \times \frac{K}{g}\left(P_\alpha - P_\beta\right), \ldots. \text{(vi)}$$

in which the value of $2b$ is substituted from (i), and for brevity P_α is written for $3\int_0^\alpha \sec^4\phi d\phi$ which $= 3\tan\alpha + \tan^3\alpha$.

Suppose u_0 the velocity at the vertex (this must be horizontal) : in this case $\beta = 0$ and $P_\beta = 0$.

$$\therefore \left(\frac{1000}{u_0}\right)^3 - \left(\frac{1000}{p}\right)^3 = \frac{d^2}{w}.\frac{K}{g}P_\alpha. \ldots\ldots\ldots\ldots \text{(vii)}$$

Substituting the value of $\left(\frac{1000}{p}\right)^3$ obtained from this equation in (vi), we have—

$$\left(\frac{1000}{q}\right)^3 = \left(\frac{1000}{u_0}\right)^3 - \frac{d^2}{w}.\frac{K}{g}P_\beta,$$

$$\text{or } q = \frac{u_0}{(1-\gamma P_\beta)^{\frac{1}{3}}}, \ldots\ldots\ldots\ldots \text{(viii)}$$

$$\text{if } \gamma = \frac{d^2}{w}.\frac{K}{g}\left(\frac{u_0}{1000}\right)^3 \text{ for brevity (ix)}$$

In like manner, we have generally $u = \dfrac{u_0}{(1-\gamma P_\phi)^{\frac{1}{3}}}. \ldots\ldots\ldots\ldots\ldots \text{(x)}$

Again (iv) may be written—

$$\frac{dt}{d\phi} = -\frac{v}{g}\sec\phi$$

$$= -\frac{u}{g}\sec^2\phi, \text{ for } v = u\sec\phi.$$

Integrating from the lower limit $t = 0$ to t, and consequently $\phi = \alpha$ to β, and substituting for u its value from (x) in terms of u_0, we obtain—

$$_{\alpha}t_{\beta} = \frac{u_0}{g}\int_{\beta}^{\alpha}\frac{\sec^2\phi}{(1-\gamma P_{\phi})^{\frac{1}{3}}}d\phi \ldots\ldots\ldots\ldots \text{(xi)}$$

Again, $$\frac{dx}{d\phi} = \frac{dx}{dt}\cdot\frac{dt}{d\phi}.$$

Now $\dfrac{dx}{dt} = u$, and substituting the value of $\dfrac{dt}{d\phi}$ from (iv) we obtain—

$$\frac{dx}{d\phi} = -u\,.\frac{v}{g}\sec\phi;$$

$$\therefore dx = -\frac{u^2}{g}\sec^2\phi d\phi.$$

Integrate this between the limits $x = 0$ to x; and consequently $\phi = \alpha$ to β, and substituting again its value for u from (x), we obtain—

$$_{\alpha}x_{\beta} = \frac{u_0^{\,2}}{g}\int_{\beta}^{\alpha}\frac{\sec^2\phi}{(1-\gamma P_{\phi})^{\frac{2}{3}}}d\phi \ldots\ldots\ldots\ldots \text{(xii)}$$

In the same way $_{\alpha}y_{\beta}$ is found; or if we know x we can find y at once, thus:—

$$\frac{dy}{d\phi} = \frac{dy}{dx}\cdot\frac{dx}{d\phi}.$$

Now $\dfrac{dy}{dx} = \tan\phi$, and substituting the value of $\dfrac{dx}{d\phi}$, we obtain—

$$_{\alpha}y_{\beta} = \frac{u_0^{\,2}}{g}\int_{\beta}^{\alpha}\frac{\tan\phi\sec^2\phi}{(1-\gamma P_{\phi})^{\frac{2}{3}}}d\phi. \ldots\ldots\ldots \text{(xiii)}$$

Mr. Bashforth has given copious tables for the integrals (**xi**,) (**xii**,) and (**xiii**) for different values of γ, and for different angles, in order to work out practical problems.

In making a calculation, the trajectory is divided into two or more arcs, and the time over each, and also the horizontal and vertical ordinates at the end, are found. When these have been calculated, the trajectory may be plotted out; the vertical height ascended and descended by the projectile being the same, or differing by some given quantity depending on the height of the gun above the target. The sum of the horizontal ordinates gives the range, and the sum of the times over the arcs gives the total time of flight.

It is important to understand the principle of this method, as it **is** probably the most scientific and accurate which has yet been employed; but it cannot be further continued in this book, as practical

examples cannot be worked without the necessary tables. These are to be found in "Bashforth's Motions of Projectiles," 1873, and "Supplement," 1881.

Niven's Method.

(*See* also p. 161).

An approximate method has been devised by Mr. Niven, F.R.S., which requires less tabular matter; it depends on a relation between the velocity and the inclination of the trajectory at any point. The theory of this method may be briefly explained as follows. Adopting the same notation, we have as before in (ii) and (iv)—

$$\frac{du}{dt} = -f\cos\phi, \quad \ldots\ldots\ldots\ldots\ldots\ldots \text{(xiv)}$$

$$\text{and } v\frac{d\phi}{dt} = -g\cos\phi. \quad \ldots\ldots\ldots\ldots\ldots\ldots \text{(xv)}$$

$$\therefore \frac{d\phi}{du} = \frac{g}{fv}.$$

Hence

$$\int_\beta^\alpha d\phi = g\int_q^p \frac{du}{vf},$$

$$\text{or } \alpha - \beta = g\int_q^p \frac{du}{vf}. \quad \ldots\ldots\ldots\ldots\ldots\ldots \text{(xvi)}$$

From (xiv) we now find the corresponding time, while α changes to β, viz.:—

$$t = \int_q^p \frac{du}{f\cos\phi}. \ldots\ldots\ldots\ldots\ldots\ldots \text{(xvii)}$$

Also since $\dfrac{dx}{dt} = u$ we derive

$$x = \int u\,dt$$

$$= \int_q^p \frac{u}{f\cos\phi}du, \quad \ldots\ldots\ldots\ldots \text{(xviii)}$$

$$\text{and } y = \int_q^p \frac{u\tan\phi}{f\cos\phi}du. \quad \ldots\ldots\ldots\ldots \text{(xix)}$$

The above quantities (xvi) (xvii) (xviii) and (xix) are the most general forms of the integrals, which give the characteristic quantities in the motion of the projectile; in order to solve them we must enter the expression for f in terms of the velocity; as already stated this cannot be done exactly, since no simple law properly expresses the relation between the two. Let us, however, assume that f is *some* function $f(v)$ of v, and, for example, let us take the integral given above for x, and examine whether it cannot be approximately calculated.

We have from (xviii) $x = \displaystyle\int_q^p \frac{u}{f(v)\cos\phi}du$

$$= \int_q^p \frac{u}{f(u\sec\phi)\cos\phi}du.$$

Now it is obvious that there must be *some* mean value of the angle of inclination between α and β, which we may denote by $\bar{\phi}$, such that

$$x = \int_q^p \frac{u}{f(u \sec \bar{\phi}) \cos \bar{\phi}} du$$

or writing herein $u = z \cos \phi$.

$$x = \cos \bar{\phi} \int_{q \sec \bar{\phi}}^{p \sec \bar{\phi}} \frac{z}{f(z)} dz.$$

If we substitute Mr. Bashforth's expression for the retardation, this is the same as

$$\frac{d^2}{w} x = \cos \bar{\phi} \int_{q \sec \bar{\phi}}^{p \sec \bar{\phi}} \frac{(1000)^3}{Kz^2} dz.$$

Table VII, drawn up by Mr. Bashforth, gives the value of the integral between the limits $v = 100$ and $v = 2900$ f.s. We may write the last expression in the form—

$$\frac{d^2}{w} x = \cos \bar{\phi} \left(S_{p \sec \bar{\phi}} - S_{q \sec \bar{\phi}}\right).$$

A similar mode of treatment applies to all the four integrals (xvi), (xvii), (xviii), and (xix), and we may now write them as follows, merely premising that instead of $\alpha - \beta$, which is in circular measure, we shall substitute the corresponding number of degrees δ,—the angle measure commonly used. We thus obtain from (xvi)—

$$\frac{d^2}{w} \delta \sec \bar{\phi} = \frac{180g}{\pi} \int_{q \sec \bar{\phi}}^{p \sec \bar{\phi}} \frac{(1000)^3 dz}{Kz^4},$$

The integral of this last expression has been tabulated by Mr. Niven, (Table VIII, p. 293), for a range of velocity from 400 to 2500 f.s.

We may write the equations (xvi) to (xix) thus:—

$$\frac{d^2}{w} \delta = \cos \bar{\phi} \left(D_{p \sec \bar{\phi}} - D_{q \sec \bar{\phi}}\right). \quad \ldots\ldots\ldots\ldots\ldots \text{(xx)}$$

$$\frac{d^2}{w} t = T_{p \sec \bar{\phi}} - T_{q \sec \bar{\phi}}, \quad \ldots\ldots\ldots\ldots\ldots\ldots \text{(xxi)}$$

$$\frac{d^2}{w} x = \cos \bar{\phi} \left(S_{p \sec \bar{\phi}} - S_{q \sec \bar{\phi}}\right), \quad \ldots\ldots\ldots\ldots \text{(xxii)}$$

and

$$\frac{d^2}{w} y = \sin \bar{\phi} \left(S_{p \sec \bar{\phi}} - S_{q \sec \bar{\phi}}\right), \quad \ldots\ldots\ldots\ldots \text{(xxiii)}$$

The value of $\bar{\phi}$ was obtained approximately by Mr. Niven, from an expanding series, but space will not allow us to follow his calculations.

In equation (xx), $\bar{\phi}$ may be taken approximately equal to $\frac{1}{2}(\alpha + \beta)$ for low angles, and equal to $\frac{1}{2}(\tan \alpha + \tan \beta)$ for higher angles of elevation. Hence q can be found.

In the other three equations this value must be changed by adding in the ascending and subtracting in the descending branch the amount $\frac{1}{3}\frac{p-q}{p+q}(\alpha-\beta)$.

For equation (**xxi**) the correction $\frac{1}{6}\frac{p-q}{p+q}(\alpha-\beta)$ is a rather more accurate approximation; but the preceding will give good results.

From the nature of the case this method gives more accurate results the less the angle interval over which the integration extends. It is found, however, to give fairly good results for considerable intervals.

The reader is referred for fuller discussion to Mr. Niven's paper, in the Proceedings of the Royal Society, No. 181, 1877, and also to a postscript to the same paper, by Professor J. C. Adams, F.R.S.

From the circumstances that the expressions for x and y given in (**xxii**) and (**xxiii**) have the same value of $\bar{\phi}$, we see that the motion of the projectile, so far as x and y are concerned (and t nearly), is the same as if it had moved along the chord joining the extremities of the arc.

Example 1.

The 8-inch howitzer is fired with a quadrant elevation of 23°. Muzzle velocity 920 f.s. (corresponding to the $10\frac{1}{2}$ lb. charge), weight of shell, 180 lbs.; diameter (with rotating gas check) 8 inches. Find the range and time of flight.

Sufficient accuracy can be obtained by dividing the whole trajectory into only two arcs, one ascending, the other descending.

For the ascending branch.

Here
$$\delta = 23°,$$
$$V = 920 \text{ f.s.},$$
$$\log\frac{d^2}{w} = \log\frac{64}{180}$$
$$= \bar{1}\cdot 5509.$$

Let the value of $\bar{\phi}$ to be used in equation (**xx**) be denoted by $\bar{\phi}_1$;

$$\therefore \tan\bar{\phi}_1 = \tfrac{1}{2}\tan 23°;$$
$$\therefore \bar{\phi}_1 = 11°\ 59'.$$

See Table XIX, pp. 318–321.

log tan 23°	=	9·6279
log 2	=	0·3010
log tan 11° 59′	=	9·3269

Now
$$p = 920\cos 23°$$
$$= 846\cdot 9 \text{ f.s.}$$

and
$$p\sec\bar{\phi}_1 = 846\cdot 9\sec 11°\ 59'$$
$$= 865\cdot 7 \text{ f.s.}$$

Also
$$\frac{d^2}{w}\delta = \cos\bar{\phi}_1\left(D_{p\sec\bar{\phi}_1} - D_{q\sec\bar{\phi}_1}\right),$$

or
$$8\cdot 3599 = D_{865\cdot 7} - D_{q\sec\bar{\phi}_1}.$$
$$\therefore D_{q\sec\bar{\phi}_1} = 66\cdot 7503.$$
$$\therefore q\sec\bar{\phi}_1 = 727\cdot 9 \text{ f.s.},$$
whence $q = 712$ f.s.

Vide Table VIII.

$D_{865\cdot 7} = 75\cdot 1102$

$D_{727\cdot 9} = 66\cdot 7503$

We can now obtain a value of $\bar{\phi}$, to employ in equations (**xxii**) and (**xxiii**), for, as we have already noticed, it is not the same as that employed in (**xx**), but

$$\begin{aligned}\bar{\phi} &= \bar{\phi}_1 + \frac{p-q}{p+q} \cdot \frac{(\alpha - \beta)}{3} \\ &= 11^\circ\, 59' + \frac{846{\cdot}9 - 712}{846{\cdot}9 + 712} \times \frac{23^\circ}{3} \\ &= 11^\circ\, 59' + 40' \\ &= 12^\circ\, 39'.\end{aligned}$$

Now $$x = \frac{\cos \bar{\phi}}{\frac{d^2}{w}} (S_{p \sec \bar{\phi}} - S_{q \sec \bar{\phi}})$$

$$= \frac{\cos 12^\circ\, 39'}{\frac{d^2}{w}} (S_{865{\cdot}7} - S_{727{\cdot}9}) \text{ Consult Table VII.}$$

The values of $p \sec \bar{\phi}$ and $q \sec \bar{\phi}$ in the expressions $S_{p \sec \bar{\phi}}$ and $S_{\sec q \bar{\phi}}$ may be taken the same as in the calculation for δ without appreciable loss of accuracy.

$$= 7784{\cdot}5 \text{ feet.}$$

Also $$y = X \tan \bar{\phi} = 1747{\cdot}2 \text{ feet,}$$

and $$\frac{d^2}{w}\, t = T_{865{\cdot}7} - T_{727{\cdot}9}. \quad \text{Consult Table VI.}$$

whence $t = 10{\cdot}0806$ seconds.

For the descending branch assume an angle of descent of $27^\circ\, 54' = 27{\cdot}9^\circ$.

Experience will tell about what angle to select: if it is not close to the correct angle it will be found that the lower end of the descending arc is a good deal either above or below the plane. An approximate result can still, however, be obtained, as shown at the end of this example.

Here $\tan \bar{\phi}_1 = \frac{1}{2} \tan 27^\circ\, 54'$
$\therefore\ \bar{\phi}_1 = 14^\circ\, 49'{\cdot}7.$

The p in this arc is the q of the last arc $= 712$ f.s., and $p \sec \bar{\phi}_1 = 712 \sec 14^\circ\, 49{\cdot}7' = 736{\cdot}6$ f.s.

To find $q \sec \bar{\phi}_1$ we have—

$$\frac{d^2}{w} \delta = \cos \bar{\phi}_1 (D_{p \sec \bar{\phi}_1} - D_{q \sec \bar{\phi}_1}),$$

whence, using Table VIII as before, we find

$$q \sec \bar{\phi}_1 = 631{\cdot}6 \text{ f.s.,}$$
whence $q = 610{\cdot}6$ f.s.

The value of $\bar{\phi}$ to employ with x and y in the descending branch is—

$$\bar{\phi} = \bar{\phi}_1 - \frac{p-q}{p+q} \cdot \frac{\alpha - \beta}{3}$$

$$= \bar{\phi}_1 - \frac{712 - 610\cdot6}{712 + 610\cdot6} \times \frac{27\cdot9}{3}$$
$$= 14^\circ\, 49\cdot7' - 42\cdot7'$$
$$= 14^\circ\, 7'.$$

Now $$x = \frac{\cos 14^\circ\; 7'}{\frac{d^2}{w}}(S_{736\cdot6} - S_{631\cdot6})$$
$$= 7040\cdot4 \text{ feet,}$$

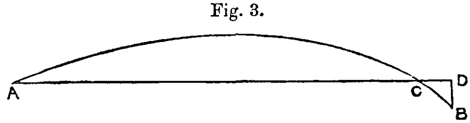
and $$y = x \tan 14^\circ\; 7'$$
$$= 1770\cdot6 \text{ feet.}$$

Also $$t = \frac{T_{736\cdot6} - T_{631\cdot6}}{\frac{d^2}{w}}$$
$$= 10\cdot6607 \text{ seconds.}$$

Hence range $= 7784\cdot5 + 7040\cdot4 = 14{,}824\cdot9$ feet $= 4941\cdot6$ yards. And total time of flight, $10\cdot0806 + 10\cdot6607 = 20\cdot7413$ seconds.

It will be noticed from the above results, that the projectile has fallen 23·4 feet $(1770\cdot6 - 1747\cdot2)$ more than it rose; if therefore the target is at the same level as the muzzle of the howitzer, it will be necessary to deduct a little from both the range and time of flight. Suppose the projectile to move in a straight line, inclined at 27° 54′, to a horizontal and with a uniform velocity, of which the horizontal component is 610·6 f.s. for the short distance considered.

Fig. 3.



We have found (Fig. 3) the range AD and the time for the projectile to reach B; it is necessary to deduct CD from AD, and the time it took to pass over CB from the time taken to travel from A to B, to find the true range and time of flight.

Now $$CD = BD \cot 27^\circ\; 54'$$
$$= 23\cdot4 \cot 27^\circ\; 54'$$
$$= 44\cdot1 \text{ feet}$$
$$= 14\cdot7 \text{ yards.}$$

The time over $CD = \frac{s}{v}$
$$= \frac{44\cdot1}{610\cdot6} \text{ approximately}$$
$$= 0\cdot0722 \text{ second.}$$

Therefore the true range would be

$$4941{\cdot}6 - 14{\cdot}7 = 4926{\cdot}9 \text{ yards,}$$

and the time of flight

$$20{\cdot}7413 - 0{\cdot}0722 = 20{\cdot}6691 \text{ seconds.}$$

The published range table gives 5000 yards as the range, and 21·5 seconds for the time of flight.

No account has been taken in this problem of the rather variable "jump" which always takes place in practice; assuming that this adds about 32 minutes to the elevation (which is a fair average), the range would be increased to almost exactly 5000 yards, and the time of flight would be made about 0·3555 seconds longer, making a total of $20{\cdot}6691 + 0{\cdot}3555 = 21{\cdot}0246$ seconds, which is 0·475 of a second less than that given in the table; the results of calculation and direct experiment are thus shown to be fairly concordant at a long range with a curved trajectory. As noted on p. 166, the barometric pressure should be taken as the mean over the trajectory, and as the projectile rises to a height of about $H = 4T^2 = 4(20)^2 = 1600$ feet, the mean barometric pressure should be that due to an elevation of $\frac{2}{3} \times 1600 = 1{\cdot}067$ feet, corresponding to a diminution of barometric pressure of about 1·067 inches; a value of τ from Table XI should be employed to correct the value of $\frac{d^2}{w}$ at the beginning of the problem on p. 271, but as the final result obtained is nearly correct, a value for σ should be taken, which will nearly neutralise the effect of this factor; in other words the coefficient of reduction $\sigma\tau = 1$ nearly.

Probability Table.

(*See* also Part I, Chapter XIV, p. 173, and Table XII, p. 306.)

The formation of the Probability Table may be briefly explained by the following outline by Lt.-Col. Kensington, R.A.

Let y be the number of errors between x and $x+dx$; then y is a function of x such that as x increases y diminishes, since the larger the errors are individually the fewer there are of them; also y is proportional to dx, since dx is infinitesimal:—

$$\therefore y = \phi(x)dx.$$

Laplace has found that $e^{-\frac{x^2}{c^2}}$ is a very close approximation to the true form of $\phi(x)$. c is a constant depending on the mode of estimating the errors.

The No. of errors between x and dx is $y = e^{-\frac{x^2}{c^2}}dx$,

and the sum of their values $= xe^{-\frac{x^2}{c^2}}dx$.

If γ be the mean error

$$= \frac{\text{total of all the values}}{\text{total number of errors}}$$

$$= \frac{\int_0^\infty xe^{-\frac{x^2}{c^2}}dx}{\int_0^\infty e^{-\frac{x^2}{c^2}}dx}$$

Integrating the upper and lower expressions (see Theory of Errors of Observations, Sir G. B. Airy),

$$\gamma = \frac{\frac{c^2}{2}}{\frac{c\sqrt{\pi}}{2}},$$

$$\text{or} \quad c = \gamma\sqrt{\pi}.$$

Hence we have found the value of c in terms of the mean error.

The *probability* of the error of some one trial not exceeding x is

$$\frac{\text{Number of these errors}}{\text{Total number of errors}} = \frac{\int_0^x e^{-\frac{x^2}{c^2}}dx}{\int_0^\infty e^{-\frac{x^2}{c^2}}dx}$$

$$\text{Let } \frac{x}{c} = z. \quad \therefore \text{ probability} = \frac{2}{c\sqrt{\pi}}\int_0^z e^{-z^2}d(cz)$$

$$= \frac{2}{\sqrt{\pi}}\int_0^z e^{-z}dz.$$

Suppose the probability is $\frac{1}{2}$, or that required for 50 per cent., we have

$$\therefore \frac{\sqrt{\pi}}{4} \text{ or } 0{\cdot}443113 = \int_0^z e^{-z^2}dz.$$

$$\text{Let } \phi'(z) = \epsilon^{-z^2}$$

$$= 1 - z^2 + \frac{z^4}{2} - \frac{z^6}{6} + \&c.$$

$$\therefore \text{ by integration } 0{\cdot}443113 = z - \frac{z^3}{3} + \frac{z^5}{10} - \frac{z^7}{42} + \&c.$$

$$\text{or} \quad \phi(z) = 0.$$

Try some likely value of z, suppose $0{\cdot}4$, then

$$z = 0{\cdot}4 + h.$$

But by Taylor's theorem

$$\phi(z+h) - \phi(z) = h\phi'(z) \text{ approximately.}$$

$$\therefore h = \frac{0{\cdot}443113 - \phi(0{\cdot}4)}{\phi'(0{\cdot}4)}.$$

Solving this, we find $\qquad h = 0{\cdot}074.$

For a second approximation take

$$z = 0{\cdot}474, \text{ the sum of the previous } z+h,$$

and, proceeding in like manner, we find

$$h = 0{\cdot}0028.$$

For a third approximation take

$$z = 0{\cdot}4768, \text{ the sum of the last } z + h,$$

we find

$$h = 0{\cdot}0001,$$

$$\therefore z = 0{\cdot}4769, \text{ very approximately.}$$

And, substituting for z its value, $\frac{x}{\gamma\sqrt{\pi}}$, we find

$$x = 0{\cdot}8453\gamma,$$

or the width, $2x$, of the probable zone which contains 50 per cent. is $1{\cdot}6906\gamma$ (γ being the mean error).

In the same way the widths of zones containing other percentages have been calculated; and hence Table XII has been compiled, all the widths being divided by $1{\cdot}6906\gamma$, thus making the 50 per cent. zone the unit for comparison.

GUNNERY TABLES.

CONTENTS.

NOTE.—In these Tables the plan of horizontal bands three lines wide has been adopted ; the eye can readily and correctly follow the horizontal line required, as it must be either the top, centre, or bottom one of a band. Vertical bands three columns wide are also employed in some tables to assist the eye in the same manner when glancing down vertical columns. It is easy to remember whether the right, centre, or left column of a band is being used.

TABLE I.

Gravimetric Density.

Gravimetric density, when each pound of a powder charge occupies any of the undermentioned number of cubic inches. Gravimetric density = 1, when one pound of powder occupies 27·73 cubic inches.

Cubic inches.	0·0	0·1	0·2	0·3	0·4	0·5	0·6	0·7	0·8	0·9
22	1·260	1·255	1·249	1·243	1·238	1·232	1·227	1·222	1·216	1·211
23	1·206	1·200	1·195	1·190	1·185	1·180	1·175	1·170	1·165	1·160
24	1·155	1·151	1·146	1·141	1·136	1·132	1·127	1·123	1·118	1·114
25	1·109	1·105	1·100	1·096	1·092	1·087	1·083	1·079	1·075	1·071
26	1·066	1·062	1·058	1·054	1·050	1·046	1·042	1·039	1·035	1·031
27	1·027	1·023	1·019	1·016	1·012	1·008	1·005	1·001	0·997	0·994
28	0·990	0·987	0·983	0·980	0·976	0·973	0·970	0·966	0·963	0·959
29	0·956	0·953	0·950	0·946	0·943	0·940	0·937	0·934	0·930	0·927
30	0·924	0·921	0·918	0·915	0·912	0·909	0·906	0·903	0·900	0·897
31	0·894	0·892	0·889	0·886	0·883	0·880	0·877	0·875	0·872	0·869
32	0·867	0·864	0·861	0·858	0·856	0·853	0·851	0·848	0·845	0·843
33	0·840	0·838	0·835	0·833	0·830	0·828	0·825	0·823	0·820	0·818
34	0·815	0·813	0·811	0·808	0·806	0·804	0·801	0·799	0·797	0·795
35	0·792	0·790	0·788	0·785	0·783	0·781	0·779	0·777	0·775	0·772
36	0·770	0·768	0·766	0·764	0·762	0·760	0·758	0·756	0·753	0·751
37	0·749	0·747	0·745	0·743	0·741	0·739	0·737	0·735	0·734	0·732
38	0·730	0·728	0·726	0·724	0·722	0·720	0·718	0·716	0·715	0·713
39	0·711	0·709	0·707	0·706	0·704	0·702	0·700	0·698	0·697	0·695
40	0·693	0·691	0·690	0·688	0·686	0·685	0·683	0·681	0·680	0·678
41	0·676	0·675	0·673	0·671	0·670	0·668	0·666	0·665	0·663	0·662
42	0·660	0·659	0·657	0·655	0·654	0·652	0·651	0·649	0·648	0·646
43	0·645	0·643	0·642	0·640	0·639	0·637	0·636	0·634	0·633	0·632
44	0·630	0·629	0·627	0·626	0·624	0·623	0·622	0·620	0·619	0·618
45	0·616	0·615	0·613	0·612	0·611	0·609	0·608	0·607	0·606	0·604
46	0·603	0·601	0·600	0·599	0·598	0·596	0·595	0·594	0·592	0·591
47	0·590	0·589	0·587	0·586	0·585	0·584	0·582	0·581	0·580	0·579
48	0·578	0·576	0·575	0·574	0·573	0·572	0·570	0·569	0·568	0·567
49	0·566	0·565	0·564	0·562	0·561	0·560	0·559	0·558	0·557	0·556
50	0·555	0·553	0·552	0·551	0·550	0·549	0·548	0·547	0·546	0·545
51	0·544	0·543	0·542	0·540	0·539	0·538	0·537	0·536	0·535	0·534
52	0·533	0·532	0·531	0·530	0·529	0·528	0·527	0·526	0·525	0·524
53	0·523	0·522	0·521	0·520	0·519	0·518	0·517	0·516	0·515	0·514
54	0·513	0·512	0·511	0·510	0·510	0·509	0·508	0·507	0·506	0·505
55	0·504	0·503	0·502	0·501	0·500	0·500	0·499	0·498	0·497	0·496

EXPLANATION OF TABLE I.—Knowing the cubic contents of the chamber of a gun and the amount of powder in the charge, the number of cubic inches allotted to each pound of powder can be found; Table I. then indicates the corresponding gravimetric density (G.D.); thus the cubic content (see Table XVI) of the chamber of the 9·2-inch B.L. gun is 5000 cubic inches, and the powder charge is 175 lbs., hence—

$5000 \div 175 = 28 \cdot 57$ cub. ins. are allotted to each lb. of the charge,

and from Table I, columns 1, 7, and 8, the G.D. $= 0 \cdot 973 - 0 \cdot 7(0 \cdot 973 - 0 \cdot 970)$

$= 0 \cdot 971$

EXPLANATION OF TABLE II.—The theoretical amount of work capable of being performed on a projectile when a powder charge *of unit gravimetric density* expands to definite amounts in the bore of a gun is stated in Table II: thus suppose a charge of 5 lbs. G.D. = 1 expands 4 times, under these circumstances each pound of powder (see Table II, columns 5 and 7) performs 82·107 ft.-tons of work, and as there are 5 lbs. of powder, the total is $82 \cdot 107 \times 5 = 410 \cdot 535$ ft.-tons; only a part, called the factor of effect, is, however, realised, owing to loss from heat given to the gun and other causes; suppose the factor is 0·8, the total work realised is $410 \cdot 535 \times 0 \cdot 8 = 328 \cdot 428$ ft.-tons.

Practically, however, the gravimetric density is seldom or never unity, and deduction must be made for the work lost in expanding from a *supposed* G.D. of unity to a G.D. equal to a tabulated density of the products of explosion (see Table II, columns 2, 6, and 10), for no work is practically performed on the projectile in this part of the expansion of the powder charge: thus, again taking the 9·2-inch B.L gun (see Table XVI) the length of the bore, less the length of the chamber, is $289 \cdot 8 - 43 \cdot 8 = 246$ inches, hence—

the capacity of the bore $289 \times \frac{1}{4}\pi(9 \cdot 2)^2 = 19{,}212$ cubic inches,

and the capacity of the chamber = 5,000 " "

∴ the total capacity = 24,212 " "

as a charge of 175 lbs. of unit gravimetric density occupies $175 \times 27 \cdot 73 =$ 4,853 " "

it must expand $24{,}212 \div 4{,}853 = 4 \cdot 99$ times.

Under these circumstances the amount of work performed (see Table II) is

$90 \cdot 565 + 0 \cdot 9 \times 0 \cdot 820 = 91 \cdot 303$ ft.-tons;

from this a deduction must be made; we have seen above in the explanation of Table I that the G.D. of this charge is 0·971; if the products of explosion had been allowed to change from G.D. = 1·0 to G.D. = 0·971, while *the powder gas is pressing on the base of the projectile*, we see from Table II, columns 2 and 3, that 2·870 ft.-tons of work would have been performed: but as expansion has taken place to this extent *without the performance of work*, this amount must be *deducted* from the total, and hence the theoretical work is

$91 \cdot 303 - 2 \cdot 870 = 88 \cdot 433$ ft.-tons per lb. of charge:

multiplying this amount by the number (175) pounds of powder in the charge, and taking a factor of effect (judged from experience) of 0·726, the total energy realised is

$88 \cdot 433 \times 175 \times 0 \cdot 726 = 11{,}240$ ft.-tons.

Now $\frac{wV^2}{2g \times 2240}$ ft.-tons is equal to this same amount; substitute the value 380 lbs. for w and we have—

$$\frac{380V^2}{2 \times 32 \cdot 19 \times 2240} = 11{,}240,$$

whence V = 2065 f.s.

Table II.

Work capable of being done by Exploding Gunpowder.

(See Noble and Abel, "Researches on Explosives," Phil. Trans. Roy. Soc., 29th May, 1879.)

Number of volumes of expansion.	Corresponding density of products of combustion.	Work that powder is capable of performing. Per lb. burned in foot-tons.	Work that powder is capable of performing. Difference.	Number of volumes of expansion.	Corresponding density of products of combustion.	Work that powder is capable of performing. Per lb. burned in foot-tons.	Work that powder is capable of performing. Difference.	Number of volumes of expansion.	Corresponding density of products of combustion.	Work that powder is capable of performing. Per lb. burned in foot-tons.	Work that powder is capable of performing. Difference.
1·00	1·000	...	...	1·84	0·543	44·394	0·625	4·90	0·204	90·565	0·841
1·01	0·990	0·980	0·980	1·86	0·537	45·009	0·615	5·00	0·200	91·385	0·820
1·02	0·980	1·936	0·956	1·88	0·532	45·614	0·605	5·10	0·196	92·186	0·801
1·03	0·971	2·870	0·934	1·90	0·526	46·209	0·595	5·20	0·192	92·968	0·782
1·04	0·962	3·782	0·912	1·92	0·521	46·795	0·586	5·30	0·188	93·732	0·764
1·05	0·952	4·674	0·892	1·94	0·515	47·372	0·577	5·40	0·185	94·479	0·747
1·06	0·943	5·547	0·873	1·96	0·510	47·940	0·568	5·50	0·182	95·210	0·731
1·07	0·935	6·399	0·852	1·98	0·505	48·499	0·559	5·60	0·178	95·925	0·715
1·08	0·926	7·234	0·835	2·00	0·500	49·050	0·551	5·70	0·175	96·625	0·700
1·09	0·917	8·051	0·817	2·05	0·488	50·383	1·333	5·80	0·172	97·310	0·685
1·10	0·909	8·852	0·810	2·10	0·476	51·673	1·290	5·90	0·169	97·981	0·671
1·11	0·901	9·637	0·785	2·15	0·465	52·922	1·249	6·00	0·166	98·638	0·657
1·12	0·893	10·406	0·769	2·20	0·454	54·132	1·210	6·10	0·164	99·282	0·644
1·13	0·885	11·160	0·754	2·25	0·444	55·304	1·172	6·20	0·161	99·915	0·633
1·14	0·877	11·899	0·739	2·30	0·435	56·439	1·135	6·30	0·159	100·536	0·621
1·15	0·870	12·625	0·726	2·35	0·425	57·539	1·100	6·40	0·156	101·145	0·609
1·16	0·862	13·338	0·713	2·40	0·417	58·605	1·066	6·50	0·154	101·744	0·599
1·17	0·855	14·038	0·700	2·45	0·408	59·639	1·034	6·60	0·151	102·333	0·589
1·18	0·847	14·725	0·687	2·50	0·400	60·642	1·003	6·70	0·149	102·912	0·579
1·19	0·840	15·400	0·675	2·55	0·392	61·616	0·974	6·80	0·147	103·480	0·568
1·20	0·833	16·063	0·663	2·60	0·384	62·563	0·947	6·90	0·145	104·038	0·558
1·21	0·826	16·716	0·653	2·65	0·377	63·486	0·923	7·00	0·143	104·586	0·548
1·22	0·820	17·359	0·643	2·70	0·370	64·385	0·899	7·10	0·141	105·125	0·539
1·23	0·813	17·992	0·633	2·75	0·363	65·262	0·877	7·20	0·139	105·655	0·530
1·24	0·806	18·614	0·622	2·80	0·357	66·119	0·857	7·30	0·137	106·176	0·521
1·25	0·800	19·226	0·612	2·85	0·351	66·955	0·836	7·40	0·135	106·688	0·512
1·26	0·794	19·828	0·602	2·90	0·345	67·771	0·816	7·50	0·133	107·192	0·504
1·27	0·787	20·420	0·592	2·95	0·339	68·568	0·797	7·60	0·131	107·688	0·496
1·28	0·781	21·001	0·581	3·00	0·333	69·347	0·779	7·70	0·130	108·177	0·489
1·29	0·775	21·572	0·571	3·05	0·328	70·109	0·762	7·80	0·128	108·659	0·482
1·30	0·769	22·133	0·561	3·10	0·322	70·854	0·745	7·90	0·126	109·133	0·474
1·32	0·758	23·246	1·113	3·15	0·317	71·585	0·731	8·00	0·125	109·600	0·467
1·34	0·746	24·324	1·078	3·20	0·312	72·301	0·716	8·10	0·123	110·060	0·460
1·36	0·735	25·371	1·047	3·25	0·308	73·002	0·701	8·20	0·122	110·514	0·454
1·38	0·725	26·389	1·018	3·30	0·303	73·690	0·688	8·30	0·120	110·962	0·448
1·40	0·714	27·380	0·991	3·35	0·298	74·365	0·675	8·40	0·119	111·404	0·442
1·42	0·704	28·348	0·968	3·40	0·294	75·027	0·662	8·50	0·117	111·840	0·436
1·44	0·694	29·291	0·943	3·45	0·290	75·677	0·650	8·60	0·116	112·270	0·430
1·46	0·685	30·211	0·920	3·50	0·286	76·315	0·638	8·70	0·115	112·695	0·425
1·48	0·676	31·109	0·898	3·55	0·282	76·940	0·625	8·80	0·114	113·114	0·419
1·50	0·667	31·986	0·877	3·60	0·278	77·553	0·613	8·90	0·112	113·528	0·414
1·52	0·658	32·843	0·857	3·65	0·274	78·156	0·603	9·00	0·111	113·937	0·409
1·54	0·649	33·681	0·838	3·70	0·270	78·749	0·593	9·10	0·110	114·341	0·404
1·56	0·641	34·500	0·819	3·75	0·266	79·332	0·583	9·20	0·109	114·739	0·398
1·58	0·633	35·301	0·801	3·80	0·263	79·905	0·573	9·30	0·108	115·133	0·394
1·60	0·625	36·086	0·785	3·85	0·260	80·469	0·564	9·40	0·106	115·521	0·388
1·62	0·617	36·855	0·769	3·90	0·256	81·024	0·555	9·50	0·105	115·905	0·384
1·64	0·610	37·608	0·753	3·95	0·253	81·570	0·546	9·60	0·104	116·284	0·379
1·66	0·602	38·346	0·738	4·00	0·250	82·107	0·537	9·70	0·103	116·659	0·375
1·68	0·595	39·069	0·723	4·10	0·244	83·157	1·050	9·80	0·102	117·029	0·370
1·70	0·588	39·778	0·709	4·20	0·238	84·176	1·019	9·90	0·101	117·395	0·366
1·72	0·581	40·474	0·696	4·30	0·232	85·166	0·990	10·00	0·100	117·757	0·362
1·74	0·575	41·156	0·682	4·40	0·227	86·128	0·962	11·00	0·091	121·165	3·408
1·76	0·568	41·827	0·671	4·50	0·222	87·064	0·936	12·00	0·083	124·239	3·074
1·78	0·562	42·486	0·659	4·60	0·217	87·975	0·911	13·00	0·077	127·036	2·797
1·80	0·555	43·133	0·647	4·70	0·213	88·861	0·886	14·00	0·071	129·602	2·566
1·82	0·549	43·769	0·636	4·80	0·208	89·724	0·863	15·00	0·066	131·970	2·368

ation of these two tables *see* p. 23.

Table III.

Stability of Rotation of Projectiles.

(Calculated from Professor Greenhill's formula by Major Cundill, R.A., and extended by Mr. A. G. Hadcock, R.A., Inspector of Ordnance Machinery, *vide* Pro. R. A. I., vol. xi, No. 2, and vol. xiv, No. 3.)

Length of projectile in calibres.	Minimum twist at muzzle of gun requisite to give stability of rotation = 1 turn in n calibres.			
	Cast-iron common shell; cavity $= \frac{8}{27}$ths vol. of shell, (Density of cast iron 7·207).	Palliser shell; cavity $= \frac{1}{8}$th vol. of shell, (Density of chilled iron 8·000).	Solid steel bullet; (Density of steel 8·000).	Solid lead and tin bullets of similar composition to M.-H. bullets; (Density of alloy 10·9).
	n	n	n	n
2·0	63·87	71·08	72·21	84·29
2·1	59·84	66·59	67·66	78·98
2·2	56·31	62·67	63·67	74·32
2·3	53·19	59·19	60·14	70·20
2·4	50·41	56·10	57·00	66·53
2·5	47·91	53·32	54·17	63·24
2·6	45·65	50·81	51·62	60·26
2·7	43·61	48·53	49·30	57·55
2·8	41·74	46·45	47·19	55·09
2·9	40·02	44·54	45·25	52·72
3·0	38·45	42·79	43·47	50·74
3·1	36·99	41·16	41·82	48·82
3·2	35·64	39·66	40·30	47·04
3·3	34·39	38·27	38·84	45·38
3·4	33·22	36·97	37·56	43·84
3·5	32·13	35·75	36·33	42·40
3·6	31·11	34·62	35·17	41·05
3·7	30·15	33·55	34·09	39·79
3·8	29·25	32·55	33·07	38·61
3·9	28·40	31·61	32·11	37·48
4·0	27·60	30·72	31·21	36·43
4·1	26·85	29·88	30·36	35·43
4·2	26·13	29·08	29·55	34·49
4·3	25·45	28·33	28·78	33·59
4·4	24·81	27·61	28·05	32·74
4·5	24·20	26·93	27·36	31·94
4·6	23·65	26·32	26·74	31·21
4·7	23·06	25·66	26·08	30·44
4·8	22·53	25·08	25·48	29·74
4·9	22·03	24·51	24·91	29·07
5·0	21·56	23·98	24·36	28·44
5·1	21·08	23·46	23·84	27·83
5·2	20·64	22·97	23·34	27·24
5·3	20·22	22·50	22·86	26·68
5·4	19·81	22·05	22·40	26·14
5·5	19·42	21·61	21·96	25·63
5·6	19·04	21·19	21·53	25·13
5·7	18·68	20·79	21·12	24·66
5·8	18·33	20·40	20·73	24·20
5·9	18·00	20·03	20·35	23·75
6·0	17·67	19·67	19·98	23·33
7·0	14 99	16·68	16·95	19·78
8·0	13·02	14·48	14·72	17·18
9·0	11·50	12·80	13·00	15·18
10·0	10·31	11·47	11·65	13·60

Explanation—This table gives the minimum theoretical twist necessary to maintain the steadiness of rotation in flight of projectiles varying in length, density, and interior dimensions. *See* p. 135.

TABLE IV.

Values of K for Ogival-headed Projectiles of $1\frac{1}{2}$ diameters for the cubic law of resistance of the air.

(*From Supplement "Bashforth's Motion of Projectiles," 1881.*)

Velocity.	Value of K.	Velocity.	Value of K.	Velocity.	Value of K.	Velocity.	Value of K.	Velocity.	Value of K.
f.s.		f.s.		f.s.		f.s.		f.s.	
100	578·1	640	93·5	1180	109·6	1720	81·8	2260	66·5
110	525·5	650	91·9	1190	109·6	1730	81·2	2270	66·4
120	481·7	660	90·5	1200	109·6	1740	80·6	2280	66·2
130	444·7	670	89·1	1210	109·6	1750	80·0	2290	65·9
140	412·9	680	87·7	1220	109·6	1760	79·5	2300	65·5
150	385·4	690	86·3	1230	109·5	1770	78·9	2310	65·0
160	361·3	700	84·9	1240	109·5	1780	78·4	2320	64·4
170	340·1	710	83·7	1250	109·4	1790	77·8	2330	63·8
180	321·2	720	82·6	1260	109·3	1800	77·3	2340	63·2
190	304·3	730	81·6	1270	109·2	1810	76·8	2350	62·6
200	289·0	740	80·6	1280	109·0	1820	76·2	2360	62·0
210	275·3	750	79·6	1290	108·8	1830	75·7	2370	61·4
220	262·8	760	78·7	1300	108·6	1840	75·2	2380	60·8
230	251·3	770	78·0	1310	108·4	1850	74·7	2390	60·2
240	240·9	780	77·4	1320	108·1	1860	74·2	2400	59·6
250	231·2	790	76·8	1330	107·8	1870	73·6	2410	59·0
260	222·4	800	76·2	1340	107·5	1880	73·1	2420	58·4
270	214·1	810	75·6	1350	107·1	1890	72·6	2430	57·8
280	206·5	820	75·2	1360	106·7	1900	72·1	2440	57·2
290	199·3	830	75·1	1370	106 3	1910	71·6	2450	56·7
300	192·7	840	75·0	1380	105·8	1920	71·2	2460	56·2
310	186·5	850	75·0	1390	105·3	1930	70·8	2470	55·7
320	180·8	860	75·0	1400	104·7	1940	70·4	2480	55·2
330	175·5	870	75·0	1410	104·1	1950	70·0	2490	54·8
340	170·6	880	75·0	1420	103·5	1960	69·7	2500	54·4
350	166·0	890	75·0	1430	102·9	1970	69·4	2510	54·0
360	161·9	900	75·0	1440	102·3	1980	69·2	2520	53·7
370	158·0	910	75·0	1450	101·6	1990	69·0	2530	53·4
380	154·4	920	75·0	1460	100·9	2000	68·8	2540	53·1
390	151·1	930	75·0	1470	100·1	2010	68·6	2550	52·9
400	148·0	940	75·0	1480	99·4	2020	68·4	2560	52·7
410	145·2	950	75·0	1490	98·6	2030	68·3	2570	52·6
420	142·5	960	75·0	1500	97·9	2040	68·2	2580	52·5
430	139·8	970	75·0	1510	97·1	2050	68·1	2590	52·5
440	137·2	980	75·0	1520	96·2	2060	68·0	2600	52·4
450	134·6	990	75·0	1530	95·3	2070	67·9	2610	52·4
460	132·0	1000	75·0	1540	94·4	2080	67·9	2620	52·4
470	129·4	1010	75·1	1550	93·6	2090	67·8	2630	52·3
480	126·9	1020	75·3	1560	92·8	2100	67·8	2640	52·3
490	124·4	1030	76·7	1570	92·0	2110	67·7	2650	52·3
	121·9	1040	80·8	1580	91·2	2120	67·6	2660	52·2
510	119·6	1050	87·3	1590	90·4	2130	67·6	2670	52·2
520	117·3	1060	94·0	1600	89·7	2140	67·5	2680	52·2
530	115·0	1070	98·7	1610	89·0	2150	67·4	2690	52·1
540	112·8	1080	102·2	1620	88·3	2160	67·3	2700	52·1
550	110·7	1090	104·9	1630	87·6	2170	67·2	2710	52·1
560	108·7	1100	106·9	1640	86·9	2180	67·2	2720	52·0
570	106·7	1110	108·4	1650	86·2	2190	67·1	2730	52·0
580	104·6	1120	109·2	1660	85·5	2200	67·0	2740	52·0
590	102·5	1130	109·6	1670	84·8	2210	66·9	2750	52·0
600	100·5	1140	109·6	1680	84·2	2220	66·8	2760	52·0
610	98·6	1150	109·6	1690	83·6	2230	66·8	2770	52·0
620	96·8	1160	109·6	1700	83·0	2240	66·7	2780	52·0
630	95·1	1170	109·6	1710	82·4	2250	66·6	2800	52·0

Mr. Bashforth assumes the weight of a cubic foot of air to be 534·22 grains, which is the weight of a cubic foot of dry air at a temperature of 62° F. under a barometric pressure of 30 inches, and $g = 32·1908$. *See* pp. 139 and 255.

Table V.

Showing the Resistance of the Air in pounds (p) to a 1-inch Projectile with an Ogival head of $1\frac{1}{2}$ diameters radius; for velocities from 100 to 2800 f.s.

(Calculated by Mr. A. G. Hadcock, R.A., Inspector of Ordnance Machinery, from Mr. Bashforth's values of K, by the use of the formula $p = \frac{K}{g}\left(\frac{v}{1000}\right)^3$. *See* p. 141.)

v	p	v	p	v	p	v	p	v	p
f.s.	lbs.	f.s.	lbs.	f.s.	lbs.	f.s.	lbs.	f.s.	lbs.
100	0·0180	640	0·7615	1180	5·594	1720	12·900	2260	23·848
110	0·0217	650	0·7840	1190	5·738	1730	13·059	2270	24·132
120	0·0259	660	0·8081	1200	5·884	1740	13·191	2280	24·368
130	0·0303	670	0·8325	1210	6·032	1750	13·318	2290	24·583
140	0·0352	680	0·8565	1220	6·183	1760	13·466	2300	24·760
150	0·0404	690	0·8807	1230	6·331	1770	13·591	2310	24·887
160	0·0459	700	0·9048	1240	6·486	1780	13·733	2320	24·987
170	0·0519	710	0·9306	1250	5·637	1790	13·862	2330	25·071
180	0·0582	720	0·9577	1260	6·791	1800	14·002	2340	25·152
190	0·0648	730	0·9861	1270	6·948	1810	14·149	2350	25·242
200	0·0718	740	1·0146	1280	7·101	1820	14·269	2360	25·316
210	0·0792	750	1·0433	1290	7·256	1830	14·414	2370	25·386
220	0·0863	760	1·0733	1300	7·413	1840	14·552	2380	25·467
230	0·0950	770	1·1062	1310	7·569	1850	14·696	2390	25·529
240	0·1035	780	1·1408	1320	7·723	1860	14·832	2400	25·588
250	0·1122	790	1·1764	1330	7·879	1870	14·949	2410	25·658
260	0·1214	800	1·2119	1340	8·034	1880	15·090	2420	25·710
270	0·1310	810	1·248	1350	8·185	1890	15·224	2430	25·772
280	0·1409	820	1·288	1360	8·339	1900	15·364	2440	25·814
290	0·1511	830	1·334	1370	8·490	1910	15·496	2450	25·898
300	0·1616	840	1·381	1380	8 639	1920	15 656	2460	26·008
310	0·1727	850	1·431	1390	8·784	1930	15·809	2470	26·071
320	0·1841	860	1·482	1400	8·924	1940	15·968	2480	26·158
330	0·1959	870	1·534	1410	9·066	1950	16·127	2490	26·276
340	0·2083	880	1·588	1420	9·206	1960	16·302	2500	26·406
350	0·2211	890	1·643	1430	9·349	1970	16·484	2510	26·534
360	0·2346	900	1·699	1440	9·489	1980	16·689	2520	26·709
370	0·2485	910	1·756	1450	9·622	1990	16·888	2530	26·866
380	0·2631	920	1·814	1460	9·753	2000	17·096	2540	27·039
390	0·2784	930	1·874	1470	9·879	2010	17·305	2550	27·243
400	0·2943	940	1·935	1480	10·013	2020	17·515	2560	27·464
410	0·3110	950	1·998	1490	10·133	2030	17·752	2570	27·736
420	0·3280	960	2·061	1500	10·263	2040	17·990	2580	28·010
430	0·3453	970	2·127	1510	10·384	2050	18·229	2590	28·337
440	0·3630	980	2·193	1520	10·493	2060	18·463	2600	28·613
450	0·3810	990	2·261	1530	10·601	2070	18·706	2610	28·945
460	0·3992	1000	2·330	1540	10·712	2080	18·978	2620	29·279
470	0·4174	1010	2·404	1550	10·829	2090	19·227	2630	29·562
480	0·4360	1020	2·482	1560	10·945	2100	19·504	2640	29·899
490	0·4547	1030	2·604	1570	11·060	2110	19·755	2650	30·241
500	0·4734	1040	2·823	1580	11·175	2120	20·010	2660	30·527
510	0·4928	1050	3·139	1590	11·288	2130	20·294	2670	30·873
520	0·5124	1060	3·478	1600	11·416	2140	20·551	2680	31·221
530	0·5318	1070	3·756	1610	11·540	2150	20·811	2690	31·494
540	0·5517	1080	3·999	1620	11·662	2160	21·072	2700	31·846
550	0·5721	1090	4·221	1630	11·784	2170	21·336	2710	32·203
560	0·5931	1100	4·420	1640	11·909	2180	21·633	2720	32·500
570	0·6139	1110	4·605	1650	12·030	2190	21·889	2730	32·859
580	0·6339	1120	4·766	1660	12·150	2200	22·158	2740	33·222
590	0·6539	1130	4·913	1670	12·268	2210	22·429	2750	33·586
600	0·6743	1140	5·041	1680	12·404	2220	22·702	2760	33·955
610	0·6952	1150	5·179	1690	12·536	2230	23·010	2770	34·325
620	0·7166	1160	5·315	1700	12·666	2240	23·288	2780	34·697
630	0·7386	1170	5·454	1710	12·801	2250	23·566	2800	35·453

Explanation.—If the resistance of the air to any projectile (having the same shape of head, smoothness of surface, and steadiness in flight) is required at any velocity included in the table, it is only necessary to multiply the tabulated amount by the square of the diameter in inches.

Thus find the resistance of the air to a 7-inch projectile moving at a velocity of 1230 f.s.

From the table the resistance to a 1-inch projectile at that velocity is 6·331 lbs.; multiply this by 7^2 or 49, and we obtain 310·22 lbs. as the resistance of the air to the 7-inch projectile.

If the shape of head, &c., differs, the factor σ must be employed, *see* p. 140.

Table VI.

Ballistic Table.

Time (t in seconds) and velocity. $\frac{d^2}{w}t = T_V - T_v$.

For elongated projectiles, heads $1\frac{1}{2}$ diameters radius.
(From Supplement "Bashforth's Motion of Projectiles, 1881.")

v.	0	1	2	3	4	5	6	7	8	9	Diff.
f s.	secs.	secs.	secs.	secs.	secs.	secs.	secs.	secs.	secs.	secs.	+
10	75·399	77·111	78·790	80·437	82·052	83·636	85·190	86·715	88·212	89·682	1·584
11	91·125	92·542	93·934	95·301	96·644	97·964	99·261	*00·536	01·789*	03·021*	1·320
12	1 04·232	05·423	06·595	07·748	08·883	09·999	11·097	12·178	13·243	14·291	1·116
13	1 15·323	16·339	17·340	18·326	19·297	20·254	21·196	22·124	23·039	23·941	·957
14	24·830	25·706	26·570	27·422	28·262	29·091	29·908	30·714	31·509	32·294	·829
15	33·068	33·832	34·586	35·331	36·066	36·792	37·508	38·215	38·913	39·602	·726
16	1 40·283	40·955	41·618	42·273	42·920	43·559	44·190	44·813	45·429	46·038	·639
17	46·640	47·235	47·823	48·404	48·978	49·546	50·107	50·662	51·211	51·754	·568
18	52·291	52·822	53·347	53·867	54·381	54·890	55·393	55·890	56·382	56·869	·509
19	1 57·351	57·828	58·300	58·767	59·229	59·686	60·138	60·586	61·029	61·468	·457
20	61·902	62·332	62·758	63·180	63·598	64·012	64·422	64·828	65·230	65·628	·414
21	66·022	66·412	66·798	67·181	67·560	67·936	68·308	68·676	69·041	69·403	·376
22	1 69·762	70·118	70·470	70·819	71·165	71·508	71·848	72·185	72·519	72·850	·343
23	73·179	73·505	73·828	74·148	74·465	74·780	75·092	75·401	75·708	76·012	·315
24	76·314	76·613	76·909	77·203	77·494	77·783	78·070	78·354	78·636	78·916	·289
25	1 79·194	79·470	79·743	80·014	80·283	80·550	80·815	81·078	81·339	81·598	·267
26	81·855	82·110	82·363	82·614	82·863	83·110	83·355	83·598	83·839	84·079	·247
27	84·317	84·553	84·787	85·020	85·251	85·481	85·709	85·935	86·160	86·382	·230
28	1 86·604	86·824	87·042	87·259	87·474	87·688	87·900	88·111	88·320	88·528	·214
29	88·734	88·939	89·143	89·345	89·546	89·745	89·943	90·140	90·335	90·529	·199
30	90·721	90·912	91·102	91·291	91·478	91·664	91·849	92·033	92·216	92·397	·186
31	1 92·577	92·756	92·934	93·111	93·287	93·461	93·634	93·806	93·971	94·147	·174
32	94·316	94·484	94·651	94·817	94·982	95·146	95·309	95·471	95·632	95·792	·164
33	95·951	96·109	96·266	96·422	96·577	96·731	96·884	97·036	97·187	97·338	·154
34	1 97·488	97·637	97·785	97·932	98·078	98·223	98·367	98·510	98·652	98·794	·145
35	98·935	99·075	99·214	99·352	99·490	99·627	99·763	99·898	00·032*	00·166*	·137
36	2 00·299	00·431	00·562	00·692	00·822	00·951	01·079	01·206	01·333	01·459	·129
37	2 01·585	01·710	01·834	01·957	02·080	02·202	02·323	02·443	02·563	02·682	·122
38	02·801	02·919	03·036	03·152	03·268	03·383	03·497	03·610	03·723	03·835	·115
39	03·947	04·058	04·168	04·278	04·387	04·496	04·604	04·711	04·818	04·924	·109
40	20 5·0299	5·1349	5·2393	5·3432	5·4466	5·5494	5·6517	5·7534	5·8546	5·9553	·1028
41	6·0554	6·1550	6·2540	6·3525	6·4505	6·5480	6·6450	6·7414	6·8373	6·9327	·0975
42	7·0276	7·1220	7·2159	7·3093	7·4022	7·4947	7·5867	7·6782	7·7693	7·8599	·0925
43	20 7·9501	8·0398	8·1291	8·2179	8·3063	8·3942	8·4817	8·5687	8·6553	8·7415	·0879
44	8·8272	8·9125	8·9974	9·0819	9·1660	9·2497	9·3330	9·4159	9·4984	9·5805	·0837
45	9·6622	9·7435	9·8244	9·9050	9·9852	0·0651*	0·1446*	*0·2237	0·3025*	0·3809*	·0799
46	21 0·4590	0·5367	0·6140	0·6910	0·7677	0·8440	0·9200	0·9956	1·0709	1·1459	·0763
47	1·2205	1·2948	1·3687	1·4423	1·5156	1·5886	1·6613	1·7336	1·8056	1·8773	·0730
48	1·9487	2·0198	2·0906	2·1611	2·2313	2·3012	2·3708	2·4401	2·5091	2·5779	·0699
49	21 2·6464	2·7146	2·7825	2·8501	2·9174	2·9845	3·0513	3·1178	3·1841	3·2501	·0671
50	3·3159	3·3814	3·4466	3·5116	3·5763	3·6608	3·7050	3·7689	3·8326	3·8960	·0645
51	3·9592	4·0221	4·0848	4·1472	4·2094	4·2713	4·3330	4·3944	4·4556	4·5165	·0619

Explanation.—The *first* figures of the *velocity* are to be found under the heading v; for the *last* figure look from there horizontally until one of the other columns which indicates the last figure of the velocity is reached, when the corresponding tabulated *time* will be found. In taking out the time, the first figure, or generally the two first figures of it, are only given under the column headed 0, and they are not repeated in the other columns in order to save room.

Thus, find the time corresponding to 280 f.s., it is 186·604 seconds; for 282 f.s. it is 187·042 seconds; for 624 f.s. it is 219·8633 seconds; and for 629 f.s. it is 220·0761, the asterisk meaning that one must be added to the figure indicated under column headed 0. Proportional parts can be taken if desired; the difference for one foot in velocity being given on the same horizontal under column headed Diff.; thus, find the time corresponding to a velocity of 624·4 f.s., it is 219·8633 + 0·4 × 0·0428 = 219·8804 seconds.

In a reverse manner, if the time is given, the velocity corresponding to it can be found: thus, if the time is 216·0240 seconds, the corresponding velocity is 545 f.s. Proportional parts can be taken if desired: thus, find the velocity corresponding to a time of 217·7802 seconds; the nearest tabulated time which is *less* is 217·7456 seconds, corresponding to a velocity of 578 f.s.; the difference between these two times is found to be 0·0346 second, whilst the tabulated difference for one foot in velocity is given as 0·0499; the amount to be added is then $\frac{0·0346}{0·0499}$ f.s. = 0·7 f.s., and the velocity required is 578·7 f.s.

Proportional parts need seldom however be taken unless the velocities are low; it will generally be sufficiently accurate to work to the nearest tabulated quantity, whether time or velocity. For further description of the use of this Table, *see* p. 143.

Table VI.—*continued.*

Ballistic Table.

Time (t in seconds) and velocity. $\frac{d^2}{w}t = T_V - T_v$.

v.	0	1	2	3	4	5	6	7	8	9	Diff.
f.s.	secs.	secs.	secs.	secs.	secs.	secs.	secs.	secs.	secs.	secs.	+
52	21 4·5772	4·6377	4·6979	4·7579	4·8177	4·8773	4·9367	4·9958	5·0547	5·1134	·0596
53	5·1719	5·2302	5·2882	5·3460	5·4036	5·4610	5·5182	5·5752	5·6320	5·6886	·0574
54	5·7450	5·8012	5·8572	5·9130	5·9686	6·0240	6·0792	6·1342	6·1890	6·2436	·0554
55	21 6·2980	6·3522	6·4062	6·4600	6·5136	6·5670	6·6202	6·6732	6·7260	6·7786	·0534
56	6·8311	6·8834	6·9355	6·9874	7·0391	7·0907	7·1421	7·1933	7·2444	7·2953	·0516
57	7·3460	7·3965	7·4469	7·4971	7·5471	7·5970	7·6467	7·6962	7·7456	7·7948	·0499
58	21 7·8438	7·8928	7·9417	7·9904	8·0389	8·0873	8·1356	8·1837	8·2316	8·2793	·0483
59	8·3271	8·3746	8·4220	8·4692	8·5163	8·5632	8·6100	8·6566	8·7031	8·7494	·0468
60	8·7957	8·8417	8·8877	8·9334	8·9791	9·0246	9·0700	9·1152	9·1603	9·2052	·0454
61	21 9·2501	9·2947	9·3393	9·3837	9·4280	9·4721	9·5161	9·5600	9·6037	9·6473	·0441
62	9·6908	9·7341	9·7773	9·8204	9·8633	9·9062	9·9489	9·9914	*0·0338	0·0761*	·0428
63	22 0·1183	0·1604	0·2023	0·2441	0·2858	0·3273	0·3687	0·4100	0·4512	0·4922	·0415
64	22 0·5332	0·5740	0·6147	0·6552	0·6957	0·7360	0·7762	0·8163	0·8563	0·8962	·0403
65	0·9359	0·9755	1·0151	1·0544	1·0937	1·1328	1·1718	1·2107	1·2495	1·2881	·0391
66	1·3267	1·3651	1·4034	1·4416	1·4797	1·5177	1·5555	1·5933	1·6309	1·6684	·0379
67	22 1·7059	1·7432	1·7804	1·8175	1·8545	1·8914	1·9281	1·9648	2·0014	2·0378	·0368
68	2·0742	2·1105	2·1466	2·1827	2·2186	2·2545	2·2902	2·3259	2·3614	2·3969	·0358
69	2·4322	2·4675	2·5027	2·5377	2·5727	2·6076	2·6424	2·6771	2·7117	2·7462	·0348
70	22 2·7806	2·8150	2·8492	2·8833	2·9174	2·9513	2·9852	3·0189	3·0526	3·0862	·0339
71	3·1196	3·1530	3·1863	3·2195	3·2526	3·2856	3·3185	3·3513	3·3840	3·4167	·0330
72	3·4492	3·4816	3·5140	3·5462	3·5784	3·6105	3·6424	3·6743	3·7061	3·7378	·0320
73	22 3·7694	3·8009	3·8323	3·8636	3·8949	3·9260	3·9571	3·9881	4·0189	4·0497	·0311
74	4·0804	4·1110	4·1416	4·1720	4·2024	4·2326	4·2628	4·2929	4·3230	4·3529	·0302
75	4·3828	4·4125	4·4422	4·4719	4·5014	4·5308	4·5602	4·5895	4·6187	4·6478	·0294
76	22 4·6769	4·7058	4·7347	4·7635	4·7922	4·8208	4·8493	4·8777	4·9060	4·9343	·0286
77	4·9624	4·9905	5·0185	5·0464	5·0742	5·1020	5·1296	5·1572	5·1847	5·2121	·0277
78	5·2394	5·2666	5·2937	5·3208	5·3478	5·3747	5·4015	5·4282	5·4549	5·4814	·0268
79	22 5·5079	5·5343	5·5606	5·5869	5·6130	5·6391	5·6652	5·6911	5·7170	5·7428	·0261
80	5·7685	5·7941	5·8197	5·8452	5·8706	5·8959	5·9212	5·9463	5·9714	5·9965	·0253
81	6·0214	6·0463	6·0711	6·0959	6·1205	6·1451	6·1696	6·1941	6·2184	6·2427	·0245
82	22 6·2669	6·2910	6·3151	6·3390	6·3629	6·3867	6·4104	6·4340	6·4576	6·4810	·0237
83	6·5044	6·5277	6·5509	6·5740	6·5971	6·6201	6·6430	6·6658	6·6885	6·7111	·0229
84	6·7337	6·7562	6·7786	6·8009	6·8232	6·8454	6·8675	6·8895	6·9114	6·9333	·0221
85	22 6·9551	6·9768	6·9984	7·0200	7·0415	7·0629	7·0842	7·1055	7·1267	7·1478	·0214
86	7·1688	7·1898	7·2107	7·2315	7·2522	7·2729	7·2935	7·3140	7·3345	7·3549	·0206
87	7·3752	7·3954	7·4156	7·4357	7·4558	7·4757	7·4956	7·5155	7·5353	7·5550	·0199
88	22 7·5746	7·5942	7·6137	7·6332	7·6526	7·6719	7·6912	7·7104	7·7295	7·7486	·0193
89	7·7677	7·7866	7·8055	7·8244	7·8431	7·8618	7·8805	7·8991	7·9176	7·9360	·0187
90	7·9544	7·9727	7·9909	8·0091	8·0272	8·0452	8·0632	8·0812	8·0990	8·1168	·0180
91	22 8·1346	8·1523	8·1699	8·1875	8·2050	8·2225	8·2399	8·2573	8·2746	8·2918	·0174
92	8·3090	8·3261	8·3432	8·3602	8·3772	8·3941	8·4109	8·4277	8·4445	8·4611	·0169
93	8·4778	8·4943	8·5109	8·5273	8·5437	8·5601	8·5764	8·5927	8·6089	8·6250	·0163
94	22 8·6411	8·6572	8·6732	8·6892	8·7051	8·7209	8·7367	8·7525	8·7682	8·7838	·0158
95	8·7994	8·8150	8·8305	8·8459	8·8613	8·8767	8·8920	8·9073	8·9225	8·9376	·0153
96	8·9528	8·9678	8·9828	8·9978	9·0128	9·0276	9·0425	9·0573	9·0720	9·0867	·0149
97	22 9·1014	9·1160	9·1306	9·1451	9·1595	9·1740	9·1884	9·2027	9·2170	9·2312	·0144
98	9·2454	9·2596	9·2737	9·2878	9·3018	9·3158	9·3298	9·3437	9·3575	9·3713	·0140
99	9·3851	9·3989	9·4126	9·4262	9·4398	9·4534	9·4670	9·4805	9·4939	9·5073	·0136
100	22 9·5207	9·5340	9·5473	9·5606	9·5738	9·5869	9·6001	9·6132	9·6262	9·6392	·0132
101	9·6522	9·6651	9·6780	9·6908	9·7036	9·7164	9·7291	9·7418	9·7544	9·7670	·0127
102	9·7796	9·7921	9·8046	9·8170	9·8294	9·8417	9·8540	9·8662	9·8783	9·8904	·0123
103	22 9·9024	9·9144	9·9262	9·9380	9·9496	9·9612	9·9727	9·9841	9·9954	0·0066*	·0115
104	23 0·0177	0·0287	0·0396	0·0504	0·0610	0·0716	0·0820	0·0923	0·1025	0·1126	·0105
105	0·1226	0·1325	0·1423	0·1520	0·1615	0·1710	0·1804	0·1897	0·1988	0·2079	·0094
106	23 0·2170	0·2259	0·2347	0·2435	0·2522	0·2609	0·2694	0·2780	0·2864	0·2948	·0086
107	0·3031	0·3114	0·3196	0·3278	0·3359	0·3439	0·3520	0·3599	0·3678	0·3757	·0080
108	0·3835	0·3913	0·3990	0·4067	0·4143	0·4219	0·4295	0·4370	0·4445	0·4519	·0076
109	23 0·4593	0·4667	0·4740	0·4813	0·4885	0·4958	0·5030	0·5101	0·5172	0·5243	·0072
110	0·5314	0·5384	0·5454	0·5524	0·5593	0·5662	0·5731	0·5800	0·5868	0·5936	·0069
111	0·6004	0·6071	0·6139	0·6206	0·6272	0·6339	0·6405	0·6471	0·6537	0·6603	·0066

TABLE VI.—*continued.*

Ballistic Table.

Time (t in seconds) and velocity. $\frac{d^2}{w}t = T_V - T_v$.

v.	0	1	2	3	4	5	6	7	8	9	Diff.
f.s.	secs.	secs.	secs.	secs.	secs.	secs.	secs.	secs.	secs.	secs.	+
112	23 0·6668	0·6733	0·6798	0·6863	0·6928	0·6992	0·7056	0·7120	0·7184	0·7248	·0064
113	0·7311	0·7374	0·7437	0·7500	0·7563	0·7625	0·7688	0·7750	0·7812	0·7874	·0062
114	0·7936	0·7997	0·8059	0·8120	0·8181	0·8242	0·8303	0·8364	0·8424	0·8484	·0061
115	23 0·8545	0·8605	0·8665	0·8726	0·8787	0·8847	0·8906	0·8965	0·9024	0·9083	·0059
116	0·9142	0·9200	0·9259	0·9317	0·9375	0·9433	0·9490	0·9548	0·9605	0·9663	·0058
117	0·9720	0·9777	0·9833	0·9890	0·9947	1·0003	1·0059	1·0115	1·0171	1·0227	·0056
118	23 1·0283	1·0338	1·0394	1·0449	1·0504	1·0559	1·0614	1·0669	1·0723	1·0778	·0055
119	1·0832	1·0886	1·0940	1·0994	1·1048	1·1101	1·1154	1·1208	1·1261	1·1314	·0054
120	1·1367	1·1420	1·1473	1·1525	1·1578	1·1630	1·1682	1·1734	1·1786	1·1838	·0052
121	23 1·1889	1·1941	1·1992	1·2043	1·2095	1·2146	1·2196	1·2247	1·2298	1·2348	·0051
122	1·2399	1·2449	1·2499	1·2549	1·2599	1·2649	1·2698	1·2748	1·2797	1·2847	·0050
123	1·2896	1·2945	1·2994	1·3043	1·3091	1·3140	1·3188	1·3237	1·3285	1·3333	·0049
124	23 1·3381	1·3429	1·3477	1·3524	1·3572	1·3619	1·3667	1·3714	1·3761	1·3808	·0047
125	1·3855	1·3902	1·3948	1·3995	1·4041	1·4088	1·4134	1·4180	1·4226	1·4272	·0046
126	1·4318	1·4364	1·4410	1·4455	1·4501	1·4546	1·4591	1·4636	1·4681	1·4726	·0045
127	23 1·4771	1·4816	1·4860	1·4905	1·4949	1·4993	1·5038	1·5082	1·5126	1·5170	·0044
128	1·5214	1·5257	1·5301	1·5345	1·5388	1·5431	1·5475	1·5518	1·5561	1·5604	·0043
129	1·5647	1·5690	1·5732	1·5775	1·5818	1·5860	1·5902	1·5945	1·5987	1·6029	·0042
130	23 1·6071	1·6113	1·6155	1·6196	1·6238	1·6280	1·6321	1·6362	1·6404	1·6445	·0042
131	1·6486	1·6527	1·6568	1·6609	1·6650	1·6690	1·6731	1·6772	1·6812	1·6852	·0041
132	1·6893	1·6933	1·6973	1·7013	1·7053	1·7093	1·7133	1·7173	1·7212	1·7252	·0040
133	23 1·7291	1·7331	1·7370	1·7410	1·7449	1·7488	1·7527	1·7566	1·7605	1·7644	·0039
134	1·7682	1·7721	1·7760	1·7798	1·7837	1·7875	1·7913	1·7952	1·7990	1·8028	·0038
135	1·8066	1·8104	1·8142	1·8179	1·8217	1·8255	1·8292	1·8330	1·8367	1·8405	·0038
136	23 1·8442	1·8479	1·8517	1·8554	1·8591	1·8628	1·8665	1·8702	1·8738	1·8775	·0037
137	1·8812	1·8848	1·8885	1·8921	1·8958	1·8994	1·9030	1·9067	1·9103	1·9139	·0036
138	1·9175	1·9211	1·9247	1·9282	1·9318	1·9354	1·9390	1·9425	1·9461	1·9496	·0036
139	23 1·9532	1·9567	1·9602	1·9638	1·9673	1·9708	1·9743	1·9778	1·9813	1·9848	·0035
140	1·9883	1·9918	1·9952	1·9987	2·0022	2·0056	2·0091	2·0125	2·0160	2·0194	·0035
141	2·0228	2·0263	2·0297	2·0331	2·0365	2·0399	2·0433	2·0467	2·0501	2·0535	·0034
142	23 2·0569	2·0602	2·0636	2·0670	2·0703	2·0737	2·0770	2·0804	2·0837	2·0870	·0034
143	2·0904	2·0937	2·0970	2·1003	2·1036	2·1069	2·1102	2·1135	2·1168	2·1201	·0033
144	2·1234	2·1267	2·1299	2·1332	2·1364	2·1397	2·1430	2·1462	2·1494	2·1527	·0033
145	23 2·1559	2·1591	2·1624	2·1656	2·1688	2·1720	2·1752	2·1784	2·1816	2·1848	·0032
146	2·1880	2·1912	2·1944	2·1975	2·2007	2·2039	2·2071	2·2102	2·2134	2·2165	·0032
147	2·2197	2·2228	2·2260	2·2291	2·2322	2·2354	2·2385	2·2416	2·2447	2·2478	·0031
148	23 2·2509	2·2540	2·2571	2·2602	2·2633	2·2664	2·2695	2·2726	2·2757	2·2787	·0031
149	2·2818	2·2849	2·2879	2·2910	2·2940	2·2971	2·3001	2·3032	2·3062	2·3093	·0030
150	2·3123	2·3153	2·3183	2·3214	2·3244	2·3274	2·3304	2·3334	2·3364	2·3394	·0030
151	23 2·3424	2·3454	2·3484	2·3514	2·3543	2·3573	2·3603	2·3633	2·3662	2·3692	·0030
152	2·3722	2·3751	2·3781	2·3810	2·3840	2·3869	2·3899	2·3928	2·3958	2·3987	·0029
153	2·4016	2·4046	2·4075	2·4104	2·4133	2·4162	2·4192	2·4221	2·4250	2·4279	·0029
154	23 2·4308	2·4337	2·4366	2·4395	2·4424	2·4453	2·4481	2·4510	2·4539	2·4568	·0029
155	2·4597	2·4625	2·4654	2·4683	2·4711	2·4740	2·4768	2·4797	2·4825	2·4854	·0029
156	2·4882	2·4911	2·4939	2·4967	2·4996	2·5024	2·5052	2·5080	2·5108	2·5137	·0028
157	23 2·5165	2·5193	2·5221	2·5249	2·5277	2·5305	2·5333	2·5361	2·5389	2·5416	·0028
158	2·5444	2·5472	2·5500	2·5528	2·5555	2·5583	2·5611	2·5638	2·5666	2·5693	·0028
159	2·5721	2·5748	2·5776	2·5803	2·5831	2·5858	2·5885	2·5913	2·5940	2·5967	·0027
160	23 2·5994	2·6022	2·6049	2·6076	2·6103	2·6130	2·6157	2·6184	2·6211	2·6238	·0027
161	2·6265	2·6292	2·6319	2·6346	2·6373	2·6400	2·6426	2·6453	2·6480	2·6506	·0027
162	2·6533	2·6560	2·6586	2·6613	2·6640	2·6666	2·6693	2·6719	2·6745	2·6772	·0026
163	23 2·6798	2·6825	2·6851	2·6877	2·6903	2·6930	2·6956	2·6982	2·7008	2·7034	·0026
164	2·7061	2·7087	2·7113	2·7139	2·7165	2·7191	2·7217	2·7243	2·7268	2·7294	·0026
165	2·7320	2·7346	2·7372	2·7398	2·7423	2·7449	2·7475	2·7500	2·7526	2·7552	·0026
166	23 2·7577	2·7603	2·7628	2·7654	2·7679	2·7705	2·7730	2·7756	2·7781	2·7806	·0025
167	2·7832	2·7857	2·7882	2·7908	2·7933	2·7958	2·7983	2·8008	2·8034	2·8059	·0025
168	2·8084	2·8109	2·8134	2·8159	2·8184	2·8209	2·8234	2·8258	2·8283	2·8308	·0025
169	23 2·8333	2·8358	2·8383	2·8407	2·8432	2·8457	2·8481	2·8506	2·8531	2·8555	·0025
170	2·8580	2·8604	2·8629	2·8653	2·8678	2·8702	2·8726	2·8751	2·8775	2·8799	·0024
171	2·8824	2·8848	2·8872	2·8896	2·8921	2·8945	2·8969	2·8993	2·9017	2·9041	·0024

TABLE VI.—*continued.*

Ballistic Table.

Time (t in seconds) and velocity. $\frac{d^2}{w}t = T_V - T_v.$

$v.$	0	1	2	3	4	5	6	7	8	9	Diff.
f. s.	secs.	secs.	secs.	secs.	secs.	secs.	secs.	secs.	secs.	secs.	+
172	23 2·9065	2·9089	2·9113	2·9137	2·9161	2·9185	2·9209	2·9233	2·9257	2·9281	·0024
173	2·9304	2·9328	2·9352	2·9376	2·9399	2·9423	2·9447	2·9470	2·9494	2·9518	·0024
174	2·9541	2·9565	2·9588	2·9612	2·9635	2·9659	2·9682	2·9705	2·9729	2·9752	·0023
175	23 2·9776	2·9799	2·9822	2·9845	2·9869	2·9892	2·9915	2·9938	2·9961	2·9985	·0023
176	3·0008	3·0031	3·0054	3·0077	3·0100	3·0123	3·0146	3·0169	3·0192	3·0215	·0023
177	3·0237	3·0260	3·0283	3·0306	3·0329	3·0351	3·0374	3·0397	3·0420	3·0442	·0023
178	23 3·0465	3·0488	3·0510	3·0533	3·0555	3·0578	3·0600	3·0623	3·0645	3·0668	·0023
179	3·0690	3·0713	3·0735	3·0757	3·0780	3·0802	3·0824	3·0847	3·0869	3·0891	·0022
180	3·0913	3·0935	3·0958	3·0980	3·1002	3·1024	3·1045	3·1068	3·1090	3·1112	·0022
181	23 3·1134	3·1156	3·1178	3·1200	3·1222	3·1244	3·1266	3·1287	3·1309	3·1331	·0022
182	3·1353	3·1375	3·1396	3·1418	3·1440	3·1461	3·1483	3·1505	3·1526	3·1548	·0022
183	3·1569	3·1591	3·1613	3·1634	3·1656	3·1677	3·1698	3·1720	3·1741	3·1763	·0021
184	23 3·1784	3·1805	3·1827	3·1848	3·1869	3·1891	3·1912	3·1933	3·1954	3·1975	·0021
185	3·1997	3·2018	3·2039	3·2060	3·2081	3·2102	3·2123	3·2144	3·2165	3·2186	·0021
186	3·2207	3·2228	3·2249	3·2270	3·2291	3·2312	3·2333	3·2353	3·2374	3·2395	·0021
187	23 3·2416	3·2437	3·2457	3·2478	3·2499	3·2520	3·2540	3·2561	3·2582	3·2602	·0021
188	3·2623	3·2643	3·2664	3·2685	3·2705	3·2726	3·2746	3·2767	3·2787	3·2808	·0021
189	3·2828	3·2848	3·2869	3·2889	3·2909	3·2930	3·2950	3·2970	3·2991	3·3011	·0020
190	23 3·3031	3·3051	3·3072	3·3092	3·3112	3·3132	3·3152	3·3172	3·3192	3·3212	·0020
191	3·3233	3·3253	3·3273	3·3293	3·3313	3·3333	3·3353	3·3372	3·3392	3·3412	·0020
192	3·3432	3·3452	3·3472	3·3492	3·3511	3·3531	3·3551	3·3571	3·3590	3·3610	·0020
193	23 3·3630	3·3649	3·3669	3·3689	3·3708	3·3728	3·3747	3·3767	3·3786	3·3806	·0020
194	3·3825	3·3845	3·3864	3·3884	3·3903	3·3922	3·3942	3·3961	3·3980	3·4000	·0019
195	3·4019	3·4038	3·4057	3·4077	3·4096	3·4115	3·4134	3·4153	3·4172	3·4192	·0019
196	23 3·4211	3·4230	3·4249	3·4268	3·4287	3·4306	3·4325	3·4344	3·4362	3·4381	·0019
197	3·4400	3·4419	3·4438	3·4457	3·4476	3·4494	3·4513	3·4532	3·4550	3·4569	·0019
198	3·4588	3·4606	3·4625	3·4644	3·4662	3·4681	3·4699	3·4718	3·4736	3·4755	·0019
199	23 3·4773	3·4791	3·4810	3·4828	3·4846	3·4865	3·4883	3·4901	3·4920	3·4938	·0018
200	3·4956	3·4974	3·4992	3·5010	3·5028	3·5047	3·5065	3·5083	3·5101	3·5119	·0018
201	3·5137	3·5155	3·5172	3·5190	3·5208	3·5226	3·5244	3·5262	3·5280	3·5297	·0018
202	23 3·5315	3·5333	3·5351	3·5368	3·5386	3·5404	3·5421	3·5439	3·5456	3·5474	·0018
203	3·5492	3·5509	3·5527	3·5544	3·5561	3·5579	3·5596	3·5614	3·5631	3·5648	·0017
204	3·5666	3·5683	3·5700	3·5717	3·5735	3·5752	3·5769	3·5786	3·5803	3·5820	·0017
205	23 3·5837	3·5854	3·5871	3·5888	3·5905	3·5922	3·5939	3·5956	3·5973	3·5990	·0017
206	3·6007	3·6024	3·6040	3·6057	3·6074	3·6091	3·6107	3·6124	3·6141	3·6157	·0017
207	3·6174	3·6191	3·6207	3·6224	3·6240	3·6257	3·6273	3·6290	3·6306	3·6323	·0016
208	23 3·6339	3·6355	3·6372	3·6388	3·6404	3·6420	3·6437	3·6453	3·6469	3·6485	·0016
209	3·6502	3·6518	3·6534	3·6550	3·6566	3·6582	3·6598	3·6614	3·6630	3·6646	·0016
210	3·6662	3·6678	3·6694	3·6710	3·6726	3·6741	3·6757	3·6773	3·6789	3·6805	·0016
211	23 3·6820	3·6836	3·6852	3·6867	3·6883	3·6899	3·6914	3·6930	3·6946	3·6961	·0016
212	3·6977	3·6992	3·7008	3·7023	3·7039	3·7054	3·7070	3·7085	3·7100	3·7116	·0015
213	3·7131	3·7146	3·7162	3·7177	3·7192	3·7207	3·7223	3·7238	3·7253	3·7268	·0015
214	23 3·7283	3·7298	3·7313	3·7329	3·7344	3·7359	3·7374	3·7389	3·7404	3·7419	·0015
215	3·7434	3·7448	3·7463	3·7478	3·7493	3·7508	3·7523	3·7538	3·7552	3·7567	·0015
216	3·7582	3·7597	3·7612	3·7626	3·7641	3·7656	3·7670	3·7685	3·7700	3·7714	·0015
217	23 3·7729	3·7743	3·7758	3·7772	3.7787	3·7801	3·7816	3·7830	3·7845	3·7859	·0014
218	3·7874	3·7888	3·7902	3·7917	3·7931	3·7945	3·7960	3·7974	3·7988	3·8002	·0014
219	3·8016	3·8031	3·8045	3·8059	3·8073	3·8087	3·8101	3·8115	3·8129	3·8144	·0014
220	23 3·8158	3·8172	3·8186	3·8200	3·8214	3·8227	3·8241	3·8255	3·8269	3·8283	·0014
221	3·8297	3·8311	3·8325	3·8338	3·8352	3·8366	3·8380	3·8394	3·8407	3·8421	·0014
222	3·8435	3·8448	3·8462	3·8476	3·8489	3·8503	3·8517	3·8530	3·8544	3·8557	·0014
223	23 3·8571	3·8584	3·8598	3·8611	3·8625	3·8638	3·8651	3·8665	3·8678	3·8692	·0013
224	3·8705	3·8718	3·8732	3·8745	3·8758	3·8772	3·8785	3·8798	3·8811	3·8824	·0013
225	3·8838	3·8851	3·8864	3·8877	3·8890	3·8903	3·8916	3·8930	3·8943	3·8956	·0013
226	23 3·8969	3·8982	3·8995	3·9008	3·9021	3·9034	3·9047	3·9059	3·9072	3·9085	·0013
227	3·9098	3·9111	3·9124	3·9137	3·9150	3·9162	3·9175	3·9188	3·9201	3·9214	·0013
228	3·9226	3·9239	3·9252	3·9264	3·9277	3·9290	3·9303	3·9315	3·9328	3·9341	·0013
229	23 3·9353	3·9366	3·9378	3·9391	3·9404	3·9416	3·9429	3·9441	3·9454	3·9467	·0013
230	3·9479	3·9492	3·9504	3·9517	3·9529	3·9542	3·9554	3·9567	3·9579	3.9592	·0013
231	3·0604	3·9617	3·9629	3·9642	3·9654	3·9667	3·9679	3·9692	3·9704	3·9716	·0012

TABLE VI.—*continued*

Ballistic Table.

Time (t in seconds) and velocity. $\dfrac{d^2}{w} t = T_V - T_v$.

v.	0	1	2	3	4	5	6	7	8	9	Diff.
f.s.	secs.	secs.	secs.	secs.	secs.	secs.	secs.	secs.	secs.	secs.	+
232	23 3·9729	3·9741	3·9754	3·9766	3·9779	3·9791	3·9803	3·9816	3·9828	3·9841	·0012
233	3·9853	3·9866	3·9878	3·9890	3·9903	3·9915	3·9927	3·9940	3·9952	3·9965	·0012
234	3·9977	3·9989	4·0002	4·0014	4·0026	4·0039	4·0051	4·0063	4·0076	4·0088	·0012
235	23 4·0100	4·0113	4·0125	4·0137	4·0150	4·0162	4·0174	4·0186	4·0199	4·0211	·0012
236	4·0223	4·0236	4·0248	4·0260	4·0272	4·0284	4·0297	4·0309	4·0321	4·0334	·0012
237	4·0346	4·0358	4·0370	4·0383	4·0395	4·0407	4·0419	4·0431	4·0444	4·0456	·0012
238	23 4·0468	4·0480	4·0492	4·0505	4·0517	4·0529	4·0541	4·0553	4·0566	4·0578	·0012
239	4·0590	4·0602	4·0614	4·0626	4·0639	4·0651	4·0663	4·0675	4·0687	4·0699	·0012
240	4·0711	4·0724	4·0736	4·0748	4·0760	4·0772	4·0784	4·0796	4·0809	4·0821	·0012
241	23 4·0833	4·0845	4.0857	4·0869	4·0881	4·0893	4·0905	4·0917	4·0930	4·0942	·0012
242	4·0954	4·0966	4·0978	4·0990	4·1002	4·1014	4·1026	4·1038	4·1050	4·1062	·0012
243	4·1074	4·1087	4·1099	4·1111	4·1123	4·1135	4·1147	4·1159	4·1171	4·1183	·0012
244	23 4·1195	4·1207	4·1219	4·1231	4·1243	4·1255	4·1267	4·1279	4·1291	4·1303	·0012
245	4·1315	4·1327	4·1339	4·1351	4·1363	4·1375	4·1387	4·1399	4·1411	4·1423	·0012
246	4·1435	4·1447	4·1459	4·1471	4·1483	4·1495	4·1506	4·1518	4·1530	4·1542	·0012
247	23 4·1554	4·1566	4·1578	4·1590	4·1602	4·1614	4·1626	4·1638	4·1649	4·1661	·0012
248	4·1673	4·1685	4·1697	4·1709	4·1721	4·1733	4·1744	4·1756	4·1768	4·1780	·0012
249	4·1792	4·1804	4·1815	4·1827	4·1839	4·1851	4·1863	4·1874	4·1886	4·1898	·0012
250	23 4·1910	4·1922	4·1933	4·1945	4·1957	4·1969	4·1980	4·1992	4·2004	4·2015	·0012
251	4·2027	4·2039	4·2051	4·2062	4·2074	4·2086	4·2097	4·2109	4·2121	4·2132	·0012
252	4·2144	4·2156	4·2167	4·2179	4·2190	4·2202	4·2214	4·2225	4·2237	4·2248	·0012
253	23 4·2260	4·2272	4·2283	4·2295	4·2306	4·2318	4·2329	4·2341	4·2352	4·2364	·0012
254	4·2375	4·2387	4·2398	4·2410	4·2421	4·2433	4·2444	4·2455	4·2467	4·2478	·0011
255	4·2490	4·2501	4·2513	4·2524	4·2535	4·2547	4·2558	4·2569	4·2581	4·2592	·0011
256	23 4·2603	4·2615	4·2626	4·2637	4·2648	4·2660	4·2671	4·2682	4·2693	4·2705	·0011
257	4·2716	4·2727	4·2738	4·2749	4·2760	4·2772	4·2783	4·2794	4·2805	4·2816	·0011
258	4·2827	4·2838	4·2849	4·2860	4·2871	4·2882	4·2893	4·2904	4·2915	4·2926	·0011
259	23 4·2937	4·2948	4·2959	4·2970	4·2981	4·2992	4·3003	4·3014	4·3025	4·3036	·0011
260	4·3046	4·3057	4·3068	4·3079	4·3090	4·3101	4·3111	4·3122	4·3133	4·3144	·0011
261	4·3154	4·3165	4·3176	4·3187	4·3197	4·3208	4·3219	4·3229	4·3240	4·3250	·0011
262	23 4·3261	4·3272	4·3282	4·3293	4·3303	4·3314	4·3325	4·3335	4·3346	4·3356	·0011
263	4·3367	4·3377	4·3388	4·3398	4·3409	4·3419	4·3429	4·3440	4·3450	4·3461	·0010
264	4·3471	4·3482	4·3492	4·3502	4·3513	4·3523	4·3533	4·3544	4·3554	4·3564	·0010
265	23 4·3574	4·3585	4·3595	4·3605	4·3615	4·3626	4·3636	4·3646	4·3656	4·3667	·0010
266	4·3677	4·3687	4·3697	4·3707	4·3717	4·3728	4·3738	4·3748	4·3758	4·3768	·0010
267	4·3778	4·3788	4·3798	4·3808	4·3818	4·3828	4·3838	4·3848	4·3858	4·3868	·0010
268	23 4·3878	4·3888	4·3898	4·3908	4·3918	4·3928	4·3938	4·3948	4·3958	4·3968	·0010
269	4·3977	4·3987	4·3997	4·4007	4·4017	4·4027	4·4036	4·4046	4·4056	4·4066	·0010
270	4·4075	4·4085	4·4095	4·4105	4·4114	4·4124	4·4134	4·4143	4·4153	4·4163	·0010
271	23 4·4172	4·4182	4·4192	4·4201	4·4211	4·4220	4·4230	4·4240	4·4249	4·4259	·0010
272	4·4268	4·4278	4·4287	4·4297	4·4307	4·4316	4·4326	4·4335	4·4344	4·4354	·0010
273	4·4363	4·4373	4·4382	4·4392	4·4401	4·4411	4·4420	4·4429	4·4439	4·4448	·0009
274	23 4·4457	4·4467	4·4476	4·4485	4·4495	4·4504	4·4513	4·4523	4·4532	4·4541	·0009
275	4·4551	4·4560	4·4569	4·4578	4·4587	4·4597	4·4606	4·4615	4·4624	4·4633	·0009
276	4·4643	4·4652	4·4661	4·4670	4·4679	4·4688	4·4697	4·4706	4·4715	4·4725	·0009
277	23 4·4734	4·4743	4·4752	4·4761	4·4770	4·4779	4·4788	4·4797	4·4806	4·4815	·0009
278	4·4824	4·4833	4·4842	4·4850	4·4859	4·4868	4·4877	4·4886	4·4895	4·4904	·0009
279	4·4913	4·4922	4·4930	4·4939	4·4948	4·4957	4·4966	4·4975	4·4983	4·4992	·0009
280	23 4·5001	4·5010	4·5018	4·5027	4·5036	4·5045	4·5053	4·5062	4·5071	4·5080	·0009
281	4·5088	4·5097	4·5105	4·5114	4·5123	4·5131	4·5140	4·5148	4·5157	4·5166	·0009
282	4·5174	4·5183	4·5191	4·5200	4·5208	4·5217	4·5226	4·5234	4·5243	4·5251	·0009
283	23 4·5260	4·5268	4·5277	4·5285	4·5293	4·5302	4·5310	4·5319	4·5327	4·5336	·0008
284	4·5344	4·5352	4·5361	4·5369	4·5378	4·5386	4·5394	4·5403	4·5411	4·5419	·0008
285	4·5427	4·5436	4·5444	4·5452	4·5461	4·5469	4·5477	4·5485	4·5494	4·5502	·0008
286	23 4·5510	4·5518	4·5527	4·5535	4·5543	4·5551	4·5559	4·5567	4·5576	4·5584	·0008
287	4·5592	4·5600	4·5608	4·5616	4·5624	4·5632	4·5641	4·5648	4·5657	4·5665	·0008
288	4·5673	4·5681	4·5689	4·5697	4·5705	4·5713	4·5721	4·5729	4·5737	4·5745	·0008
289	23 4·5753	4·5761	4·5769	4·5777	4·5785	4·5793	4·5800	4·5808	4·5816	4·5824	·0008
290	4·5832										

TABLE VII.

Ballistic Table.

Distance (or space, s in feet) and velocity. $\frac{d^2}{w}s = S_V - S_v$.

For elongated projectiles, heads $1\frac{1}{2}$ diameters radius.

(From Supplement "Bashforth's Motion of Projectiles," 1881.)

v	0	1	2	3	4	5	6	7	8	9	Diff.
f.s.	feet.	feet.	feet.	feet.	feet.	feet.	feet.	feet.	feet.	feet.	+
10	1066	1238	1409	1578	1745	1910	2074	2236	2397	2557	166
11	2715	2871	3026	3180	3333	3484	3633	3782	3929	4075	151
12	4220	4363	4506	4647	4787	4926	5064	5200	5336	5471	139
13	5604	5737	5866	5999	6129	6257	6385	6511	6637	6762	129
14	6886	7009	7132	7253	7373	7493	7612	7730	7847	7964	120
15	8079	8194	8309	8422	8535	8647	8758	8868	8978	9087	112
16	9196	9304	9411	9517	9623	9728	9833	9937	10040	10142	105
17	10244	10346	10447	10546	10645	10743	10841	10939	11037	11134	98
18	11230	11326	11421	11516	11610	11704	11797	11890	11982	12074	94
19	12165	12256	12346	12436	12525	12614	12703	12791	12878	12966	89
20	13052	13139	13224	13310	13395	13480	13564	13648	13731	13814	85
21	13896	13979	14060	14142	14223	14303	14384	14463	14543	14622	81
22	14701	14779	14857	14935	15013	15090	15167	15244	15319	15395	77
23	15470	15545	15620	15694	15768	15842	15916	15989	16061	16134	74
24	16206	16278	16350	16421	16492	16563	16633	16703	16773	16843	71
25	1 6912·1	6981·2	7050·0	7118·5	7186·7	7254·7	7322·4	7389·8	7457·0	7523·9	68·0
26	7590·6	7657·0	7723·2	7789·1	7854·7	7920·1	7985·3	8050·2	8114·8	8179·3	65·4
27	8243·5	8307·5	8371·2	8434·7	8498·0	8561·0	8623·9	8686·4	8748·8	8810·9	63·0
28	1 8872·8	8934·5	8996·0	9057·2	9118·3	9179·1	9239·7	9300·1	9360·3	9420·3	60·8
29	9480·0	9539·6	9598·9	9658·1	9717·0	9775·8	9834·3	9892·6	9950·8	*0008·7	58·7
30	2 0066·5	0124·0	0181·4	0238·5	0295·5	0352·3	0409·0	0465·4	0521·7	0577·7	56·8
31	2 0633·6	0689·3	0744·8	0800·1	0855·3	0910·2	0965·0	1019·6	1074·0	1128·3	55·0
32	1182·4	1236·3	1290·0	1343·5	1396·9	1450·2	1503·2	1556·1	1608·8	1661·4	53·2
33	1713·8	1766·0	1818·1	1870·0	1921·7	1973·3	2024·7	2076·0	2127·1	2178·1	51·6
34	2 2228·9	2279·6	2330·0	2380·4	2430·6	2480·6	2530·5	2580·2	2629·7	2679·1	50·0
35	2728·4	2777·5	2826·4	2875·2	2923·8	2972·3	3020·7	3068·8	3116·9	3164·7	48·5
36	3212·5	3260·1	3307·5	3354·8	3402·0	3449·0	3495·9	3542·6	3589·2	3635·6	47·0
37	2 3682·0	3728·1	3774·2	3820·0	3865·8	3911·4	3956·9	4002·2	4047·4	4092·5	45·6
38	4137·4	4182·2	4226·8	4271·4	4315·7	4360·0	4404·1	4448·1	4491·9	4535·7	44·3
39	4579·2	4622·7	4666·0	4709·2	4752·3	4795·2	4838·1	4880·8	4923·3	4965·7	42·9
40	2 5008·0	5050·2	5092·3	5134·2	5176·0	5217·6	5259·2	5300·6	5341·9	5383·0	41·7
41	5424·0	5464·9	5505·7	5546·4	5586·9	5627·3	5667·6	5707·8	5747·8	5787·8	40·4
42	5827·6	5867·3	5906·9	5946·4	5985·8	6025·0	6064·2	6103·3	6142·2	6181·0	39·3
43	2 6219·8	6258·4	6296·9	6335·3	6373·6	6411·8	6449·9	6487·9	6525·8	6563·6	38·2
44	6601·3	6638·9	6676·4	6713·7	6751·0	6788·2	6825·3	6862·3	6899·3	6936·1	37·2
45	6972·8	7009·4	7046·0	7082·4	7118·8	7155·0	7191·2	7227·3	7263·3	7299·2	36·3
46	2 7335·1	7370·8	7406·5	7442·1	7477·6	7513·0	7548·3	7583·6	7618·8	7653·9	35·4
47	7688·9	7723·8	7758·7	7793·5	7828·2	7862·8	7897·3	7931·8	7966·2	8000·5	34·6
48	8034·7	8068·9	8103·0	8137·0	8170·9	8204·8	8238·6	8272·3	8305·9	8339·5	33·9
49	2 8373·0	8406·5	8439·8	8473·1	8506·4	8539·5	8572·6	8605·6	8638·6	8671·5	33·2
50	8704·3	8737·1	8769·8	8802·4	8835·0	8867·5	8900·0	8932·3	8964·7	8996·9	32·5
51	9029·1	9061·2	9093·2	9125·2	9157·1	9189·0	9220·8	9252·5	9284·2	9315·8	31·9

EXPLANATION.—The arrangement of this table is similar to the last, *distance* being considered instead of *time*. The first figures of the *velocity* are to be found under the heading v; for the last figure look from there horizontally until one of the other columns which indicates the last figure of the velocity is reached, when the corresponding tabulated *distance* will be found. In taking out the distance the first figure of it is only given in the column headed 0 (for velocities above 250 f.s.), and it is not repeated in the other columns in order to save room.

Thus find the distance corresponding to 540 f.s., it is 29966·3 feet; for 541 f.s. it is 29996·7 feet, and for 547 f.s. it is 30177·8 feet, the asterisk meaning that one must be added to the figure indicated under column headed 0. Proportional parts can be taken if desired; the difference for one foot in velocity being given on the same horizontal under column headed Diff., thus find the distance corresponding to a velocity of 547·3 f.s.; it is 30177·8 + 0·3 × 30·2 = 30186·9 feet.

In a reverse manner, if the distance is given the velocity corresponding to it can be found: thus if the distance is 32407·9 feet, the corresponding velocity is 626 f.s. Proportional parts can be taken if desired; thus, find the velocity corresponding to a distance of 33630·0 feet; the nearest tabulated distance which is *less* is 33619·8 feet, corresponding to a velocity of 673 f.s., the difference between these two distances is 10·2 feet; whilst the tabulated difference for one foot in velocity is given as 24·8 feet; the amount to be added is then $\frac{10{\cdot}2}{24{\cdot}8}$ f.s. = 0·4 f.s., and the velocity required is 673·4 f.s.

Proportional parts need seldom, however, be taken unless the velocities are low; it will generally be sufficiently accurate to work to the nearest tabulated quantity, whether distance or velocity. For further description of the use of this table *see* p. 147.

Note; *s* is expressed in FEET NOT in YARDS as in ordinary range tables; this should be remembered, or mistakes will occur in calculations.

Table VII.—*continued.*

Ballistic Table.

Distance (or space, s in feet) and velocity. $\dfrac{d^2}{w}s = S_V - S_v$.

v.	0	1	2	3	4	5	6	7	8	9	Diff.
f.s.	feet.	feet.	feet.	feet.	feet.	feet.	feet.	feet.	feet.	feet.	+
52	2 9347·3	9378·8	9410·3	9441·6	9472·9	9504·2	9535·4	9566·5	9597·6	9628·7	31·3
53	9659·6	9690·6	9721·4	9752·2	9783·0	9813·7	9844·3	9874·9	9905·4	9935·9	30·7
54	9966·3	9996·7	*0027·0	0057·3*	*0087·5	*0117·7	0147·8*	*0177·8	*0207·8	0237·8*	30·2
55	3 0267·6	0297·5	0327·3	0357·0	0386·7	0416·3	0445·9	0475·4	0504·9	0534·3	29·6
56	0563·6	0592·9	0622·2	0651·4	0680·6	0709·7	0738·7	0767·7	0796·7	0825·6	29·1
57	0854·5	0883·3	0912·1	0940·9	0969·6	0998·2	1026·8	1055·4	1083·9	1112·4	28·6
58	3 1140·8	1169·2	1197·6	1226·0	1254·3	1282·5	1310·8	1339·0	1367·1	1395·2	28·3
59	1423·3	1451·3	1479·3	1507·3	1535·2	1563·0	1590·9	1618·7	1646·4	1674·2	27·9
60	1701·8	1729·5	1757·1	1784·6	1812·2	1839·6	1867·1	1894·5	1921·9	1949·2	27·5
61	3 1976·5	2003·7	2031·0	2058·1	2085·3	2112·4	2139·4	2166·4	2193·4	2220·4	27·1
62	2247·3	2274·2	2301·0	2327·8	2354·5	2381·3	2407·9	2434·6	2461·2	2487·7	26·7
63	2514·3	2540·8	2567·2	2593·6	2620·0	2646·3	2672·6	2698·9	2725·1	2751·3	26·3
64	3 2777·5	2803·6	2329·7	2855·7	2881·7	2907·7	2933·7	2959·6	2985·4	3011·2	26·0
65	3037·0	3062·8	3088·5	3114·2	3139·8	3165·4	3191·0	3216·5	3242·0	3267·4	25·6
66	3292·8	3318·2	3343·5	3368·8	3394·1	3419·3	3444·5	3469·6	3494·7	3519·8	25·2
67	3 3544·8	3569·8	3594·8	3619·8	3644·7	3669·5	3694·3	3719·1	3743·9	3768·6	24·8
68	3793·3	3818·0	3842·6	3867·2	3891·7	3916·2	3940·7	3965·2	3989·6	4014·0	24·5
69	4038·4	4062·7	4087·0	4111·3	4135·6	4159·8	4184·0	4208·1	4232·2	4256·3	24·2
70	3 4280·4	4304·5	4328·5	4352·4	4376·4	4400·3	4424·1	4448·0	4471·8	4495·5	23·9
71	4519·3	4543·0	4566·6	4590·2	4613·8	4637·4	4660·9	4684·4	4707·8	4731·3	23·5
72	4754·7	4777·9	4801·3	4824·6	4847·9	4871·1	4894·2	4917·4	4940·5	4963·6	23·2
73	3 4986·6	5009·6	5032·6	5055·5	5078·4	5101·3	5124·1	5146·9	5169·6	5192·4	22·8
74	5215·1	5237·7	5260·3	5282·9	5305·5	5328·0	5350·5	5373·0	5395·4	5417·8	22·5
75	5440·2	5462·5	5484·8	5507·1	5529·3	5551·5	5573·7	5595·8	5617·9	5640·0	22·2
76	3 5662·1	5684·1	5706·0	5728·0	5749·9	5771·7	5793·5	5815·3	5837·0	5858·7	21·8
77	5880·4	5902·0	5923·6	5945·1	5966·6	5988·1	6009·5	6030·9	6052·2	6073·6	21·5
78	6094·8	6116·1	6137·3	6158·4	6179·6	6200·7	6221·7	6242·7	6263·7	6284·6	21·1
79	3 6305·5	6326·4	6347·2	6368·0	6388·8	6409·5	3430·2	6450·8	6471·4	6492·0	20·7
80	6512·6	6533·1	6553·6	6574·0	6594·4	6614·8	6635·1	6655·4	6675·7	6695·9	20·4
81	6716·1	6736·3	6756·4	6776·5	6796·5	6816·5	6836·5	6856·4	6876·3	6896·1	20·0
82	3 6916·0	6935·7	6955·5	6975·1	6994·8	7014·4	7033·9	7053·4	7072·9	7092·3	19·6
83	7111·7	7131·0	7150·3	7169·6	7188·8	7207·9	7227·1	7246·1	7265·2	7284·1	19·1
84	7303·1	7322·0	7340·8	7359·6	7378·4	7397·1	7415·8	7434·4	7453·0	7471·5	18·7
85	3 7490·0	7508·5	7526·9	7545·3	7563·6	7581·8	7600·0	7618·2	7636·3	7654·4	18·2
86	7672·4	7690·5	7708·4	7726·4	7744·2	7762·0	7779·9	7797·6	7815·4	7833·0	17·8
87	7850·6	7868·2	7885·8	7903·3	7920·8	7938·2	7955·6	7973·0	7990·3	8007·6	17·4
88	3 8024·8	8042·0	8059·2	8076·3	8093·4	8110·4	8127·4	8144·4	8161·3	8178·2	17·0
89	8195·0	8211·9	8228·6	8245·4	8262·1	8278·7	8295·4	8312·0	8328·5	8345·0	16·6
90	8361·5	8377·9	8394·3	8410·7	8427·0	8443·3	8459·6	8475·8	8492·0	8508·2	16·3
91	3 8524·3	8540·4	8556·4	8572·4	8588·4	8604·3	8620·3	8636·1	8652·0	8667·8	15·9
92	8683·5	8699·3	8715·0	8730·7	8746·3	8761·9	8777·5	8793·0	8808·5	8824·0	15·6
93	8839·4	8854·8	8870·2	8885·5	8900·8	8916·1	8931·3	8946·5	8961·7	8976·8	15·3
94	3 8991·9	9007·0	9022·0	9037·0	9052·0	9066·9	9081·9	9096·7	9111·6	9126·4	15·0
95	9141·2	9156·0	9170·7	9185·4	9200·1	9214·7	9229·3	9243·9	9258·4	9272·9	14·6
96	9287·4	9301·9	9316·3	9330·7	9345·0	9359·4	9373·7	9387·9	9402·2	9416·4	14·3
97	3 9430·6	9444·7	9458·9	9473·0	9487·0	9501·1	9515·1	9529·1	9543·0	9557·0	14·0
98	9570·8	9584·7	9598·6	9612·4	9626·1	9639·9	9653·6	9667·3	9681·0	9694·6	13·7
99	9708·3	9721·9	9735·4	9749·0	9762·5	9775·9	9789·4	9802·8	9816·2	9829·6	13·5
100	3 9842·9	9856·3	9869·6	9882·9	9896·1	9909·3	9922·5	9935·3	9948·8	9961·9	13·2
101	9975·0	9988·1	*0001·1	0014·1*	*0027·1	*0040·0	0052·9*	*0065·8	*0078·7	0091·5*	12·9
102	4 0104·3	0117·1	0129·8	0142·5	0155·2	0167·8	0180·4	0192·9	0205·4	0217·8	12·6
103	4 0230·1	0242·4	0254·6	0266·8	0278·8	0290·8	0302·7	0314·5	0326·2	0337·8	11·9
104	0349·4	0360·8	0372·2	0383·4	0394·5	0405·6	0416·5	0427·3	0438·1	0448·7	11·0
105	0459·2	0469·6	0479·9	0490·0	0500·1	0510·1	0520·0	0529·8	0539·5	0549·2	9·9
106	4 0558·7	0568·2	0577·6	0586·9	0596·2	0605·4	0614·5	0623·6	0632·6	0641·6	9·2
107	0650·5	0659·3	0668·1	0676·9	0685·6	0694·2	0702·8	0711·4	0719·9	0728·4	8·6
108	0736·8	0745·2	0753·6	0761·9	0770·2	0778·4	0786·6	0794·8	0802·9	0811·0	8·2
109	4 0819·0	0827·1	0835·0	0843·0	0850·9	0858·9	0866·7	0874·6	0882·4	0890·2	7·9
110	0897·9	0905·7	0913·4	0921·1	0928·7	0936·4	0944·0	0951·5	0959·1	0966·6	7·6
111	0974·2	0981·6	0989·1	0996·6	1004·0	1011·4	1018·8	1026·2	1033·5	1040·9	7·4

TABLE VII.—*continued.*

Ballistic Table.

Distance (or space, s in feet) and velocity. $\frac{d^2}{w}s = S_V - S_v$.

v.	0	1	2	3	4	5	6	7	8	9	Diff.
f.s.	feet.	feet.	feet.	feet.	feet.	feet.	feet.	feet.	feet.	feet.	+
112	4 1048·2	1055·5	1062·8	1070·0	1077·3	1084·5	1091·7	1099·0	1106·1	1113·3	7·2
113	1120·5	1127·6	1134·8	1141·9	1149·0	1156·1	1163·2	1170·2	1177·3	1184·4	7·1
114	1191·4	1198·4	1205·4	1212·4	1219·4	1226·4	1233·3	1240·3	1247·2	1254·1	6·9
115	4 1261·0	1267·9	1274·8	1281·7	1288·6	1295·4	1302·3	1309·1	1315·9	1322·7	6·8
116	1329·5	1336·3	1343·1	1349·8	1356·6	1363·3	1370·0	1376·7	1383·4	1390·1	6·7
117	1396·8	1403·5	1410·1	1416·8	1423·4	1430·0	1436·6	1443·2	1449·8	1456·4	6·6
118	4 1462·9	1469·5	1476·0	1482·6	1489·1	1495·6	1502·1	1508·6	1515·1	1521·5	6·5
119	1528·0	1534·4	1540·9	1547·3	1553·7	1560·1	1566·5	1572·9	1579·2	1585·6	6·4
120	1591·9	1598·3	1604·6	1610·9	1617·2	1623·5	1629·8	1636·1	1642·3	1648·6	6·3
121	4 1654·8	1661·1	1667·3	1673·5	1679·7	1685·9	1692·1	1698·2	1704·4	1710·5	6·2
122	1716·7	1722·8	1728·9	1735·0	1741·1	1747·2	1753·3	1759·4	1765·4	1771·5	6·1
123	1777·5	1783·6	1789·6	1795·6	1801·6	1807·6	1813·6	1819·6	1825·6	1831·5	6·0
124	4 1837·5	1843·4	1849·4	1855·3	1861·2	1867·1	1873·0	1878·9	1884·8	1890·6	5·9
125	1896·5	1902·3	1908·2	1914·0	1919·8	1925·6	1931·5	1937·3	1943·0	1948·8	5·8
126	1954·6	1960·4	1966·1	1971·9	1977·6	1983·3	1989·0	1994·8	2000·5	2006·2	5·7
127	4 2011·8	2017·5	2023·2	2028·9	2034·5	2040·2	2045·8	2051·4	2057·0	2062·7	5·6
128	2068·3	2073·9	2079·5	2085·0	2090·6	2096·2	2101·8	2107·3	2112·9	2118·4	5·6
129	2123·9	2129·4	2135·0	2140·5	2146·0	2151·5	2157·0	2162·4	2167·9	2173·4	5·5
130	4 2178·8	2184·3	2189·7	2195·1	2200·6	2206·0	2211·4	2216·8	2222·2	2227·6	5·4
131	2233·0	2288·4	2243·7	2249·1	2254·5	2259·8	2265·1	2270·5	2275·8	2281·1	5·3
132	2286·4	2291·8	2297·1	2302·4	2307·6	2312·9	2318·2	2323·5	2328·7	2334·0	5·3
133	4 2339·2	2344·5	2349·7	2355·0	2360·2	2365·4	2370·6	2375·8	2381·0	2386·2	5·2
134	2391·4	2396·6	2401·8	2406·9	2412·1	2417·3	2422·4	2427·6	2432·7	2437·8	5·2
135	2443·0	2448·1	2453·2	2458·3	2463·4	2468·5	2473·6	2478·7	2483·8	2488·9	5·1
136	4 2493·9	2499·0	2504·1	2509·1	2514·2	2519·2	2524·3	2529·3	2534·3	2539·4	5·0
137	2544·4	2549·4	2554·4	2559·4	2564·4	2569·4	2574·4	2579·4	2584·3	2589·3	5·0
138	2594·3	2599·2	2604·2	2609·1	2614·1	2619·0	2624·0	2628·9	2633·8	2638·8	4·9
139	4 2643·7	2648·6	2653·5	2658·4	2663·3	2668·2	2673·1	2678·0	2682·9	2687·8	4·9
140	2692·6	2697·5	2702·4	2707·2	2712·1	2717·0	2721·8	2726·7	2731·5	2736·3	4·9
141	2741·2	2746·0	2750·8	2755·7	2760·5	2765·3	2770·1	2774·9	2779·7	2784·5	4·8
142	4 2789·3	2794·1	2798·9	2803·7	2808·5	2813·2	2818·0	2822·8	2827·5	2832·3	4·8
143	2837·1	2841·8	2846·6	2851·3	2856·0	2860·8	2865·5	2870·2	2875·0	2879·7	4·7
144	2884·4	2889·1	2893·8	2898·6	2903·3	2908·0	2912·7	2917·4	2922·1	2926·7	4·7
145	4 2931·4	2936·1	2940·8	2945·5	2950·1	2954·8	2959·5	2964·1	2968·8	2973·5	4·7
146	2978·1	2982·8	2987·4	2992·1	2996·7	3001·3	3006·0	3010·6	3015·2	3019·9	4·6
147	3024·5	3029·1	3033·7	3038·4	3043·0	3047·6	3052·2	3056·8	3061·4	3066·0	4·6
148	4 3070·6	3075·2	3079·8	3084·4	3089·0	3093·5	3098·1	3102·7	3107·3	3111·8	4·6
149	3116·4	3121·0	3125·6	3130·1	3134·7	3139·2	3143·8	3148·3	3152·9	3157·4	4·6
150	3162·0	3166·5	3171·0	3175·6	3180·1	3184·6	3189·2	3193·7	3198·2	3202·7	4·5
151	4 3207·2	3211·8	3216·3	3220·8	3225·3	3229·8	3234·3	3238·8	3243·3	3247·8	4·5
152	3252·3	3256·8	3261·3	3265·8	3270·3	3274·8	3279·3	3283·8	3288·3	3292·8	4·5
153	3297·2	3301·7	3306·2	3310·6	3315·1	3319·6	3324·1	3328·5	3333·0	3337·5	4·5
154	4 3342·0	3346·4	3350·9	3355·3	3359·8	3364·3	3368·7	3373·2	3377·6	3382·1	4·5
155	3386·5	3391·0	3395·4	3399·9	3404·3	3408·7	3413·2	3417·6	3422·0	3426·5	4·4
156	3430·9	3435·3	3439·8	3444·2	3448·6	3453·0	3457·4	3461·9	3466·3	3470·7	4·4
157	4 3475·1	3479·5	3483·9	3488·3	3492·7	3497·1	3501·5	3505·9	3510·3	3514·7	4·4
158	3519·1	3523·5	3527·9	3532·3	3536·7	3541·1	3545·4	3549·8	3554·2	3558·6	4·4
159	3563·0	3567·3	3571·7	3576·1	3580·4	3584·8	3589·1	3593·5	3597·9	3602·2	4·4
160	4 3606·6	3610·9	3615·3	3619·6	3624·0	3628·3	3632·6	3637·0	3641·3	3645·7	4·3
161	3650·0	3654·3	3658·7	3663·0	3667·3	3671·6	3676·0	3680·3	3684·6	3688·9	4·3
162	3693·3	3697·6	3701·9	3706·1	3710·5	3714·8	3719·1	3723·4	3727·7	3732·0	4·3
163	4 3736·3	3740·6	3744·9	3749·2	3753·5	3757·8	3762·1	3766·4	3770·6	3774·9	4·3
164	3779·2	3783·5	3787·8	3792·0	3796·3	3800·6	3804·9	3809·1	3813·4	3817·6	4·3
165	3881·9	3826·2	3830·4	3334·7	3838·9	3843·2	3847·4	3851·7	3855·9	3860·2	4·3
166	4 3864·4	3868·7	3872·9	3377·2	3881·4	3885·6	3889·9	3894·1	3898·3	3902·5	4·2
167	3906·8	3911·0	3915·2	3919·5	3923·7	3927·9	3932·1	3936·3	3940·5	3944·7	4·2
168	3949·0	3953·2	3957·4	3961·6	3965·8	3970·0	3974·2	3978·4	3982·6	3986·7	4·2
169	4 3990·9	3995·1	3999·3	4003·5	4007·7	4011·9	4016·0	4020·2	4024·4	4028·6	4·2
170	4032·7	4036·9	4041·1	4045·2	4049·4	4053·6	4057·7	4061·9	4066·0	4070·2	4·2
171	4074·3	4078·5	4082·6	4086·8	4090·9	4095·1	4099·2	4103·3	4107·5	4111·6	4·1

TABLE VII.—*continued.*

Ballistic Table.

Distance (or space, s in feet) and velocity. $\frac{d^2}{w}s = S_V - S_v$.

v.	0	1	2	3	4	5	6	7	8	9	Diff.
f.s.	feet.	feet.	feet.	feet.	feet.	feet.	feet.	feet.	feet.	feet.	+
172	4 4115·7	4119·9	4124·0	4128·1	4132·3	4136·4	4140·5	4144·6	4148·7	4152·9	4·1
173	4157·0	4161·1	4165·2	4169·3	4173·4	4177·5	4181·6	4185·7	4189·8	4193·9	4·1
174	4198·0	4202·1	4206·2	4210·3	4214·4	4218·5	4222·6	4226·7	4230·8	4234·8	4·1
175	4 4238·9	4243·0	4247·1	4251·2	4255·3	4259·3	4263·4	4267·5	4271·5	4275·6	4·1
176	4279·6	4283·7	4287·8	4291·8	4295·9	4300·0	4304·0	4308·0	4312·1	4316·1	4·1
177	4320·2	4324·2	4328·3	4332·3	4336·4	4340·4	4344·4	4348·5	4352·5	4356·5	4·0
178	4 4360·5	4364·6	4368·6	4372·6	4376·6	4380·7	4384·7	4388·7	4392·7	4396·7	4·0
179	4400·7	4404·7	4408·8	4412·8	4416·8	4420·8	4424·8	4428·8	4432·8	4436·8	4·0
180	4440·8	4444·7	4448·7	4452·7	4456·7	4460·7	4464·7	4468·7	4472·6	4476·6	4·0
181	4 4480·6	4484·6	4488·5	4492·5	4496·5	4500·5	4504·4	4508·4	4512·4	4516·3	4·0
182	4520·3	4524·2	4528·2	4532·2	4536·1	4540·1	4544·0	4548·0	4551·9	4555·9	4·0
183	4559·8	4563·7	4567·7	4571·6	4575·6	4579·5	4583·4	4587·4	4591·3	4595·2	3·9
184	4 4599·2	4603·1	4607·0	4610·9	4614·9	4618·8	4622·7	4626·6	4630·5	4634·4	3·9
185	4638·4	4642·3	4646·2	4650·1	4654·0	4657·9	4661·8	4665·7	4669·6	4673·5	3·9
186	4677·4	4681·3	4685·2	4689·1	4693·0	4696·9	4700·8	4704·6	4708·5	4712·4	3·9
187	4 4716·3	4720·2	4724·1	4727·9	4731·8	4735·7	4739·6	4743·4	4747·3	4751·2	3·9
188	4755·0	4758·9	4762·8	4766·7	4770·5	4774·4	4778·2	4782·1	4786·0	4789·8	3·9
189	4793·7	4797·5	4801·4	4805·2	4809·1	4812·9	4816·8	4820·6	4824·5	4828·3	3·8
190	4 4832·2	4836·0	4439·8	4843·7	4847·5	4851·4	4855·2	4859·0	4862·8	4866·7	3·8
191	4870·5	4874·3	4878·1	4882·0	4885·8	4889·6	4893·4	4897·3	4901·1	4904·9	3·8
192	4908·7	4912·5	4916·3	4920·1	4923·9	4927·7	4931·5	4935·3	4939·1	4942·9	3·8
193	4 4946·7	4950·5	4954·3	4958·1	4961·9	4965·7	4969·4	4973·2	4977·0	4980·7	3·8
194	4984·5	4988·3	4992·1	4995·8	4999·6	5003·4	5007·1	5010·9	5014·7	5018·4	3·8
195	5022·2	5025·9	5029·7	5033·4	5037·2	5040·9	5044·7	5048·4	5052·1	5055·9	3·7
196	4 5059·6	5063·4	5067·1	5070·8	5074·6	5078·3	5082·0	5085·7	5089·4	5093·1	3·7
197	5096·9	5100·6	5104·3	5108·0	5111·7	5115·4	5119·1	5122·8	5126·5	5130·2	3·7
198	5133·9	5137·5	5141·2	5144·9	5148·6	5152·3	5156·0	5159·6	5163·3	5166·9	3·7
199	4 5170·6	5174·3	5177·9	5181·6	5185·2	5188·9	5192·5	5196·2	5199·8	5203·4	3·6
200	5207·1	5210·7	5214·3	5218·0	5221·6	5225·2	5228·8	5232·5	5236·1	5239·7	3·6
201	5243·3	5246·9	5250·5	5254·1	5257·7	5261·3	5264·9	5268·5	5272·1	5275·7	3·6
202	4 5279·2	5282·8	5286·4	5290·0	5293·6	5297·2	5300·7	5304·3	5307·8	5311·4	3·6
203	5314·9	5318·5	5322·0	5325·6	5329·1	5332·7	5336·2	5339·7	5343·3	5346·8	3·5
204	5350·3	5353·8	5357·3	5360·9	5364·4	5367·9	5371·4	5374·9	5378·4	5381·9	3·5
205	4 5385·4	5388·9	5392·4	5395·9	5399·4	5402·9	5406·3	5409·8	5413·3	5416·7	3·5
206	5420·2	5423·7	5427·1	5430·6	5434·1	5437·5	5441·0	5444·4	5447·8	5451·3	3·5
207	5454·7	5458·1	5461·6	5465·0	5468·4	5471·9	5475·3	5478·7	5482·1	5485·5	3·4
208	4 5488·9	5492·3	5495·7	5499·1	5502·5	5505·9	5509·3	5512·7	5516·1	5519·4	3·4
209	5522·8	5526·2	5529·6	5532·9	5536·3	5539·7	5543·0	5546·4	5549·7	5553·1	3·4
210	5556·4	5559·8	5563·1	5566·4	5569·8	5573·1	5576·5	5579·8	5583·1	5586·4	3·3
211	4 5589·7	5593·0	5596·4	5599·7	5603·0	5606·3	5609·6	5612·9	5616·2	5619·5	3·3
212	5622·8	5626·1	5629·3	5632·6	5635·9	5639·2	5642·5	5645·7	5649·0	5652·3	3·3
213	5655·5	5658·8	5662·0	5665·3	5668·6	5671·8	5675·1	5678·3	5681·5	5684·8	3·2
214	4 5688·0	5691·2	5694·5	5697·7	5700·9	5704·2	5707·4	5710·6	5713·8	5717·0	3·2
215	5720·2	5723·4	5726·6	5729·9	5733·1	5736·3	5739·5	5742·6	5745·8	5749·0	3·2
216	5752·2	5755·4	5758·6	5761·8	5764·9	5768·1	5771·3	5774·4	5777·6	5780·8	3·2
217	4 5783·9	5787·1	5790·2	5793·4	5796·6	5799·7	5802·9	5806·0	5809·1	5812·2	3·1
218	5815·4	5818·5	5821·6	5824·8	5827·9	5831·0	5834·1	5837·3	5840·4	5843·5	3·1
219	5846·6	5849·7	5852·8	5855·9	5859·0	5862·1	5865·2	5868·3	5871·4	5874·4	3·1
220	4 5877·5	5880·6	5883·7	5886·8	5889·9	5893·0	5896·0	5899·1	5902·1	5905·2	3·1
221	5908·3	5911·3	5914·4	5917·4	5920·5	5923·6	5926·6	5929·6	5932·7	5935·7	3·0
222	5938·7	5941·8	5944·8	5947·8	5950·9	5953·9	5956·9	5959·9	5963·0	5966·0	3·0
223	4 5969·0	5972·0	5975·0	5978·0	5981·0	5984·0	5987·0	5990·0	5993·0	5996·0	3·0
224	5999·0	6002·0	6004·9	6007·9	6010·9	6013·9	6016·9	6019·8	6022·8	6025·8	3·0
225	6028·7	6031·7	6034·6	6037·6	6040·5	6043·5	6046·5	6049·4	6052·4	6055·3	3·0
226	4 6058·3	6061·2	6064·1	6067·1	6070·0	6072·9	6075·9	6078·8	6081·7	6084·7	2·9
227	6087·6	6090·5	6093·4	6096·3	6099·3	6102·2	6105·1	6108·0	6110·9	6113·8	2·9
228	6116·7	6119·6	6122·5	6125·4	6128·3	6131·2	6134·1	6137·0	6139·9	6142·8	2·9
229	4 6145·7	6148·6	6151·5	6154·4	6157·3	6160·2	6163·1	6166·0	6168·8	6171·7	2·9
230	6174·6	6177·5	6180·4	6183·3	6186·2	6189·1	6191·9	6194·8	6197·7	6200·6	2·9
231	6203·5	6206·4	6209·3	6212·1	6215·0	6217·9	6220·8	6223·7	6226·6	6229·5	2·9

TABLE VII.—*continued.*

Ballistic Table.

Distance (or space, s in feet) and velocity. $\frac{d^2}{w}s = S_V - S_v$.

v.	0	1	2	3	4	5	6	7	8	9	Diff.
f.s.	feet.	feet.	feet.	feet.	feet.	feet.	feet.	feet.	feet.	feet.	+
232	4 6232·3	6235·2	6238·1	6241·0	6243·9	6246·8	6249·7	6252·6	6255·4	6258·3	2·9
233	6261·2	6264·1	6267·0	6269·9	6272·8	6275·7	6278·6	6281·5	6284·3	6287·2	2·9
234	6290·1	6293·0	6295·9	6298·8	6301·7	6304·6	6307·5	6310·4	6313·3	6316·2	2·9
235	4 6319·0	6322·0	6324·9	6327·7	6330·6	6333·5	6336·4	6339·3	6342·2	6345·1	2·9
236	6348·0	6350·9	6353·8	6356·7	6359·6	6362·5	6365·4	6368·3	6371·2	6374·1	2·9
237	6377·0	6379·9	6382·8	6385·7	6388·6	6391·5	6394·4	6397·3	6400·2	6403·1	2·9
238	4 6406·0	6408·9	6411·8	6414·8	6417·7	6420·6	6423·5	6426·4	6429·3	6432·2	2·9
239	6435·1	6438·0	6440·9	6443·8	6446·8	6449·7	6452·6	6455·5	6458·4	6461·3	2·9
240	6464·2	6467·1	6470·1	6473·0	6475·9	6478·8	6481·7	6484·6	6487·6	6490·5	2·9
241	4 6493·4	6496·3	6499·2	6502·2	6505·1	6508·0	6510 9	6513·8	6516·8	6519·7	2·9
242	6522·6	6525·6	6528·5	6531·4	6534·3	6537·3	6540·2	6543·1	6546·1	6549·0	2·9
243	6551·9	6554·9	6557·8	6560·7	6563·7	6566·6	6569·5	6572·5	6575·4	6578·3	2·9
244	4 6581·3	6584·2	6587·2	6590·1	6593·0	6596·0	6598·9	6601·8	6604·8	6607·7	2·9
245	6610·6	6613·6	6616·5	6619·5	6622·4	6625·3	6628·3	6631·2	6634·2	6637·1	2·9
246	6640·1	6643·0	6645·9	6748·9	6651·8	6654·8	6657·7	6660·6	6663·6	6666·5	2·9
247	4 6669·5	6672·4	6675·4	6678·3	6681·3	6684·2	6687·2	6690·1	6693·0	6696·0	2·9
248	6698·9	6701·9	6704·8	6707·8	6710·7	6713·7	6716·6	6719·6	6722·5	6725·5	2·9
249	6728·4	6731·3	6734·3	6737·2	6740·2	6743·1	6746·1	6749·0	6752·0	6754·9	2·9
250	4 6757·8	6760·7	6763·7	6766·7	6769·6	6772·6	6775·5	6778·4	6781·4	6784·3	2·9
251	6787·3	6790·2	6793·1	6796·1	6799·0	6802·0	6804·9	6807·8	6810·8	6813·7	2·9
252	6816·6	6819·6	6822·5	6825·4	6828·4	6831·3	6834·2	6837·1	6840·1	6843·0	2·9
253	4 6845·9	6848·8	6851·8	6854·7	6857·6	6860·5	6863·5	6866·4	6869·3	6872·2	2·9
254	6875·1	6878·1	6881·0	6883·9	6886·8	6889·7	6892·6	6895·6	6898·5	6901·4	2·9
255	6904·3	6907·2	6910·1	6913·0	6915·9	6918·8	6921·7	6924·6	6927·5	6930·4	2·9
256	4 6933·3	6936·2	6939·1	6942·0	6944·9	6947·8	6950·6	6953·5	6956·4	6959·3	2·9
257	6962·2	6965·0	6967·9	6970·8	6973·7	6976·5	6979·4	6982·3	6985·1	6988·0	2·9
258	6990·9	6993·7	6996·6	6999·4	7002·3	7005·1	7008·0	7010·8	7013·7	7016·5	2·9
259	4 7019·4	7022·2	7025·0	7027·9	7030·7	7033·5	7036·4	7039·2	7042·0	7044·8	2·8
260	7047·7	7050·5	7053·3	7056·1	7058·9	7061·7	7064·5	7067·4	7070·2	7073·0	2·8
261	7075·8	7078·6	7081·4	7084·2	7087·0	7089·7	7092·5	7095·3	7098·1	7100·9	2·8
262	4 7103·7	7106·5	7109·2	7112·0	7114·8	7117·6	7120·3	7123·1	7125·9	7128·6	2·8
263	7131·4	7134·2	7136·9	7139·7	7142·4	7145·2	7147·9	7150·7	7153·4	7156·2	2·8
264	7158·9	7161·7	7164·4	7167·1	7169·9	7172·6	7175·4	7178·1	7180·8	7183·5	2·7
265	4 7186·3	7189·0	7191·7	7194·4	7197·1	7199·9	7202·6	7205·3	7208·0	7210·7	2·7
266	7213·4	7216·1	7218·8	7221·5	7224·2	7226·9	7229·6	7232·3	7235·0	7237·7	2·7
267	7240·4	7243·1	7245·8	7248·5	7251·2	7253·8	7256·5	7259·2	7261·9	7264·5	2·7
268	4 7267·2	7269·9	7272·5	7275·2	7277·9	7280·5	7283·2	7285·9	7288·5	7291·2	2·7
269	7293·8	7296·5	7299·1	7301·8	7304·4	7307·1	7309·7	7312·3	7315·0	7317·6	2·6
270	7320·2	7322·9	7325·5	7328·1	7330·8	7333·4	7336·0	7338·6	7341·2	7343·9	2·6
271	4 7346·5	7349·1	7351·7	7354·3	7356·9	7359·5	7362·1	7364·7	7367·3	7369·9	2·6
272	7372·5	7375·1	7377·7	7380·3	7382·9	7385·5	7388·1	7390·7	7393·3	7395·8	2·6
273	7398·4	7401·0	7403·6	7406·2	7408·7	7411·3	7413·9	7416·4	7419·0	7421·6	2·6
274	4 7424·1	7426·7	7429·3	7431·8	7434·4	7436·9	7439·5	7442·0	7444·6	7447·1	2·6
275	7449·7	7452·2	7454·8	7457·3	7459·8	7462·4	7464·9	7467·4	7470·0	7472·5	2·5
276	7475·0	7477·5	7480·1	7482·6	7485·1	7487·6	7490·1	7492·7	7495·2	7497·7	2·5
277	4 7500·2	7502·7	7505·2	7507·7	7510·2	7512·7	7515·2	7517·7	7520·2	7522·7	2·5
278	7525·2	7527·7	7530·1	7532·6	7535·1	7537·6	7540·1	7542·6	7545·0	7547·5	2·5
279	7550·0	7552·4	7554·9	7557·4	7559·9	7562·3	7564·8	7567·2	7569·7	7572·2	2·5
280	4 7574·6	7577·1	7579·5	7582·0	7584·4	7586·8	7589·3	7591·7	7594·2	7596·6	2·4
281	7599·0	7601·5	7603·9	7606·4	7608·8	7611·2	7613·6	7616·1	7618·5	7620·9	2·4
282	7623·3	7625·7	7628·2	7630·6	7633·0	7635·4	7637·8	7640·2	7642·6	7645·0	2·4
283	4 7647·4	7649·8	7652·2	7654·6	7657·0	7659·4	7661·8	7664·2	7666·6	7669·0	2·4
284	7671·3	7673·7	7676·1	7678·5	7680·9	7683·3	7685·6	7688·0	7690·4	7692·7	2·4
285	7695·1	7697·5	7699·8	7702·2	7704·6	7706·9	7709·3	7711·6	7714·0	7716·4	2·4
286	4 7718·7	7721·1	7723·4	7725·8	7728·1	7730·4	7732·8	7735·1	7737·5	7739·8	2·3
287	7742·1	7744·5	7746·8	7749·1	7751·5	7753·8	7756·1	7758·4	7760·8	7763·1	2·3
288	7765·4	7767·7	7770·0	7772·4	7774·7	7777·0	7779·3	7781·6	7783·9	7786·2	2·3
289	4 7788·5	7790·8	7793·1	7795·4	7797·7	7800·0	7802·3	7804·6	7806·9	7809·2	2·3
290	7811·5										

Table VIII.

Ballistic Table.

Inclination (δ in degrees) and velocity. $\frac{d^2}{w}\delta = D_V - D_v$.

For elongated projectiles, heads 1½ diameters radius.

(By W. D. Niven, Esq., F.R.S.)

v.	0	1	2	3	4	5	6	7	8	9
f.s	degs.	degs.	degs.	degs.	degs.	degs.	degs.	degs.	degs.	degs.
40	0·0000	0·4838	0·9640	1·4407	1·9137	2·3830	2·8488	3·3110	3·7689	4·2240
41	4·6757	5·1240	5·5688	6·0101	6·4482	6·8828	7·3141	7·7421	8·1660	8·5874
42	9·0056	9·4207	9·8327	10·2410	10·6467	11·0496	11·4494	11·8462	12·2397	12·6306
43	13·0187	13·4039	13·7862	14·1652	14·5419	14·9159	15·2872	15·6557	16·0211	16·3843
44	16·7450	17·1030	17·4585	17·8110	18·1614	18·5094	18·8549	19·1980	19·5383	19·8766
45	20·2125	20·5460	20·8772	21·2054	21·5320	21·8565	22·1788	22·4989	22·8169	23·1327
46	23·4463	23·7578	24·0671	24·3736	24·6788	24·9821	25·2834	25·5827	25·8801	26·1756
47	26·4691	26·7607	27·0503	27·3376	27·6234	27·9075	28·1897	28·4702	28·7486	29·0254
48	29·3006	29·5739	29·8455	30·1151	30·3833	30·6498	30·9147	31·1779	31·4393	31·6993
49	31·9576	32·2143	32·4695	32·7227	32·9747	33·2253	33·4743	33·7219	33·9679	34·2125
50	34·4557	34·6973	34·9375	35·1761	35·4134	35·6493	35·8837	36·1167	36·3480	36·5783
51	36·8073	37·0349	37·2613	37·4862	37·7099	37·9323	38·1534	38·3731	38·5914	38·8086
52	39·0246	39·2394	39·4529	39·6651	39·8762	40·0860	40·2947	40·5022	44·7083	40·9135
53	41·1175	41·3204	41·5221	41·7225	41·9221	42·1205	42·3179	42·5142	42·7095	42·9037
54	43·0967	43·2887	43·4795	43·6690	43·8578	44·0456	44·2324	44·4182	44·6031	44·7870
55	44·9698	45·1516	45·3325	45·5122	45·6910	45·8689	46·0457	46·2217	46·3964	46·5705
56	46·7437	46·9160	47·0874	47·2581	47·4277	47·5965	47·7644	47·9314	48·0973	48·2625
57	48·4270	48·5906	48·7534	48·9153	49·0764	49·2368	49·3963	49·5551	49·7130	49·8701
58	50·0265	50·1822	50·3370	50·4909	50·6442	50·7968	59·9487	51·0999	51·2505	51·4002
59	51·5492	51·6975	51·8451	51·9917	52·1378	52·2832	52·4280	52·5721	52·7155	52·8583
60	53·0003	53·1417	53·2825	53·4224	53·5618	53·7005	53·8386	53·9761	54·1130	54·2492
61	54·3847	54·5196	54·6539	54·7875	54·9205	55·0529	55·1846	55·3158	55·4462	55·5761
62	55·7054	55·8342	55·9623	56·0899	56·2169	56·3433	56·4690	56·5942	56·7188	56·8428
63	56·9663	57·0891	57·2114	57·3330	57·4542	57·5749	57·6950	57·8146	57·9338	58·0523
64	58·1703	58·2878	58·4046	58·5209	58·6367	58·7521	58·8669	58·9832	59·0949	59·2081
65	59·3209	59·4332	59·5449	59·6562	59·7669	59·8772	59·9869	60·0961	60·2047	60·3130
66	60·4207	60·5280	60·6348	60·7411	60·8470	60·9523	61·0572	61·1616	61·2654	61·3688
67	61·4719	61·5744	61·6766	61·7783	61·8796	61·9804	62·0808	62·1807	62·2802	62·3793
68	62·4779	62·5761	62·6739	62·7711	62·8680	62·9646	63·0607	63·1565	63·2519	63·3468
69	63·4414	63·5356	63·6294	63·7227	63·8157	63·9084	64·0006	64·0924	64·1838	64·2749
70	64·3656	64·4559	64·5459	64·6356	64·7249	64·8137	64·9022	64·9903	65·0779	65·1652
71	65·2522	65·3388	65·4250	65·5107	65·5962	65·6813	65·7660	65·8504	65·9345	66·0182
72	66·1015	66·1845	66·2671	66·3494	66·4313	66·5128	66·5940	66·6749	66·7553	66·8355
73	66·9153	66·9949	67·0740	67·1529	67·2314	67·3096	67·3875	67·4649	67·5422	67·6190
74	67·6955	67·7717	67·8476	67·9231	67·9983	68·0733	68·1479	68·2223	68·2964	68·3702
75	68·4436	68·5168	68·5896	68·6620	68·7342	68·8062	68·8778	68·9492	69·0204	69·0912
76	69·1617	69·2318	69·3017	69·3712	69·4404	69·5094	69·5780	69·6464	69·7145	69·7823
77	69·8497	69·9169	69·9838	70·0503	70·1166	70·1826	70·2483	70·3137	70·3787	70·4436
78	70·5082	70·5725	70·6365	70·7004	70·7639	70·8271	70·8901	70·9527	71·0149	71·0770
79	71·1388	71·2004	71·2617	71·3228	71·3837	71·4442	71·5045	71·5646	71·6244	71·6839
80	71·7432	71·8023	71·8611	71·9196	71·9779	72·0359	72·0937	72·1513	72·2086	72·2656
81	72·3225	72·3791	72·4354	72·4915	72·5473	72·6030	72·6584	72·7135	72·7685	72·8232

Explanation.—The arrangement of this table is very similar to the last two, inclination in degrees taking the place of time and distance respectively. The *first* figures of the *velocity* are to be found under the heading *v*; for the last figure look from there horizontally until one of the other columns which indicates the last figure of the velocity is reached when the corresponding tabulated *inclination* will be found.

Thus find the inclination corresponding to 950 f.s., it is 78·1841 degrees; for 976 f.s. it is 78·9280 degrees. A column of difference for one foot per second in velocity is not given in this table as in the others, but proportional parts can be taken if desired: thus find the inclination corresponding to a velocity of 976·4 f.s. The tabulated amounts for 976 f.s. and 977 f.s. are found to be 78·9280 and 78·9551 respectively, their difference is 0·0271, and 0·4 of this difference, or 0·0108, must be added to 78·9280, making a result of 78·9388 degrees.

In a reverse manner, if the inclination is given the velocity corresponding to it can be found: thus if the inclination is 77·7807 the corresponding velocity is 937 f.s. Proportional parts can be taken if desired: thus find the velocity corresponding to an inclination of 77·8011 degrees; the nearest tabulated amounts are 77·7807 and 77·8125, their difference is 0·0318, and the given amount differs from the least of these (which corresponds to a velocity of 937 f.s.) by 0·0204; hence the proportional part to be added is $\frac{0·0204}{0·0318} = 0·6$, and the velocity required is 937·6 f.s.

Proportional parts need, however, seldom be taken; it will generally be sufficiently accurate to work to the nearest tabulated quantity, whether inclination or velocity. For further description of the use of this table see p. 151.

Table VIII.—*continued.*

Ballistic Table.

Inclination (δ in degrees) and velocity. $\frac{d^2}{w}\delta = D_V - D_v$.

v.	0	1	2	3	4	5	6	7	8	9
f.s.	degs.	degs.	degs.	degs.	degs.	degs.	degs.	degs.	degs.	degs.
82	72·8776	72·9317	72·9856	73·0393	73·0927	73·1458	73·1988	73·2514	73·3038	73·3560
83	73·4079	73·4596	73·5111	73·5622	73·6132	73·6639	73·7145	73·7648	73·8149	73·8647
84	73·9143	73·9636	74·0127	74·0615	74·1101	74·1585	74·2067	74·2546	74·3023	74·3498
85	74·3971	74·4441	74·4910	74·5376	74·5839	74·6301	74·6760	74·7217	74·7670	74·8123
86	74·8573	74·9022	74·9468	74·9912	75·0355	75·0795	75·1233	75·1669	75·2104	75·2536
87	75·2966	75·3395	75·3821	75·4246	75·4668	75·5089	75·5507	75·5924	75·6339	75·6752
88	75·7163	75·7572	75·7980	75·8385	75·8788	75·9190	75·9590	75·9988	76·0384	76·0778
89	76·1171	76·1562	76·1952	76·2339	76·2725	76·3109	76·3492	76·3873	76·4252	76·4629
90	76·5005	76·5379	76·5751	76·6121	76·6490	76·6857	76·7223	76·7588	76·7951	76·8312
91	76·8671	76·9029	76·9385	76·9739	77·0092	77·0444	77·0794	77·1142	77·1489	77·1835
92	77·2179	77·2522	77·2863	77·3203	77·3541	77·3878	77·4213	77·4547	77·4879	77·5210
93	77·5540	77·5868	77·6195	77·6520	77·6844	77·7167	77·7488	77·7807	77·8125	77·8442
94	77·9757	77·9071	77·9384	77·9695	78·0005	78·0314	78·0622	78·0929	78·1234	78·1538
95	78·1841	78·2142	78·2442	78·2741	78·3039	78·3335	78·3630	78·3924	78·4216	78·4508
96	78·4798	78·5087	78·5375	78·5622	78·5947	78·6231	78·6514	78·6796	78·7076	78·7356
97	78·7634	78·7911	78·8188	78·8463	78·8736	78·9009	78·9280	78·9551	78·9819	79·0087
98	79·0354	79·0621	79·0886	79·1150	79·1413	79·1675	79·1936	79·2195	79·2454	79·2712
99	79·2968	79·3224	79·3478	79·3731	79·3983	79·4234	79·4484	79·4734	79·4982	79·5230
100	79·5476	79·5722	79·5966	79·6210	79·6543	79·6695	79·6935	79·7175	79·7414	79·7652
101	79·7889	79·8124	79·8359	79·8593	79·8826	79·9058	79·9289	79·9519	79·9748	79·9976
102	80·0203	80·0430	80·0655	80·0879	80·1102	80·1324	80·1544	80·1763	80·1981	80·2197
103	80·2412	80·2625	80·2837	80·3048	80·3256	80·3462	80·3667	80·3869	80·4071	80·4270
104	80·4466	80·4661	80·4854	80·5045	80·5234	80·5420	80·5605	80·5787	80·5967	80·6145
105	80·6321	80·6495	80·6667	80·6835	80·7003	80·7169	80·7333	80·7495	80·7654	80·7813
106	80·7970	80·8126	80·8280	80·8432	80·8583	80·8733	80·8882	80·9029	80·9175	80·9319
107	80·9463	80·9606	80·9747	80·9886	81·0026	81·0164	81·0301	81·0437	81·0573	81·0707
108	81·0841	81·0973	81·1105	81·1236	81·1366	81·1495	81·1624	81·1751	81·1877	81·2003
109	81·2129	81·2253	81·2377	81·2501	81·2623	81·2745	81·2866	81·2986	81·3105	81·3224
110	81·3342	81·3460	81·3571	81·3695	81·3811	81·3927	81·4042	81·4156	81·4269	81·4382
111	81·4495	81·4607	81·4719	81·4829	81·4939	81·5049	81·5159	81·5268	81·5377	81·5485
112	81·5593	81·5700	81·5807	81·5913	81·6019	81·6124	81·6230	81·6334	81·6439	81·6543
113	81·6647	81·6750	81·6853	81·6955	81·7057	81·7159	81·7260	81·7361	81·7462	81·7562
114	81·7662	81·7761	81·7861	81·7960	81·8058	81·8156	81·8254	81·8351	81·8448	81·8545
115	81·8641	81·8737	81·8833	81·8929	81·9024	81·9119	81·9213	81·9307	81·9401	81·9495
116	81·9588	81·9681	81·9774	81·9866	81·9958	82·0049	82·0141	82·0232	82·0322	82·0413
117	82·0503	82·0592	82·0682	82·0771	82·0860	82·0948	82·1036	82·1124	82·1212	82·1299
118	82·1386	82·1473	82·1559	82·1645	82·1731	82·1817	82·1902	82·1988	82·2073	82·2157
119	82·2241	82·2325	82·2408	82·2492	82·2575	82·2657	82·2740	82·2822	82·2903	82·2985
120	82·3066	82·3147	82·3228	82·3309	82·3389	82·3469	82·3549	82·3629	82·3708	82·3787
121	82·3865	82·3944	82·4022	82·4100	82·4178	82·4255	82·4333	82·4410	82·4486	82·4563
122	82·4639	82·4715	82·4790	82·4865	82·4940	82·5015	82·5090	82·5164	82·5238	82·5312
123	82·5386	82·5459	82·5533	82·5606	82·5679	82·5751	82·5824	82·5896	82·5968	82·6040
124	82·6112	82·6183	82·6254	82·6324	82·6395	82·6465	82·6535	82·6605	82·6675	82·6744
125	82·6814	82·6883	82·6951	82·7019	82·7088	82·7156	82·7224	82·7291	82·7359	82·7427
126	82·7494	82·7561	82·7627	82·7694	82·7760	82·7826	82·7892	82·7957	82·8023	82·8088
127	82·8153	82·8218	82·8283	82·8348	82·8412	82·8477	82·8541	82·8604	82·8668	82·8731
128	82·8794	82·8857	82·8920	82·8983	82·9045	82·9107	82·9169	82·9231	82·9292	82·9354
129	82·9415	82·9477	82·9538	82·9599	82·9660	82·9720	82·9780	82·9840	82·9900	82·9960
130	83·0019	83·0079	83·0138	83·0197	83·0256	83·0315	83·0373	83·0432	83·0490	83·0548
131	83·0606	83·0664	83·0721	83·0779	83·0836	83·0893	83·0950	83·1007	83·1063	83·1119
132	83·1176	83·1232	83·1288	83·1344	83·1400	83·1455	83·1511	83·1566	83·1621	83·1676
133	83·1730	83·1785	83·1840	83·1894	83·1949	83·2003	83·2057	83·2110	83·2164	83·2217
134	83·2271	83·2324	83·2377	83·2430	83·2483	83·2536	83·2588	83·2641	83·2693	83·2745
135	83·2797	83·2849	83·2900	83·2951	83·3003	83·3054	83·3105	83·3156	83·3207	83·3257
136	83·3308	83·3359	83·3409	83·3459	83·3509	83·3560	83·3609	83·3659	83·3709	83·3759
137	83·3808	83·3857	83·3906	83·3955	83·4004	83·4053	83·4101	83·4150	83·4198	83·4247
138	83·4295	83·4343	83·4391	83·4438	83·4486	83·4533	83·4581	83·4628	83·4676	83·4723
139	83·4770	83·4817	83·4863	83·4910	83·4956	83·5003	83·5049	83·5095	83·5141	83·5187
140	83·5233	83·5279	83·5325	83·5371	83·5417	83·5462	83·5507	83·5553	83·5598	83·5642
141	83·5687	83·5732	83·5777	83·5821	83·5866	83·5910	83·5954	83·5999	83·6043	83·6087

Table VIII.—*continued.*

Ballistic Table.

Inclination (δ in degrees) and velocity. $\frac{d^2}{w}\delta = D_V - D_v$.

v.	0	1	2	3	4	5	6	7	8	9
f.s.	degs.	degs.	degs.	degs.	degs.	degs.	degs.	degs.	degs.	degs.
142	83·6130	83·6174	83·6218	83·6261	83·6305	83·6348	83·6392	83·6435	83·6478	83·6522
143	83·6565	83·6607	83·6650	83·6693	83·6735	83·6778	83·6820	83·6862	83·6904	83·6946
144	83·6988	83·7030	83·7072	83·7114	83·7156	83·7197	83·7239	83·7280	83·7321	83·7362
145	83·7403	83·7444	83·7485	83·7526	83·7567	83·7608	83·7649	83·7689	83·7730	83·7770
146	83·7810	83·7850	83·7891	83·7930	83·7970	83·8010	83·8050	83·8090	83·8130	83·8170
147	83·8209	83·8249	83·8288	83·8327	83·8366	83·8406	83·8445	83·8484	83·8522	83·8561
148	83·8600	83·8639	83·8677	83·8715	83·8754	83·8792	83·8830	83·8869	83·8907	83·8945
149	83·8983	83·9021	83·9059	83·9096	83·9134	83·9172	83·9209	83·9247	83·9285	83·9322
150	83·9359	83·9396	83·9433	83·9470	83·9507	83·9544	83·9581	83·9617	83·9654	83·9691
151	83·9727	83·9764	83·9800	83·9837	83·9873	83·9909	83·9946	83·9982	84·0018	84·0054
152	84·0090	84·0126	84·0161	84·0197	84·0233	84·0269	84·0304	84·0340	84·0375	84·0410
153	84·0446	84·0481	84·0516	84·0551	84·0587	84·0622	84·0657	84·0692	84·0727	84·0762
154	84·0796	84·0831	84·0866	84·0900	84·0935	84·0969	84·1004	84·1038	84·1072	84·1106
155	84·1140	84·1174	84·1208	84·1242	84·1276	84·1310	84·1344	84·1378	84·1412	84·1445
156	84·1479	84·1513	84·1546	84·1579	84·1613	84·1646	84·1679	84·1713	84·1746	84·1779
157	84·1812	84·1845	84·1878	84·1911	84·1943	84·1976	84·2009	84·2041	84·2074	84·2107
158	84·2139	84·2172	84·2204	84·2237	84·2269	84·2301	84·2333	84·2366	84·2398	84·2430
159	84·2461	84·2493	84·2525	84·2557	84·2588	84·2620	84·2652	84·2683	84·2715	84·2746
160	84·2778	84·2809	84·2840	84·2871	84·2902	84·2933	84·2965	84·2996	84·3027	84·3058
161	84·3088	84·3119	84·3150	84·3180	84·3210	84·3242	84·3272	84·3302	84·3333	84·3363
162	84·3394	84·3424	84·3454	84·3484	84·3514	84·3544	84·3574	84·3604	84·3634	84·3664
163	84·3694	84·3724	84·3753	84·3783	84·3813	84·3843	84·3872	84·3902	84·3931	84·3960
164	84·3990	84·4019	84·4048	84·4078	84·4107	84·4136	84·4165	84·4194	84·4223	84·4252
165	84·4281	84·4310	84·4339	84·4367	84·4396	84·4425	84·4453	84·4482	84·4510	84·4539
166	84·4567	84·4595	84·4624	84·4652	84·4680	84·4709	84·4737	84·4765	84·4793	84·4821
167	84·4849	84·4877	84·4905	84·4933	84·4961	84·4988	84·5016	84·5044	84·5070	84·5099
168	84·5127	84·5154	84·5181	84·5209	84·5236	84·5263	84·5291	84·5318	84·5345	84·5372
169	84·5399	84·5426	84·5453	84·5480	84·5508	84·5534	84·5561	84·5588	84·5615	84·5641
170	84·5668	84·5695	84·5721	84·5748	84·5775	84·5801	84·5828	84·5854	84·5880	84·5907
171	84·5933	84·5959	84·5985	84·6012	84·6038	84·6064	84·6090	84·6116	84·6142	84·6168
172	84·6193	84·6219	84·6245	84·6271	84·6297	84·6322	84·6348	84·6373	84·6399	84·6424
173	84·6449	84·6475	84·6500	84·6525	84·6550	84·6575	84·6601	84·6626	84·6651	84·6676
174	84·6701	84·6726	84·6750	84·6776	84·6800	84·6825	84·6850	84·6875	84·6899	84·6924
175	84·6948	84·6973	84·6997	84·7022	84·7046	84·7071	84·7095	84·7119	84·7144	84·7168
176	84·7192	84·7216	84·7240	84·7264	84·7288	84·7312	84·7336	84·7360	84·7384	84·7408
177	84·7432	84·7455	84·7479	84·7503	84·7526	84·7550	84·7574	84·7597	84·7621	84·7645
178	84·7668	84·7692	84·7715	84·7739	84·7762	84·7785	84·7809	84·7832	84·7855	84·7878
179	84·7902	84·7925	84·7948	84·7972	84·7994	84·8017	84·8040	84·8063	84·8086	84·8109
180	84·8131	84·8154	84·8177	84·8199	84·8222	84·8244	84·8267	84·8289	84·8312	84·8334
181	84·8357	84·8379	84·8401	84·8424	84·8446	84·8468	84·8490	84·8513	84·8535	84·8557
182	84·8579	85·8601	84·8623	84·8645	84·8667	84·8689	84·8711	84·8732	84·8754	84·8776
183	84·8798	84·8819	84·8841	84·8863	84·8884	84·8906	84·8927	84·8949	84·8970	84·8992
184	84·9013	84·9035	84·9056	84·9077	84·9099	84·9120	84·9141	84·9162	84·9184	84·9205
185	84·9226	84·9476	84·9268	84·9289	84·9310	84·9331	84·9351	84·9372	84·9393	84·9414
186	84·9435	84·9456	84·9476	84·9497	84·9518	84·9538	84·9559	84·9580	84·9600	84·9621
187	84·9641	84·9662	84·9682	84·9702	84·9723	84·9743	84·9763	84·9784	84·9804	84·9824
188	84·9845	84·9865	84·9885	84·9905	84·9925	84·9946	84·9966	84·9986	85·0006	85·0026
189	85·0045	85·0065	85·0085	85·0105	85·0125	85·0145	85·0165	85·0185	85·0204	85·0224
190	85·0244	85·0263	85·0283	85·0303	85·0322	85·0342	85·0361	85·0380	85·0400	85·0419
191	85·0438	85·0458	85·0477	85·0496	85·0515	85·0535	85·0554	85·0573	85·0592	85·0611
192	85·0630	85·0650	85·0669	85·0687	85·0706	85·0725	85·0744	85·0763	85·0782	85·0801
193	85·0820	85·0838	85·0857	85·0876	85·0895	85·0913	85·0932	85·0951	85·0969	85·0988
194	85·1006	85·1025	85·1043	85·1062	85·1080	85·1099	85·1117	85·1136	85·1154	85·1172
195	85·1190	85·1208	85·1227	85·1245	85·1263	85·1281	85·1299	85·1317	85·1335	85·1353
196	85·1371	85·1389	85·1407	85·1425	85·1443	85·1460	85·1478	85·1496	85·1514	85·1531
197	85·1549	85·1567	85·1584	85·1602	85·1619	85·1637	85·1654	85·1672	85·1689	85·1707
198	85·1724	85·1741	85·1759	85·1776	85·1793	85·1810	85·1827	85·1844	85·1862	85·1879
199	85·1896	85·1913	85·1930	85·1947	85·1964	85·1981	85·1998	85·2014	85·2031	85·2048
200	85·2065	85·2081	85·2098	85·2115	85·2131	85·2148	85·2165	85·2181	85·2198	85·2214
201	85·2231	85·2247	85·2264	85·2280	85·2296	85·2313	85·2329	85·2346	85·2362	85·2378

Table VIII.—*continued.*

Ballistic Table.

Inclination (δ in degrees) and velocity. $\dfrac{d^2}{w}\delta = D_V - D_v$.

v.	6	1	2	3	4	5	6	7	8	9
f.s.	degs.	degs.	degs.	degs.	degs.	degs.	degs.	degs.	degs.	degs.
202	85·2394	85·2411	85·2427	85·2443	85·2459	85·2476	85·2492	85·2507	85·2524	85·2540
203	85·2556	85·2572	85·2588	85·2604	85·2620	85·2635	85·2651	85·2667	85·2682	85·2698
204	85·2714	85·2729	85·2745	85·2760	85·2776	85·2791	85·2807	85·2822	85·2838	85·2853
205	85·2868	85·2884	85·2899	85·2915	85·2930	85·2945	85·2960	85·2975	85·2990	85·3005
206	85·3020	85·3035	85·3051	85·3066	85·3081	85·3095	85·3110	85·3125	85·3140	85·3155
207	85·3170	85·3184	85·3199	85·3214	85·3229	85·3244	85·3258	85·3273	85·3287	85·3302
208	85·3316	85·3331	85·3345	85·3360	85·3373	85·3388	85·3403	85·3417	85·3431	85·3446
209	85·3460	85·3474	85·3488	85·3503	85·3517	85·3531	85·3545	85·3559	85·3573	85·3587
210	85·3601	85·3615	85·3629	85·3643	85·3657	85·3671	85·3685	85·3698	85·3712	85·3726
211	85·3740	85·3754	85·3767	85·3781	85·3795	85·3808	85·3822	85·3836	85·3849	85·3863
212	85·3876	85·3890	85·3903	85·3917	85·3930	85·3943	85·3957	85·3970	85·3983	85·3996
213	85·4010	85·4023	85·4036	85·4049	85·4063	85·4076	85·4089	85·4102	85·4115	85·4128
214	85·4141	85·4154	85·4167	85·4180	85·4913	85·4206	85·4219	85·4232	85·4245	85·4258
215	85·4271	85·4284	85·4297	85·4309	85·4322	85·4335	85·4348	85·4360	85·4373	85·4385
216	85·4398	85·4411	85·4423	85·4436	85·4448	85·4461	85·4473	85·4485	85·4498	85·4510
217	85·4523	85·4535	85·4547	85·4560	85·4572	85·4584	85·4597	85·4609	85·4621	85·4633
218	85·4645	85·4658	85·4670	85·4682	85·4694	85·4706	85·4718	85·4730	85·4742	85·4754
219	85·4766	85·4778	85·4790	85·4802	85·4814	85·4825	85·4837	85·4849	85·4861	85·4873
220	85·4885	85·4896	85·4908	85·4920	85·4932	85·4943	85·4955	85·4967	85·4978	85·4990
221	85·5001	85·5013	85·5024	85·5036	85·5047	85·5059	85·5070	85·5082	85·5093	85·5105
222	85·5116	85·5128	85·5139	85·5150	85·5162	85·5173	85·5184	85·5195	85·5207	85·5218
223	85·5229	85·5240	85·5251	85·5262	85·5273	85·5285	85·5296	85·5307	85·5318	85·5329
224	85·5340	85·5351	85·5362	85·5373	85·5384	85·5394	85·5405	85·5416	85·5427	85·5438
225	85·5449	85·5460	85·5470	85·5481	85·5492	85·5502	85·5513	85·5524	85·5534	85·5545
226	85·5556	85·5566	85·5577	85·5588	85·5598	85·5609	85·5619	85·5630	85·5640	85·5651
227	85·5661	85·5672	85·5682	85·5693	85·5703	85·5713	85·5724	85·5734	85·5744	85·5755
228	85·5765	85·5775	85·5785	85·5796	85·5806	85·5816	85·5826	85·5836	85·5846	85·5856
229	85·5866	85·5876	85·5886	85·5896	85·5906	85·5916	85·5926	85·5936	85·5946	85·5956
230	85·5966	85·5976	85·5986	85·5996	85·6006	85·6015	85·6025	85·6035	85·6045	85·6055
231	85·6064	85·6074	85·6084	85·6094	85·6103	85·6113	85·6123	85·6132	85·6142	85·6151
232	85·6161	85·6171	85·6180	85·6190	85·6199	85·6209	85·6218	85·6228	85·6237	85·6247
233	85·6256	85·6265	85·6275	85·6284	85·6294	85·6303	85·6312	85·6321	85·6331	85·5340
234	85·6349	85·6358	85·6367	85·6377	85·6386	85·6395	85·6404	85·6413	85·6422	85·6431
235	85·6441	85·6450	85·6459	85·6468	85·6477	85·6486	85·6495	85·6504	85·6513	85·6522
236	85·6531	85·6540	85·6549	85·6558	85·6566	85·6575	85·6584	85·6593	85·6602	85·6611
237	85·6619	85·6628	85·6637	85·6646	85·6654	85·6663	85·6672	85·6680	85·6689	85·6698
238	85·6706	85·6715	85·6724	85·6732	85·6741	85·6749	85·6758	85·6766	85·6775	85·6783
239	85·6792	85·6800	85 6809	85·6817	85·6826	85·6834	85·6843	85·6851	85·6859	85·6868
240	85·6876	85·6885	85·6893	85·6901	85·6909	85·6918	85·6926	85·6934	85·6942	85·6951
241	85·6959	85·6967	85·6975	85·6984	85·6992	85·7000	85·7008	85·7016	85·7024	85·7032
242	85·7041	85·7049	85·7057	85·7065	85·7073	85·7081	85·7089	85·7097	85·7105	85·7113
243	85·7121	85·7128	85·7136	85·7144	85·7152	85·7160	85·7168	85·7176	85·7184	85·7192
244	85·7200	85·7207	85·7215	85·7223	85·7231	85·7239	85·7246	85·7254	85·7262	85·7270
245	85·7277	85·7285	85·7293	85·7301	85·7308	85·7316	85·7324	85·7331	85·7339	85·7346
246	85·7354	85·7362	85·7369	85·7377	85·7384	85·7392	85·7399	85·7407	85·7414	85·7422
247	85·7429	85·7436	85·7444	85·7451	85·7459	85·7466	85·7474	85·7481	85·7488	85·7496
248	85·7503	85·7510	85·7517	85·7525	85·7532	85·7539	85·7547	85·7554	85·7561	85·7568
249	85·7575	85·7583	85·7590	85·7597	85·7604	85·7611	85·7618	85·7626	85·7633	85·7640

TABLE IX.

Abridged Ballistic Table.

For elongated projectiles, heads $1\frac{1}{2}$ diameters radius.
(Recalculated by Prof. Greenhill from Bashforth's data.)

v	ΔT	T_v	ΔS	S_v	ΔD	D_v
f.s.						
100	15·6890	0·0000	1647·35	0·00	—	—
110	13·0823	15·6890	1504·46	1647·35	—	—
120	11·0703	28·7713	1383·78	3151·81	—	—
130	9·4919	39·8416	1281·40	4535·59	—	—
140	8·2272	49·3335	1192·94	5816·99	—	—
150	7·2013	57·5606	1116·21	7009·93	—	—
160	6·3531	64·7620	1048·26	8126·14	—	—
170	5·6474	71·1150	988·29	9174·40	—	—
180	5·0540	76·7624	934·99	10162·69	—	—
190	4·5485	81·8164	886·96	11097·68	—	—
200	4·1161	86·3649	843·81	11984·64	—	—
210	3·7419	90·4810	804·51	12828·45	—	—
220	3·4173	94·2230	768·90	13632·96	—	—
230	3·1323	97·6403	736·09	14401·86	—	—
240	2·8813	100·7726	705·92	15137·95	—	—
250	2·6603	103·6539	678·37	15843·87	—	—
260	2·4627	106·3142	652·61	16522·24	—	—
270	2·2875	108·7768	629·07	17174·85	—	—
280	2·1301	111·0644	607·08	17803·92	—	—
290	1·9884	113·1945	586·57	18411·00	—	—
300	1·8589	115·1828	566·97	18997·57	—	—
310	1·7426	117·0418	548·92	19564·54	—	—
320	1·6356	118·7844	531·58	20113·46	—	—
330	1·5375	120·4200	515·07	20645·04	—	—
340	1·4470	121·9575	499·20	21160·11	—	—
350	1·3638	123·4045	484·13	21659·31	—	—
360	1·2861	124·7682	469·42	22143·44	—	—
370	1·2148	126·0543	455·55	22612·86	—	—
380	1·1476	127·2691	441·81	23068·41	—	—
390	1·0853	128·4167	428·71	23510·22	—	—
400	1·0268	129·5020	415·87	23938·93	4·6763	0·0000
410	0·9723	130·5289	403·50	24354·80	4·3212	4·6763
420	0·9226	131·5012	392·09	24758·30	4·0037	8·9975
430	0·8772	132·4237	381·57	25150·39	3·7192	13·0012
440	0·8350	133·3009	371·59	25531·96	3·4610	16·7203
450	0·7964	134·1359	362·37	25903·55	3·2283	20·1812
460	0·7610	134·9323	353·85	26265·92	3·0183	23·4096
470	0·7278	135·6933	345·72	26619·77	2·8261	26·4279
480	0·6973	136·4211	338·21	26965·49	2·6519	29·2540

EXPLANATION.—Velocities are to be found under the heading v. The abridgements of the tables of $\frac{d^2}{w}t$, $\frac{d^2}{w}s$, and $\frac{d^2}{v}\delta$, are in the 1st, 2nd, and 3rd double columns respectively, the tabulated quantities being on the right, and the difference for 10 feet velocity being on the left in each case.

Thus find the tabulated time, distance, and inclination in degrees corresponding to 500 f.s. They are found to be respectively 137·7877 seconds, 27634·97 feet, and 34·3995 degrees respectively. Proportional parts must be taken when the velocity is not an exact multiple of 10; thus find the time, distance, and inclination for 624·4 f.s.; they are respectively—

$$144{\cdot}1599 + 0{\cdot}24 \times 0{\cdot}4271 = 144{\cdot}2624 \text{ seconds},$$
$$31177{\cdot}19 + 0{\cdot}24 \times 266{\cdot}95 = 31241{\cdot}26 \text{ feet},$$
$$55{\cdot}6232 + 0{\cdot}24 \times 1{\cdot}2604 = 55{\cdot}9257 \text{ degrees}.$$

In a reverse manner, if the tabulated quantities are given the corresponding velocities can be found. thus find the velocities corresponding to—

	145·3050 seconds,	31777·62 feet, and	58·1233 degrees;
the nearest tabulated quantities which are *less* are	145·0012 seconds,	31707·13 feet, and	58·0866 degrees;
the original quantities differ from these by	0·3038 seconds,	70·49 feet, and	0·0367 degrees;

the velocities consequently are respectively—

$$640 + 10 \times \frac{0{\cdot}3038}{0{\cdot}4020} = 647{\cdot}6 \text{ f.s.}$$

$$640 + 10 \times \frac{70{\cdot}49}{259{\cdot}30} = 642{\cdot}7 \text{ f.s.}$$

$$640 + 10 \times \frac{0{\cdot}0367}{1{\cdot}1496} = 640{\cdot}3 \text{ f.s.}$$

The values of ΔD and D_v are not given for velocities less than 400 f.s., as with low velocities the trajectory is very much curved, and this approximation is not then applicable.

TABLE IX.

Abridged Ballistic Table—*continued.*

v	ΔT	T_v	ΔS	S_v	ΔD	D_v
f s.						
490	0·6692	137·1185	331·27	27303·70	2·4936	31·9059
500	0·6433	137·7877	324·87	27634·97	2·3495	34·3995
510	0·6183	138·4310	318·45	27959·84	2·2145	36·7490
520	0·5952	139·0493	312·50	28278·29	2·0912	38·9635
530	0·5733	139·6446	306·74	28590·79	1·9766	41·0547
540	0·5526	140·2179	301·14	28897·53	1·8699	43·0312
550	0·5332	140·7705	295·94	29198·67	1·7721	44·9012
560	0·5148	141·3037	290·86	29494·61	1·6805	46·6732
570	0·4977	141·8185	286·15	29785·47	1·5963	48·3538
580	0·4821	142·3162	282·05	30071·62	1·5201	49·9500
590	0·4677	142·7983	278·32	30353·67	1·4498	51·4701
600	0·4539	143·2660	274·58	30631·99	1·3836	52·9200
610	0·4400	143·7199	270·62	30906·57	1·3197	54·3036
620	0·4271	144·1599	266·95	31177·19	1·2604	55·6232
630	0·4142	144·5870	262·99	31444·14	1·2030	56·8836
640	0·4020	145·0012	259·30	31707·13	1·1496	58·0866
650	0·3902	145·4032	255·58	31966·43	1·0987	59·2362
660	0·3787	145·7934	251·81	32222·01	1·0503	60·3349
670	0·3678	146·1720	248·28	32473·82	1·0051	61·3852
680	0·3576	146·5399	244·96	32722·10	0·9629	62·3902
690	0·3480	146·8975	241·86	32967·06	0·9235	63·3531
700	0·3385	147·2455	238·67	33208·92	0·8857	64·2766
710	0·3292	147·5840	235·39	33447·59	0·8492	65·1623
720	0·3196	147·9132	231·73	33682·98	0·8131	66·0115
730	0·3105	148·2328	228·25	33914·71	0·7793	66·8246
740	0·3019	148·5434	224·93	34142·96	0·7475	67·6039
750	0·2938	148·8453	221·78	34367·89	0·7176	68·3514
760	0·2823	149·1391	218·23	34589·67	0·6878	69·0690
770	0·2765	149·4243	214·28	34807·90	0·6580	69·7568
780	0·2681	149·7008	210·48	35022·18	0·6300	70·4148
790	0·2602	149·9689	206·83	35232·66	0·6036	71·0447
800	0·2526	150·2291	203·31	35439·49	0·5787	71·6483
810	0·2450	150·4817	199·67	35642·80	0·5544	72·2270
820	0·2371	150·7266	195·64	35842·47	0·5302	72·7814
830	0·2290	150·9638	191·23	36038·11	0·5047	73·3116
840	0·2210	151·1928	186·73	36229·34	0·4824	73·8163
850	0·2133	151·4138	182·39	36416·07	0·4602	74·2986
860	0·2060	151·6271	178·20	36598·46	0·4393	74·7588
870	0·1990	151·8331	174·15	36776·66	0·4195	75·1981
880	0·1924	152·0322	170·24	36950·81	0·4009	75·6176
890	0·1860	152·2245	166·43	37121·05	0·3833	76·0185
900	0·1799	152·4105	162·80	37287·48	0·3666	76·4018
910	0·1741	152·5904	159·26	37450·28	0·3508	76·7684
920	0·1685	152·7644	155·83	37609·54	0·3359	77·1192
930	0·1631	152·9329	152·52	37765·37	0·3218	77·4551
940	0·1580	153·0960	149·31	37917·89	0·3091	77·7769
950	0·1531	153·2540	146·20	38067·20	0·2957	78·0860
960	0·1484	153·4071	143·18	38213·40	0·2836	78·3816
970	0·1439	153·5555	140·26	38356·58	0·2721	78·6652
980	0·1395	153·6993	137·43	38496·84	0·2613	78·9374
990	0·1354	153·8388	134·68	38634·27	0·2509	79·1986
1000	0·1314	153·9742	132·01	38768·95	0·2411	79·4495
1010	0·1272	154·1055	129·08	38900·96	0·2311	79·6906
1020	0·1227	154·2327	125·74	39030·04	0·2207	79·9217
1030	0·1150	154·3554	119·07	39155·78	0·2050	80·1424
1040	0·1046	154·4704	109·28	39274·85	0·1846	80·3474
1050	0·0938	154·5750	98·95	39384·13	0·1640	80·5320
1060	0·0857	154·6688	91·27	39483·08	0·1484	80·6959
1070	0·0800	154·7545	86·02	39574·35	0·1373	80·8443
1080	0·0755	154·8345	81·91	39660·37	0·1283	80·9816

Table IX.

Abridged Ballistic Table—*continued.*

v	ΔT	T_v	ΔS	S_v	ΔD	D_v
f.s.						
1090	0·0719	154·9100	78·75	39742·28	0·1211	81·1100
1100	0·0688	154·9819	75·97	39821·03	0·1148	81·2311
1110	0·0663	155·0507	73·93	39897·00	0·1097	81·3459
1120	0·0641	155·1170	72·16	39970·93	0·1052	81·4555
1130	0·0624	155·1811	70·83	40043·09	0·1014	81·5607
1140	0·0608	155·2435	69·60	40113·92	0·0979	81·6621
1150	0·0592	155·3043	68·40	40183·52	0·0946	81·7600
1160	0·0577	155·3635	67·23	40251·92	0·0914	81·8546
1170	0·0562	155·4212	66·09	40319·15	0·0883	81·9459
1180	0·0548	155·4775	64·98	40385·24	0·0853	82·0342
1190	0·0535	155·5323	63·89	40450·22	0·0825	82·1196
1200	0·0522	155·5858	62·84	40514·11	0·0798	82·2021
1210	0·0509	155·6379	61·81	40576·95	0·0772	82·2819
1220	0·0496	155·6888	60·80	40638·76	0·0747	82·3591
1230	0·0485	155·7384	59·88	40699·56	0·0724	82·4339
1240	0·0473	155·7869	58·92	40759·44	0·0701	82·5673
1250	0·0462	155·8342	58·04	40818·36	0·0680	82·5764
1260	0·0452	155·8805	57·17	40876·40	0·0659	82·6443
1270	0·0442	155·9257	56·38	40933·57	0·0640	82·7102
1280	0·0433	155·9699	55·61	40989·95	0·0621	82·7742
1290	0·0424	156·0132	54·86	41045·56	0·0603	82·8363
1300	0·0415	156·0555	54·12	41100·42	0·0586	82·8967
1310	0·0406	156·0970	53·45	41154·54	0·0570	82·9553
1320	0·0398	156·1377	52·79	41207·99	0·0555	83·0123
1330	0·0391	156·1775	52·15	41260·78	0·0540	83·0677
1340	0·0383	156·2165	51·52	41312·93	0·0525	83·1217
1350	0·0376	156·2548	50·95	41364·45	0·0512	83·1742
1360	0·0369	156·2924	50·39	41415·40	0·0499	83·2254
1370	0·0363	156·3294	49·90	41465·79	0·0487	83·2753
1380	0·0357	156·3656	49·41	41515·69	0·0475	83·3240
1390	0·0351	156·4013	48·94	41565·10	0·0464	83·3715
1400	0·0345	156·4364	48·52	41614·04	0·0453	83·4179
1410	0·0340	156·4709	48·12	41662·56	0·0443	83·4632
1420	0·0335	156·5049	47·72	41710·68	0·0433	83·5075
1430	0·0330	156·5384	47·33	41758·40	0·0424	83·5509
1440	0·0325	156·5714	47·00	41805·73	0·0415	83·5933
1450	0·0321	156·6039	46·68	41852·73	0·0407	83·6348
1460	0·0317	156·6360	46·36	41899·41	0·0398	83·6754
1470	0·0313	156·6677	46·10	41945·77	0·0391	83·7253
1480	0·0309	156·6989	45·80	41991·87	0·0383	83·7544
1490	0·0305	156·7298	45·52	42037·67	0·0376	83·7927
1500	0·0301	156·7602	45·28	42083·19	0·0369	83·8302
1510	0·0298	156·7903	45·10	42128·47	0·0362	83·8671
1520	0·0295	156·8201	44·93	42173·57	0·0356	83·9033
1530	0·0292	156·8496	44·77	42218·50	0·0350	83·9390
1540	0·0289	156·8787	44·57	42263·27	0·0344	83·9740
1550	0·0285	156·9076	44·37	42307·84	0·0339	84·0086
1560	0·0282	156·9361	44·19	42352·21	0·0333	84·0423
1570	0·0279	156·9643	44·01	42396·40	0·0327	84·0756
1580	0·0277	156·9923	43·84	42440·41	0·0322	84·1083
1590	0·0274	157·0199	43·68	42484·25	0·3217	84·1405
1600	0·0271	157·0473	43·47	42527·93	0·0311	84·1721
1610	0·0268	157·0744	43·27	42571·40	0·0306	84·2033
1620	0·0265	157·1012	43·08	42614·67	0·0301	84·2339
1630	0·0262	157·1277	42·90	42657·75	0·0296	84·2640
1640	0·0260	157·1539	42·72	42700·65	0·0291	84·2936
1650	0·0257	157·1799	42·55	42743·37	0·0287	84·3227
1660	0·0255	157·2056	42·39	42785·92	0·0282	84·3513
1670	0·0252	157·2311	42·18	42828·31	0·0277	84·3795
1680	0·0249	157·2563	41·98	42870·49	0·0273	84·4073

Table IX.

Abridged Ballistic Table—*continued.*

v	ΔT	T_v	ΔS	S_v	ΔD	D_v
f.s.						
1690	0·0247	157·2812	41·78	42912·47	0·0268	84·4345
1700	0·0244	157·3058	41·60	42954·25	0·0264	84·4613
1710	0·0242	157·3302	41·41	42995·85	0·0260	84·4877
1720	0·0239	157·3544	41·23	43037·26	0·0256	84·5137
1730	0·0237	157·3783	41·06	43078·49	0·0252	84·5393
1740	0·0234	157·4019	40·90	43119·55	0·0248	84·5644
1750	0·0232	157·4254	40·69	43160·45	0·0244	84·5892
1760	0·0230	157·4486	40·53	43201·14	0·0240	84·6136
1770	0·0227	157·4715	40·33	43241·67	0·0236	84·6376
1780	0·0225	157·4942	40·19	43282·00	0·0233	84·6612
1790	0·0223	157·5168	40·00	43322·19	0·0229	84·6844
1800	0·0221	157·5390	39·81	43362·19	0·0225	84·7073
1810	0·0219	157·5611	39·68	43402·00	0·0222	84·7299
1820	0·0217	157·5830	39·51	43441·68	0·0219	84·7521
1830	0·0214	157·6046	39·34	43481·19	0·0216	84 7740
1840	0·0212	157·6260	39·17	43520·53	0·0212	84·7955
1850	0·0210	157·6473	39·01	43559·70	0·0209	84·8167
1860	0·0209	157·6683	38·90	43598·71	0·0206	84·8376
1870	0·0207	157·6892	38·75	43637·61	0·0203	84·8583
1880	0·0205	157·7098	38·61	43676·36	0·0200	84·8786
1890	0·0203	157·7303	38·46	43714·97	0·0198	84·8986
1900	0·0201	157·7506	38·32	43753·43	0·0195	84·9184
1910	0·0199	157·7707	38·19	43791·75	0·0192	84·9379
1920	0·0197	157·7907	38·01	43829·94	0·0189	84·9571
1930	0·0196	157·8104	37·83	43867·95	0·0186	84·9760
1940	0·0194	157·8300	37·66	43905·78	0·0184	84·9946
1950	0·0192	157·8493	37·48	43943·44	0·0181	85·0130
1960	0·0190	157·8685	37·26	43980·92	0·0178	85·0311
1970	0·0187	157·8875	36·99	44018·18	0·0175	85·0489
1980	0·0185	157·9062	36·73	44055·17	0·0172	85·0664
1990	0·0183	157·9247	36·47	44091·90	0·0169	85·0836
2000	0·0181	157·9430	36·21	44128·37	0·0166	85·1005
2010	0·0178	157·9610	35·95	44164·58	0·0163	85·1171
2020	0·0176	157·9789	35·65	44200·53	0·0160	85·1334
2030	0·0174	157·9965	35·35	44236·18	0·0158	85·1494
2040	0·0171	158·0138	35·06	44271·53	0·0155	85·1652
2050	0·0169	158·0310	34·77	44306·59	0·0152	85·1807
2060	0·0167	158·0479	34·49	44341·36	0·0149	85·1958
2070	0·0165	158·0646	34·21	44375·85	0·0147	85·2108
2080	0·0163	158·0811	33·93	44410·06	0·0144	85·2254
2090	0·0160	158·0974	33·60	44443·99	0·0141	85·2398
2100	0·0158	158·1134	33·34	44477·59	0·0139	85·2539
2110	0·0156	158·1292	33·02	44510·93	0·0139	85·2678
2120	0·0154	158·1448	32·76	44543·95	0·0134	85·2814
2130	0·0152	158·1603	32·50	44576·71	0·0132	85·2948
2140	0·0150	158·1755	32·25	44609·21	0·0129	85·3080
2150	0·0149	158·1905	32·00	44641·46	0·0127	85·3209
2160	0·0147	158·2054	31·75	44673·46	0·0125	85·3336
2170	0·0145	158·2200	31·46	44705·21	0·0123	85·3461
2180	0·0143	158·2345	31·22	44736·67	0·0121	85·3583
2190	0·0141	158·2488	30·98	44767·89	0·0119	85·3704
2200	0·0139	158·2629	30·74	44798·87	0·0117	85·3823
2210	0·0138	158·2768	30·51	44829·61	0·0115	85·3939
2220	0·0136	158·2906	30·23	44860·12	0·0113	85·4054
2230	0·0134	158·3042	30·01	44890·35	0·0111	85·4167
2240	0·0133	158·3176	29·79	44920·36	0·0109	85·4277
2250	0·0131	158·3309	29·53	44950·15	0·0107	85·4386
2260	0·0130	158·3440	29·31	44979·68	0·0105	85·4494
2270	0·0128	158·3569	29·14	45008·99	0·0104	85·4599
2280	0·0127	158·3697	28·98	45038·13	0·0102	85·4701

TABLE IX.

Abridged Ballistic Table—*continued.*

v	ΔT	T_v	ΔS	S_v	ΔD	D_v
f.s.						
2290	0·0126	158·3824	28·90	45067·11	0·0101	85·4805
2300	0·0125	158·3950	28·82	45096·01	0·0100	85·4906
2310	0·0125	158·4075	28·84	45124·83	0·0099	85·5006
2320	0·0124	158·4200	28·85	45153·67	0·0098	85·5106
2330	0·0124	158·4324	28·88	45182·52	0·0098	85·5203
2340	0·0123	158·4447	28·91	45211·40	0·0097	85·5301
2350	0·0123	158·4571	28·94	45240·31	0·0096	85·5398
2360	0·0123	158·4694	28·98	45269·25	0·0096	85·5494
2370	0·0122	158·4816	29·02	45298·23	0·0095	85·5589
2380	0·0122	158·4938	29·06	45327·25	0·0094	85·5684
2390	0·0122	158·5060	29·10	45356·31	0·0094	85·5778
2400	0·0121	158·5182	29·16	45385·41	0·0093	85·5872
2410	0·0121	158·5303	29·21	45414·57	0·0092	85·5965
2420	0·0121	158·5424	29·27	45443·78	0·0092	85·6057
2430	0·0121	158·5545	29·33	45473·05	0·0091	85·6149
2440	0·0120	158·5665	29·35	45502·38	0·0091	85·6240
2450	0·0120	158·5785	29·37	45531·73	0·0090	85·6331
2460	0·0119	158·5905	29·39	45561·10	0·0089	85·6420
2470	0·0119	158·6024	29·41	45590·49	0·0089	85·6510
2480	0·0119	158·6143	29·44	45619·90	0·0088	85·6598
2490	0·0118	158·6261	29·42	45649·34	0·0087	85·6686
2500	0·0117	158·6379	29·40	45678·76	0·0086	85·6773
2510	0·0117	158·6496	29·39	45708·16	0·0086	85·6859
2520	0·0116	158·6613	29·32	45737·55	0·0085	85·6945
2530	0·0115	158·6729	29·25	45766·87	0·0084	85·7030
2540	0·0115	158·6845	29·13	45796·12	0·0083	85·7114
2550	0·0114	158·6959	29·01	45825·25	0·0082	85·7197
2560	0·0112	158·7073	28·84	45854·26	0·0081	85·7279
2570	0·0111	158·7185	28·67	45883·10	0·0080	85·7359
2580	0·0110	158·7297	28·50	44911·77	0·0079	85·7439
2590	0·0109	158·7407	28·29	45940·27	0·0077	85·7518
2600	0·0108	158·7516	28·12	45968·56	0·0076	85·7595
2610	0·0107	158·7624	27·91	45996·68	0·0075	85·7671
2620	0·0106	158·7731	27·70	46024·59	0·0074	85·7746
2630	0·0105	158·7837	27·54	46052·29	0·0073	85·7820
2640	0·0103	158·7942	27·33	46079·83	0·0072	85·7893
2650	0·0102	158·8045	27·18	46107·16	0·0071	85·7965
2660	0·0101	158·8147	26·97	46134·34	0·0070	85·8036
2670	0·0100	158·8248	26·77	46161·31	0·0069	85·8106
2680	0·0099	158·8348	26·62	46188·08	0·0068	85·8175
2690	0·0098	158·8447	26·43	46214·70	0·0067	85·8243
2700	0·0097	158·8545	26·23	46241·13	0·0066	85·8310
2710	0·0096	158·8642	26·09	46267·36	0·0065	85·8376
2720	0·0095	158·8738	25·90	46293·45	0·0064	85·8441
2730	0·0094	158·8833	25·71	46319·35	0·0063	85·8505
2740	0·0093	158·8927	25·52	46345·06	0·0062	85·8568
2750	0·0092	158·9020	25·34	46370·58	0·0062	85·8630
2760	0·0091	158·9112	25·15	46395·92	0·0061	85·8692
2770	0·0090	158·9203	24·97	46421·07	0·0060	85·8753
2780	0·0089	158·9293	24·79	46446·04	0·0059	85·8813
2790	0·0088	158·9382	24·62	46470·83	0·0058	85·8872
2800	0·0087	158·9470	24·32	46495·45	0·0057	85·8930

TABLE X.

Ballistic Table for Spherical Projectiles.

(Recalculated by Mr. Hadcock, R.A., from Bashforth's data, and extended to low velocities).

For lower velocities this table is provisional, pending the results of further experiments.

v.	ΔT.	T.	ΔS.	S.	ΔD.	D.
f.s.						
300	1·2232	0·0000	366·91	0·00	7·5191	0·0000
310	1·1505	1·2232	356·67	366·91	6·8454	7·5191
320	1·0824	2·3737	346·37	723·58	6·2387	14·3645
330	1·0217	3·4561	337·22	1069·95	5·7113	20·6032
340	0·9647	4·4778	328·01	1407·17	5·2335	26·3145
350	0·9137	5·4425	319·78	1735·18	4·8148	31·5480
360	0·8653	6·3562	311·51	2054·96	4·4333	36·3628
370	0·8218	7·2215	304·07	2366·47	4·0967	40·7961
380	0·7805	8·0433	296·60	2670·54	3·7884	44·8928
390	0·7432	8·8238	289·84	2967·14	3·5147	48·6812
400	0·7076	9·5670	283·05	3256·98	3·2629	52·1959
410	0·6753	10·2746	276·88	3540·03	3·0380	55·4588
420	0·6445	10·9499	270·69	3816·91	2·8303	58·4968
430	0·6151	11·5944	264·51	4087·60	2·6385	61·3271
440	0·5763	12·2095	253·59	4352·11	2·4159	63·9656
450	0·5508	12·7858	247·86	4605·70	2·2575	66·3815
460	0·5265	13·3366	242·20	4853·56	2·1111	68·6390
470	0·5035	13·8631	236·64	5095·76	1·9758	70·7501
480	0·4816	14·3666	231·18	5332·40	1·8506	72·7259
490	0·4609	14·8482	225·84	5563·58	1·7349	74·5765
500	0·4413	15·3091	220·63	5789·42	1·6277	76·3114
510	0·4227	15·7504	215·55	6010·05	1·5285	77·9391
520	0·4050	16·1731	210·61	6225·60	1·4366	79·4676
530	0·3883	16·5781	205·80	6436·21	1·3513	80·9042
540	0·3725	16·9664	201·14	6642·01	1·2722	82·2555
550	0·3575	17·3389	196·61	6843·15	1·1988	83·5277
560	0·3429	17·6964	192·01	7039·76	1·1293	84·7265
570	0·3291	18·0393	187·57	7231·77	1·0648	85·8558
580	0·3157	18·3684	183·11	7419·34	1·0039	86·9206
590	0·3028	18·6841	178·64	7602·45	0·9465	87·9245
600	0·2903	18·9869	174·19	7781·09	0·8925	88·8710
610	0·2786	19·2772	169·95	7955·28	0·8424	89·7635
620	0·2673	19·5558	165·75	8125·23	0·7953	90·6059
630	0·2567	19·8231	161·74	8290·98	0·7516	91·4012
640	0·2467	20·0798	157·92	8452·72	0·7111	92·1528
650	0·2371	20·3265	154·14	8610·64	0·6729	92·8639
660	0·2281	20·5636	150·53	8764·78	0·6374	93·5368
670	0·2195	20·7917	147·09	8915·31	0·6044	94·1742
680	0·2115	21·0112	143·80	9062·40	0·5736	94·7786
690	0·2038	21·2227	140·65	9206·20	0·5449	95·3522
700	0·1966	21·4265	137·63	9346·85	0·5180	95·8971
710	0·1898	21·6231	134·73	9484·48	0·4930	96·4151
720	0·1832	21·8129	131·88	9619·21	0·4692	96·9081
730	0·1770	21·9961	129·22	9751·09	0·4472	97·3773
740	0·1711	22·1731	126·59	9880·31	0·4264	97·8245
750	0·1653	22·3442	123·99	10006·90	0·4066	98·2509
760	0·1600	22·5095	121·57	10130·89	0·3882	98·6575
770	0·1547	22·6695	119·12	10252·46	0·3706	99·0457
780	0·1496	22·8242	116·72	10371·58	0·3539	99·4163
790	0·1447	22·9738	114·30	10488·30	0·3378	99·7702
800	0·1399	23·1185	111·89	10602·60	0·3225	100·1080

EXPLANATION.—This table is used in the same manner as the last, except that it is applied spherical projectiles—whether shot or shell from smooth-bore guns, or to the balls of case shot, or the bullets of shrapnel shell.

TABLE X.

Ballistic Table for Spherical Projectiles—*continued.*

v.	Δ*T.*	*T.*	Δ*S.*	*S.*	Δ*D.*	*D.*
f.s.						
810	0·1352	23·2584	109·50	10714·49	0·3078	100·4305
820	0·1306	23·3936	107·07	10823·99	0·2937	100·7383
830	0·1261	23·5242	104·68	10931·06	0·2803	101·0320
840	0·1218	23·6503	102·33	11035·74	0·2675	101·3123
850	0·1177	23·7721	100·01	11138·07	0·2553	101·5798
860	0·1137	23·8898	97·76	11238·08	0·2438	101·8351
870	0·1098	24·0035	95·53	11335·84	0·2328	102·0789
880	0·1062	24·1133	93·44	11431·37	0·2225	102·3117
890	0·1026	24·2195	91·35	11524·81	0·2127	102·5342
900	0·0993	24·3221	89·33	11616·16	0·2034	102·7469
910	0·0959	24·4214	87·32	11705·49	0·1945	102·9503
920	0·0928	24·5173	85·37	11792·81	0·1860	103·1448
930	0·0898	24·6101	83·48	11878·18	0·1780	103·3308
940	0·0869	24·6999	81·65	11961·66	0·1704	103·5088
950	0·0840	24·7868	79·83	12043·31	0·1631	103·6792
960	0·0813	24·8708	78·01	12123·14	0·1561	103·8423
970	0·0785	24·9521	76·19	12201·15	0·1493	103·9984
980	0·0759	25·0306	74·43	12277·34	0·1429	104·1477
990	0·0734	25·1065	72·67	12351·77	0·1368	104·2906
1000	0·0709	25 1799	70·87	12424·44	0·1307	104·4274
1010	0·0684	25·2508	69·08	12495·31	0·1249	104·5581
1020	0·0660	25·3192	67·31	12564·39	0·1193	104·6830
1030	0·0636	25·3852	65·55	12631·70	0·1140	104·8023
1040	0·0614	25·4488	63·81	12697·25	0·1088	104·9163
1050	0·0591	25·5102	62·08	12761·06	0·1039	105·0251
1060	0·0570	25·5693	60·42	12823·14	0·0992	105·1290
1070	0·0550	25·6263	58·82	12883·56	0·0948	105·2282
1080	0·0531	25·6813	57·31	12942·38	0·0906	105·3230
1090	0·0513	25·7344	55·89	12999·69	0·0868	105·4136
1100	0·0496	25·7857	54·59	13055·58	0·0832	105·5004
1110	0·0481	25·8353	53·36	13110·17	0·0799	105·5836
1120	0·0466	25·8834	52·21	13163·53	0·0768	105·6635
1130	0·0453	25·9300	51·15.	13215·74	0·0739	105·7403
1140	0·0440	25·9753	50·16	13266·89	0·0712	105·8142
1150	0·0428	26·0193	49·23	13317·05	0·0687	105·8854
1160	0·0417	26·0621	48·35	13366·28	0·0663	105·9541
1170	0·0406	26·1038	47·53	13414·63	0·0640	106·0204
1180	0·0396	26·1444	46·73	13462·16	0·0619	106·0844
1190	0·0386	26·1840	45·97	13508·89	0·0599	106·1463
1200	0·0377	26·2226	45·27	13554·86	0·0580	106·2062
1210	0·0369	26·2603	44·61	13600·13	0·0562	106·2642
1220	0·0361	26·2972	44·00	13644·74	0·0545	106·3204
1230	0·0353	26·3333	43·43	13688·74	0·0529	106·3749
1240	0·0346	26·3686	42·87	13732·17	0·0514	106·4278
1250	0·0339	26·4032	42·36	13775·04	0·0500	106·4792
1260	0·0332	26·4371	41·85	13817·40	0·0486	106·5292
1270	0·0326	26·4703	41·39	13859·25	0·0473	106·5778
1280	0·0320	26·5029	40·94	13900·64	0·0461	106·6251
1290	0·0314	26·5349	40·49	13941·58	0·0449	106·6712
1300	0·0308	26·5663	40·04	13982·07	0·0437	106·7161
1310	0·0302	26·5971	39·59	14022·11	0·0425	106·7598
1320	0·0297	26·6273	39·18	14061·70	0·0415	106·8023
1330	0·0291	26·6570	38·75	14100·88	0·0404	106·8438
1340	0·0286	26·6861	38·33	14139·63	0·0394	106·8842
1350	0·0281	26·7147	37·92	14177·96	0·0384	106·9236
1360	0·0276	26·7428	37·52	14215·88	0·0374	106·9620
1370	0·0271	26·7704	37·15	14253·40	0·0365	106·9994
1380	0·0267	26·7975	36·80	14290·55	0·0356	107·0359
1390	0·0262	26·8242	36·45	14327·35	0·0348	107·0715
1400	0·0258	26·8504	36·11	14363·80	0·0340	107·1063

TABLE X.

Ballistic Table for Spherical Projectiles—*continued.*

v.	ΔT.	T.	ΔS.	S.	ΔD.	D.
f.s.						
1410	0·0254	26·8762	35·77	14399·91	0·0332	107·1403
1420	0·0250	26·9016	35·48	14435·68	0·0324	107·1735
1430	0·0246	26·9266	35·16	14471·16	0·0317	107·2059
1440	0·0242	26·9512	34·85	14506·32	0·0310	107·2376
1450	0·0238	26·9754	34·54	14541·17	0·0303	107·2686
1460	0·0235	26·9992	34·24	14575·71	0·0296	107·2989
1470	0·0231	27·0227	33·98	14609·95	0·0290	107·3285
1480	0·0228	27·0458	33·69	14643·93	0·0284	107·3575
1490	0·0224	27·0686	33·41	14677·62	0·0278	107·3859
1500	0·0221	27·0910	33·14	14711·03	0·0272	107·4137
1510	0·0218	27·1131	32·85	14744·17	0·0266	107·4409
1520	0·0214	27·1349	32·59	14777·02	0·0260	107·4675
1530	0·0211	27·1563	32·34	14809·61	0·0255	107·4935
1540	0·0208	27·1774	32·06	14841·95	0·0249	107·5190
1550	0·0205	27·1982	31·82	14874·01	0·0244	107·5439
1560	0·0202	27·2187	31·58	14905·83	0·0239	107·5683
1570	0·0200	27·2389	31·33	14937·41	0·0234	107·5922
1580	0·0197	27·2589	31·10	14968·74	0·0230	107·6156
1590	0·0194	27·2786	30·86	14999·84	0·0225	107·6386
1600	0·0191	27·2980	30·64	15030·70	0·0221	107·6611
1610	0·0189	27·3171	30·42	15061·34	0·0216	107·6832
1620	0·0186	27·3360	30·19	15091·76	0·0212	107·7048
1630	0·0184	27·3546	29·99	15121·95	0·0208	107·7260
1640	0·0182	27·3730	29·79	15151·94	0·0204	107·7468
1650	0·0179	27·3912	29·60	15181·73	0·0201	107·7672
1660	0·0177	27·4091	29·38	15211·33	0·0197	107·7873
1670	0·0175	27·4268	29·20	15240·71	0·0193	107·8070
1680	0·0173	27·4443	29·02	15269·91	0·0190	107·8263
1690	0·0171	27·4616	28·84	15298·93	0·0186	107·8453
1700	0·0168	27·4787	28·64	15327·77	0·0183	107·8639
1710	0·0167	27·4955	28·47	15356·41	0·0180	107·8822
1720	0·0165	27·5122	28·31	15384·88	0·0176	107·9002
1730	0·0163	27·5287	28·13	15413·19	0·0173	107·9178
1740	0·0161	27·5450	27·97	15441·32	0·0170	107·9351
1750	0·0159	27·5611	27·81	15469·29	0·0168	107·9521
1760	0·0157	27·5770	27·64	15497·10	0·0165	107·9689
1770	0·0155	27·5927	27·49	15524·74	0·0162	107·9854
1780	0·0154	27·6082	27·33	15552·23	0·0159	108·0016
1790	0·0152	27·6236	27·16	15579·56	0·0156	108·0175
1800	0·0150	27·6388	27·03	15606·72	0·0154	108·0331
1810	0·0148	27·6538	26·87	15633·75	0·0151	108·0485
1820	0·0147	27·6686	26·72	15660·62	0·0149	108·0636
1830	0·0145	27·6833	26·54	15687·34	0·0146	108·0785
1840	0·0143	27·6978	26·40	15713·88	0·0144	108·0931
1850	0·0142	27·7121	26·25	15740·28	0·0141	108·1075
1860	0·0140	27·7263	26·09	15766·53	0·0139	108·1216
1870	0·0139	27·7403	25·93	15792·62	0·0137	108·1355
1880	0·0137	27·7542	25·79	15818·55	0·0135	108·1492
1890	0·0136	27·7679	25·64	15844·34	0·0132	108·1627
1900	0·0134	27·7815	25·48	15869·98	0·0130	108·1759

TABLE XI.

Correction τ of K for Temperature and Pressure of Atmosphere two-thirds Saturated with Moisture.

(*From the Rev. F. Bashforth's paper.* "Pro. R. A. I.," *Vol.* XIII, *No.* 10.)

F.	26in.	27 in.	28 in.	29 in.	30 in.	31 in.	Δ +	F.	26in.	27in.	28in.	29in.	30 in.	31 in.	Δ +
0°	·983	1·021	1·059	1·097	1·134	1·172	38	50°	·884	·919	·953	·987	1·021	1·055	34
1	·981	1·019	1·056	1·094	1·132	1·170	38	51	·883	·917	·951	·985	1·019	1·053	34
2	·979	1·017	1·054	1·092	1·130	1·167	38	52	·881	·915	·949	·983	1·017	1·051	34
3	·977	1·015	1·052	1·090	1·127	1·165	38	53	·879	·913	·947	·981	1·015	1·048	34
4	·975	1·012	1·050	1·087	1·125	1·162	38	54	·877	·911	·945	·978	1·012	1·046	34
5	·973	1·010	1·047	1·085	1·122	1·160	37	55	·875	·909	·943	·976	1·010	1·044	34
6	·971	1·008	1·045	1·083	1·120	1·157	37	56	·874	·907	·941	·974	1·008	1·042	34
7	·969	1·006	1·043	1·080	1·118	1·155	37	57	·872	·905	·939	·972	1·006	1·039	34
8	·966	1·004	1·041	1·078	1·115	1·152	37	58	·870	·904	·937	·970	1·004	1·037	34
9	·964	1·001	1·039	1·076	1·113	1·150	37	59	·868	·902	·935	·968	1·002	1·035	33
10	·962	·999	1·036	1·073	1·110	1·147	37	60	·866	·900	·933	·966	1·000	1·033	33
11	·960	·997	1·034	1·071	1·108	1·145	37	61	·865	·898	·931	·964	·998	1·031	33
12	·958	·995	1·032	1·069	1·105	1·142	37	62	·863	·896	·929	·962	·996	1·029	33
13	·956	·993	1·029	1·066	1·103	1·140	37	63	·861	·894	·927	·960	·993	1·027	33
14	·954	·991	1·027	1·064	1·101	1·137	37	64	·859	·892	·925	·958	·991	1·024	33
15	·952	·989	1·025	1·062	1·098	1·135	37	65	·857	·890	·923	·956	·989	1·022	33
16	·950	·986	1·023	1·060	1·096	1·133	37	66	·856	·889	·921	·954	·987	1·020	33
17	·948	·984	1·021	1·057	1·094	1·130	37	67	·854	·887	·919	·952	·985	1·018	33
18	·946	·982	1·019	1·055	1·091	1·128	36	68	·852	·885	·918	·950	·983	1·016	33
19	·944	·980	1·017	1·053	1·089	1·125	36	69	·850	·883	·916	·949	·981	1·014	33
20	·942	·978	1·014	1·051	1·087	1·123	36	70	·849	·881	·914	·946	·979	1·012	33
21	·940	·976	1·012	1·048	1·084	1·121	36	71	·847	·879	·912	·944	·977	1·010	33
22	·938	·974	1·010	1·046	1·082	1·118	36	72	·845	·878	·910	·943	·975	1·008	33
23	·936	·972	1·008	1·044	1·080	1·116	36	73	·843	·876	·908	·941	·973	1·006	32
24	·934	·970	1·006	1·042	1·078	1·114	36	74	·842	·874	·906	·939	·971	1·004	32
25	·932	·968	1·004	1·039	1·075	1·111	36	75	·840	·872	·904	·937	·969	1·001	32
26	·930	·966	1·001	1·037	1·073	1·109	36	76	·838	·870	·902	·935	·967	·999	32
27	·928	·964	·999	1·035	1·071	1·106	36	77	·836	·868	·901	·933	·965	·997	32
28	·926	·962	·997	1·033	1·069	1·104	36	78	·834	·867	·899	·931	·963	·995	32
29	·924	·960	·995	1·031	1·066	1·102	36	79	·833	·865	·897	·929	·961	·993	32
30	·922	·958	·993	1·028	1·064	1·099	36	80	·831	·863	·895	·927	·959	·991	32
31	·920	·956	·991	1·026	1·062	1·097	35	81	·829	·861	·893	·925	·957	·989	32
32	·918	·954	·989	1·024	1·059	1·095	35	82	·827	·859	·891	·923	·955	·987	32
33	·916	·952	·987	1·022	1·057	1·093	35	83	·826	·858	·889	·921	·953	·985	32
34	·914	·950	·985	1·020	1·055	1·090	35	84	·824	·856	·887	·919	·951	·983	32
35	·913	·948	·983	1·018	1·053	1·088	35	85	·822	·854	·885	·917	·949	·980	32
36	·911	·946	·981	1·016	1·051	1·086	35	86	·821	·852	·884	·915	·947	·978	32
37	·909	·944	·979	1·013	1·048	1·083	35	87	·819	·850	·882	·913	·945	·976	32
38	·907	·942	·977	1·011	1·046	1·081	35	88	·817	·848	·880	·911	·943	·974	31
39	·905	·940	·974	1·009	1·044	1·079	35	89	·815	·847	·878	·909	·941	·972	31
40	·903	·938	·973	1·007	1·042	1·077	35	90	·814	·845	·876	·908	·939	·970	31
41	·901	·936	·971	1·005	1·040	1·075	35	91	·812	·843	·874	·905	·937	·968	31
42	·899	·934	·968	1·003	1·038	1·072	35	92	·810	·841	·872	·903	·935	·966	31
43	·898	·932	·967	1·001	1·036	1·070	35	93	·808	·839	·870	·902	·933	·964	31
44	·896	·930	·964	0·999	1·033	1·068	34	94	·806	·837	·868	·900	·931	·962	31
45	·894	·928	·963	0·997	1·031	1·066	34	95	·805	·836	·867	·898	·929	·960	31
46	·892	·926	·960	0·995	1·029	1·063	34	96	·803	·834	·865	·896	·926	·957	31
47	·890	·924	·958	0·993	1·027	1·061	34	97	·801	·832	·863	·893	·924	·955	31
48	·888	·923	·957	·991	1·025	1·059	34	98	·799	·830	·861	·891	·922	·953	31
49	·886	·920	·955	·989	1·023	1·057	34	99	·797	·828	·859	·889	·920	·951	31
50	·884	·919	·953	·987	1·021	1·055	34	100	·796	·826	·857	·888	·918	·949	31

EXPLANATION.—To find the value of τ for given atmospheric conditions: under the heading F. the temperature in degrees Fahrenheit will be found; glance from thence horizontally until a column is reached which is headed with the given barometric pressure, and the required value will be found: differences must often be taken. Thus, find the value of τ when the thermometer is at 78° F., barometer 29·5. In the horizontal of 78° F. and under column 29 inches barometer, 0·931 is indicated; for the extra 0·5 of barometric pressure add 0·5 × 0·032 (or half the tabulated difference Δ) = 0·016, and the value of τ required = 0·931 + 0·016 = 0·947.

For further description of this Table *see* p. 140.

Table XII.

Probability Factors.

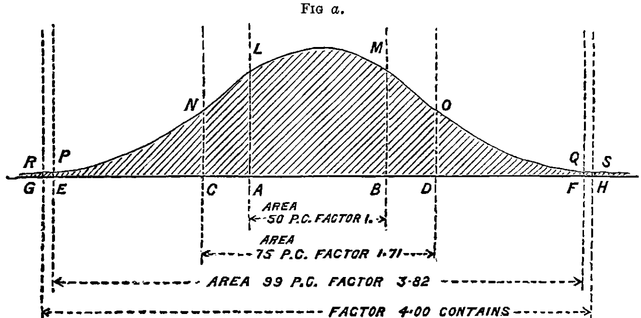
The following gives the proportional width of other zones (containing a different percentage of hits) to one of 50 per cent. as unity.

Per cent.	Factor.	Per cent.	Factor.	Per cent.	Factor.	Per cent.	Factor.	Per cent.	Factor.
1	0·02	21	0·40	41	0·80	61	1·27	81	1·94
2	0·04	22	0·41	42	0·82	62	1·30	82	1·98
3	0·06	23	0·43	43	0·84	63	1·33	83	2·03
4	0·07	24	0·45	44	0·86	64	1·36	84	2·08
5	0·09	25	0·47	45	0·89	65	1·39	85	2·13
6	0·11	26	0·49	46	0·91	66	1·42	86	2·18
7	0·13	27	0·51	47	0·93	67	1·45	87	2·24
8	0·15	28	0·53	48	0·95	68	1·48	88	2·30
9	0·17	29	0·55	49	0·98	69	1·51	89	2·37
10	0·18	30	0·57	50	1·00	70	1·54	90	2·44
11	0·20	31	0·59	51	1·02	71	1·57	91	2·52
12	0·22	32	0·61	52	1·04	72	1·60	92	2·60
13	0·24	33	0·63	53	1·07	73	1·64	93	2·69
14	0·26	34	0·65	54	1·09	74	1·67	94	2·78
15	0·28	35	0·67	55	1·12	75	1·71	95	2·91
16	0·30	36	0·70	56	1·14	76	1·74	96	3·04
17	0·32	37	0·72	57	1·17	77	1·78	97	3·22
18	0·34	38	0·74	58	1·19	78	1·82	98	3·45
19	0·36	39	0·76	59	1·22	79	1·86	99	3·82
20	0·38	40	0·78	60	1·25	80	1·90	100	∞*

* As a factor of 4 contains more than 99 per cent. of the rounds fired, for practical purposes it may be taken to contain the total of 100 per cent.

Explanation.—Taking the width of a 50 p.c. zone as unity, the factors in the above table are the widths of other zones containing different percentages: thus 80 p.c. and 20 p.c. zones are respectively 1·90 and 0·38 times as wide as the 50 p.c. zone. If the width of the 50 p.c. zone is given in yards or feet, the widths of other zones containing different percentages can be obtained by *multiplying* by their corresponding factors: thus if the width of a 50 p.c. zone is 3 yards, the widths of 25 p.c. and 72 p.c. zones are 0·47 × 3 = 1·41 yards and 1·60 × 3 = 4·80 yards respectively. Conversely, if it is required to find what percentages will fall in zones of given widths, the factors must be obtained by *dividing* each by the width of the 50 p.c. zone: thus with the same 50 p.c. zone (3 yards wide) as before, what percentages will fall in zones 2 yards and 6 yards wide? the factors are $\frac{2}{3}$ = 0·67 and $\frac{6}{3}$ = 2·00, and they correspond to 35 p.c. and 82·4 p.c. respectively. The annexed fig. (*a*) represents the probability curve, the horizontal being asymptotic to it; the total area contained between the curve and the horizontal is proportional to the total number of rounds or 100 p.c. The central area ABML represents half the total area, or 50 p.c. of the rounds fired. Areas of different widths contain percentages according to the table, the widths being the same as the factors in the table: thus the 75 p.c. area, CDON, is 1·71 times as wide as the 50 p.c. area, and the 99 p.c. area, EFQP, is 3·82 times as wide: it is a fair approximation to assume that an area 4 times as wide as the 50 p.c. area contains all the area, though there is a small portion outside.

Fig *a*.



This table, and others similar to it, were prepared for use in astronomy for determining the probable accuracy of observations: if, for instance, the latitude of a place is carefully found several times independently there will be slight differences in the results—the mean is taken as the latitude: if all the observations nearly agree, it is probable that this mean is very near the truth: but if the observations differ considerably among themselves there is a greater liability to error in the average; this can be expressed numerically: thus the latitude of a place may be found from several observations with a pocket sextant as 51° 28′ ± 1·4′, but with a good astronomical instrument it may be determined as 51° 27′ 35″ ± 0·6″, meaning that in the first case the latitude is probably correct within 1·4 minutes, in the second case that it is probably correct within 0·6 seconds.

This probability table has since been adopted for gunnery purposes: for further description of its employment *see* p. 173, and for an outline explaining the formation of the Table, *see* p. 274.

Table XIII.

Programme of Firing

To obtain the necessary Data for a good Range Table for a Heavy Gun.

(*Vide* paper by Commander May, R.N., "Pro. R.A.I.," vol. xiv, No. 10.)

Readings of the barometer and thermometer and of the direction and velocity of the wind should be taken periodically.

Series.	Number of rounds to be fired at each range.	Range.	Elevation.	Projectile.	Object to be determined.	Remarks.
		yds.	° ′			
(a)	7	500	...	Palliser shot	Velocity, regularity of velocity and jump; also elevation for this range.	Fired at a target.
(b)	10	2000	...	Plugged common shell	Elevation, angle of descent, and accuracy.	Fired at a target. Remaining velocities at long ranges have not yet been directly observed at long ranges in England, but probably they will be taken at this range.
(c)	5	...	3 30	Shrapnel shell	To determine range and accuracy at the different elevations, and also if the various projectiles range similarly.	Common and shrapnel shell may if desired be fused with percussion, and with the double acting time and percussion fuze respectively, and the action of fuzes on graze may thus be tested at the same time as the range and accuracy is determined.
	5	...	3 30	Common shell		
	10	...	6 0	Palliser shot		
	5	...	9 0	Palliser shot		
	5	...	9 0	Common shell		
	10	...	14 0	Common shell		
(d)	5	500	...	Shrapnel shell	To test shrapnel shell, to obtain data for a fuze scale, and to test the regularity of time fuzes.	The ranges indicated here are those at about which the fuzes are set to act; as the shells burst in the air, the actual ranges in this series would be somewhat longer.
	5	2000	...	Shrapnel shell		
	5	3500	...	Shrapnel shell		
	5	5000	...	Shrapnel shell		
Total number of rounds fired.	77					

Explanation. Series (a). The range of 500 yards is selected because projectiles steady down in flight at that distance, and the jump recorded is fairly regular; 7 rounds are sufficient for a good average.

Series (b). A range of 2000 yards is taken because it is a good average range: it is about the limit at which vertical targets can be fired at, when the angle of descent can be found from direct observation.

Series (c). The elevations are chosen to give ranges not more than about 2000 yards different from each other.

Series (d). A group of 5 rounds at each range is necessary to give a reliable average, and 4 groups are required in order to judge the rate of burning of fuzes at different ranges.

Table XIV.

Range Tables.

(A) For a Gun.

Extracts from Provisional Range Table for the 12-inch B.L. Gun, Marks I, II, III, and IV (based on Practice of 24 and 25/2/85, 17/4/85, and 23/6/85; from 6,000 to 8,000 yards, on calculation).

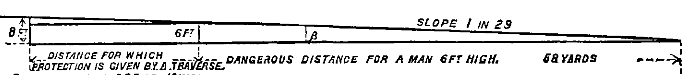
Charge, 295 lb. brown prism.[1]; gravimetric density, $\frac{33 \cdot 100}{0 \cdot 838}$ | Muzzle velocity, 1892 f.s.
Projectile, Weight 714 lb. | Mounting, Garrison "yoke."
Jump, 6 minutes.

Range.	Elevation.	Angle of descent.	Slope of descent.	To hit an object 10 feet high, range must be known within	Remaining velocity.	Penetration, wrought iron.	50 per cent. of rounds should fall within			Time of flight.	Estimated fuze scale for medium time and concussion fuze.	Fuze scale.
							Length.	Breadth.	Height.			
yds.	° ′	° ′	1 in	yds.	f. s.	ins.	yds.	yds.	feet.	secs.		
0	—	—	—	—	1892	22·5	—	—	—	—	—	—
100	—	0 5	688	1146	1876	22·3	18	0·05	0·1	0·16	—	1·0
200	0 4	0 10	344	573	1860	22·1	18	0·05	0·2	0·32	—	1·8
300	0 9	0 15	229	382	1845	21·9	18	0·1	0·3	0·48	—	2·6
400	0 13	0 20	172	286	1829	21·7	18	0·1	0·3	0·64	—	3·35
500	0 18	0 25	137	229	1814	21·5	18	0·1	0·4	0·81	—	4·1
1500	1 10	1 23	41	69	1667	19·6	18	0·35	1·3	2·54	1·14	10·2
1600	1 16	1 29	39	64	1653	19·4	18	0·35	1·4	2·72	1·28	10·8
1700	1 21	1 36	36	60	1639	19·2	18	0·4	1·5	2·90	1·42	11·35
1800	1 27	1 43	33	56	1625	19·0	18	0·4	1·6	3·09	1·55	11·95
1900	1 32	1 50	31	52	1611	18·8	18	0·45	1·7	3·28	1·69	12·5
2000	1 38	1 57	29	49	1597	18·7	18	0·45	1·8	3·47	1·82	13·1
2100	1 44	2 4	28	46	1583	18·5	18	0·5	1·9	3·66	1·95	13·65
2200	1 50	2 11	26	44	1570	18·3	18	0·5	2·0	3·85	2·08	14·2
2300	1 56	2 19	25	41	1556	18·1	18	0·55	2·2	4·04	2·21	14·8
7500	9 2	13 22	4·2	7	1045	11·6	—	—	—	16·8	—	—
7600	9 12	13 41	4·1	7	1040	11·5	—	—	—	17·1	—	—
7700	9 23	14 0	4·0	7	1035	11·4	—	—	—	17·4	—	—
7800	9 34	14 20	3·9	7	1030	11·4	—	—	—	17·7	—	—
7900	9 45	14 40	3·8	6	1025	11·3	—	—	—	18·0	—	—
8000	9 57	15 0	3·7	6	1021	11·2	—	—	—	18·3	—	—

Explanation.—The first column gives the range in yards; the second the corresponding angle of elevation; the third column gives the angle of descent; and the fourth the slope of descent, which is derived from the last, as the number of the slope is the natural cotangent of the angle of descent. In this form it is useful for finding the dangerous distance, or the distance for which protection is given by a traverse of given height; or it can be employed to find whether the crest of a glacis protects an escarp from fire.

Thus, range 2000 yards: we find from the range-table that the slope of descent is 1 in 29. For a man 6 feet high the dangerous distance is $6 \times 29 = 174$ feet $= 58$ yards (Fig. *a*). A traverse 8 feet high will give protection to a man 6 feet high for a distance in rear of it $(8 - 6) \times 29 = 58$ feet $= 19$ yards (see Fig. *a*), but it must be remembered that the top of the traverse is liable to be shot away.

Fig. *a*.



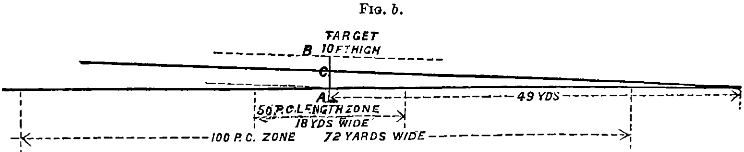
Note.—The vertical scale is 3 times as great as the horizontal one in both Figs. *a* and *b*.

The fifth column also depends on the angle of descent, as it is constructed by multiplying half the height of the object (5 feet) by the number of the slope of descent: thus at 2000 yards the range should be known within $5 \times 29 = 145$ feet $= 49$ yards; this only, however, refers to the *mean* trajectory (see Fig. *b*).

The remaining velocity in the sixth column is obtained by the use of Bashforth's tables, and the seventh column of penetration of wrought iron is found by calculation, or by Maitland's diagram.

The next three columns give the width of the 50 per cent. zones: thus the length happens to be always 18 yards wide. Suppose the range of 2000 yards has been underestimated by 49 yards; if the centre of the target is aimed at, the point of mean impact will be at A, the foot of the target, AB, which is supposed to be 10 feet high (Fig. *b*),

Fig. *b*.



instead of at C, the centre; in this case 50 per cent. of all the shots fired, *i.e.*, all those which fly further than the mean, will still fall upon the target if it is wide enough; and if the point of mean impact is anywhere on the ground short of the target, but within a zone four times the width of the 50 per cent. length zone (as that practically contains all the rounds) a certain percentage will still strike the target. It is only when the point of mean impact falls short of the 100 per cent. zone, or $\frac{1}{2} \times 18 \times 4 = 36$ *yards short of the target, that no rounds except possible ricochets can* fall on it; this is equivalent to an error in range of $49 + 36 = 85$ yards in place of the 49 yards mentioned in Column 5, which must be understood to refer to the mean trajectory only.

The time of flight is found from actual firing trials, as are also the fuze scales, since the composition burns at different rates when at rest and when fired from this gun.

Table XIV.

Range Tables.

(B.) For a Howitzer.

Extracts from Range Table for the 8-inch R.M.L. Howitzer of 70 cwt. (Mark I).

(Based on practice of July, September, and December, 1879. Projectile 180 lbs., with rotating gas-checks.)

Charge.	Muzzle velocity.	Range.	Elevation.	Deflection left.	Drift right.	Angle of descent.	Five minutes alteration of elevation increases or decreases the range by	Five minutes will alter point of impact vertically or laterally at each range by	50 per cent. of rounds should fall within			Remaining velocity.	Time of flight.	Fuze scale.
									Length.	Breadth.	Height.			
lbs. R.L.G.[2]	f.s.	yds.	° ′	° ′	yds.	° ′	yds.	yds.	yds.	yds.	yds.	f.s.	secs	
11½	956	1200	3 44	0 6	1·9	4 0	25·0	1·74	16·4	0·40	1·21	876	4·0	...
		1600	5 6	0 9	3·8	5 36	23·8	2·32	21·3	0·56	2·14	852	5·4	...
		2300	7 33	0 13	8·7	8 42	23·8	3·34	29·7	0·87	4·64	814	7·9	...
		2400	7 54	0 13	9·5	9 12	22·7	3·49	30·9	0·92	5·07	809	8·2	...
		2500	8 16	0 14	10·4	9 42	22·7	3·63	32·1	0·98	5·50	804	8·6	...
10½	920	1200	4 12	0 7	2·3	4 36	20·8	1·74	15·5	0·45	1·31	845	4·1	...
		1600	5 48	0 10	4·5	6 21	20·8	2·32	20·3	0·62	2·31	823	5·6	...
		2300	8 36	0 16	10·5	9 51	20·8	3·34	28·3	0·99	4·99	787	8·3	...
		2400	9 0	0 17	11·5	10 24	20·8	3·49	29·4	1·05	5·44	782	8·7	...
		2500	9 24	0 18	12·7	10 57	20·8	3·63	30·5	1·12	5·90	777	9·1	...
9½	875	1200	4 45	0 8	2·9	5 18	18·5	1·74	14·7	0·50	1·44	807	4·2	...
		1600	6 33	0 12	5·6	7 18	18·5	2·32	19·2	0·70	2·52	787	5·8	...
		2300	9 48	0 19	12·5	11 0	17·8	3·34	26·9	1·10	5·33	752	8·6	...
		2400	10 16	0 20	14·0	11 36	17·8	3·49	28·0	1·17	5·82	747	9·1	...
		2500	10 44	0 21	15·5	12 12	16·6	3·63	29·0	1·25	6·31	742	9·6	...
8½	825	1200	5 24	0 10	3·6	6 9	16·6	1·74	13·8	0·57	1·57	763	4·2	...
		1600	7 28	0 15	7·0	8 24	15·6	2·32	18·2	0·78	2·69	744	5·8	...
		2300	11 20	0 23	15·2	12 44	14·7	3·34	25·4	1·22	5·82	712	9·2	...
		2400	11 54	0 24	16·7	13 36	13·9	3·49	26·4	1·30	6·46	708	9·7	...
		2500	12 30	0 25	18·3	14 18	13·9	3·63	27·4	1·38	7·10	704	10·2	...
7½	770	1200	6 24	0 14	4·8	7 6	13·9	1·74	13·2	0·61	1·64	713	4·5	...
		1600	8 48	0 19	8·8	9 42	13·9	2·32	17·2	0·85	2·86	695	6·2	...
		2300	13 8	0 29	19·2	14 48	12·5	3·34	24·0	1·34	6·46	665	9·7	...
		2400	13 48	0 30	21·0	15 36	12·5	3·49	24·9	1·42	7·04	661	10·3	...
		2500	14 18	0 32	23·0	16 24	11·9	3·63	25·8	1·51	7·63	657	10·9	...
7	715	1200	6 56	0 16	5·7	7 42	13·1	1·74	12·4	0·69	1·77	662	4·6	...
		1600	9 32	0 22	10·5	10 42	12·5	2·32	16·4	0·94	3·16	646	6·6	...
		2300	14 12	0 33	21·6	16 42	11·3	3·34	22·7	1·46	6·98	622	10·2	...
		2400	14 56	0 34	23·5	17 42	10·8	3·49	23·6	1·54	7·65	619	10·8	...
		2500	15 42	0 35	25·5	18 42	10·4	3·63	24·5	1·63	8·32	616	11·4	...
6½	710	1200	7 36	0 19	6·5	8 27	11·9	1·74	11·9	0·74	1·84	660	4·8	...
		1600	10 24	0 24	11·2	11 33	11·9	2·32	15·5	1·02	3·23	644	6·8	...
		2300	15 24	0 35	23·7	17 42	10·8	3·34	21·5	1·58	7·08	616	10·0	...
		2400	16 10	0 37	25·7	18 42	10·0	3·49	22·3	1·67	7·74	612	11·4	...
		2500	17 0	0 39	28·0	19 42	9·6	3·63	23·1	1·76	8·40	608	12·0	...
5½	641	1200	9 12	0 24	8·5	10 6	9·2	1·74	10·6	0·87	2·00	593	5·7	...
		1600	12 48	0 32	15·0	14 18	9·2	2·32	13·8	1·19	3·61	579	7·9	...
		2300	19 6	0 46	30·5	23 0	8·3	3·34	19·2	1·79	8·38	558	12·2	...
		2400	20 6	0 47	33·0	24 24	7·6	3·49	19·9	1·88	9·24	556	12·9	...
		2500	21 12	0 49	35·7	25 54	5·9	3·63	20·6	1·97	10·10	554	13·6	...
4½	556	1200	11 48	0 32	11·2	12 48	6·9	1·74	9·3	1·00	2·22	516	6·8	...
		1600	16 48	0 45	21·1	18 24	6·0	2·32	12·1	1·35	4·15	504	9·5	...
		2300	27 12	1 10	46·6	30 48	4·8	3·34	16·9	1·98	10·37	487	15·2	...
		2400	28 56	1 14	51·4	33 12	4·7	3·49	17·5	2·07	11·50	486	16·2	...
3½	473	1200	16 33	0 33	11·4	17 30	4·6	1·74	21·7	1·50	6·9	438	8·4	...
		1600	24 21	0 48	22·3	26 0	3·6	2·32	30·4	2·18	14·9	433	11·8	...

Explanation.—This represents 10 tables: each of the 10 different charges in the original has a separate table devoted to it.

In the first column is the powder charge, and in the second the corresponding muzzle velocity, in block type. The third column gives the range in yards, the fourth the angle of elevation: as the sights are placed vertically in howitzers, the fifth column records the deflection which must be given in order to compensate for the drift which is stated in the sixth column: the next column gives the angle of descent, and the two following the changes in range and position of point of impact on a vertical target respectively corresponding to alterations of five minutes in the elevation at the various ranges. The next three columns give the width of the 50 per cent. zones, in which it will be noticed that the height is given in yards; in the previous range table it is given in feet; this is an unfortunate want of uniformity, and it is well to note in any particular range table whether the height is given in yards or in feet. The last three columns give the remaining velocity and times of flight under the varied conditions of elevation and charge, and a blank space is left for thef uze scale to be completed when the graduations are known.

TABLE XV.

Variations in Range and Vertical Accuracy due to various causes.

(*Vide* Paper by Commander May, R.N., Pro. R.A.I., vol. xiv, No. 10.)

Nature of Gun.	Projectile.		Range.	A variation of				(c) Half a gale of wind, 50 f.s., blowing up or down the range.		(d) A variation of 1 inch in the barometer, or of 16° F. in the thermometer.		(e) Error of 100 yards in judging the range.
				(a) 50 f.s. in M.V.		(b) 5 mins. in jump or laying.						
	M.V.	Weight.		Increase or decrease of range.	Variation in height on a vertical target.	Increase or decrease of range.	Variation in height on a vertical target.	Increase or decrease of range.	Variation in height on a vertical target.	Increase or decrease of range.	Variation in height on a vertical target.	Variation in height on a vetical target.
	f.s.	lbs.	yds.	yds.	feet.	yds.	feet.	yds.	feet.	yds.	feet.	feet.
4-inch B.L.	1900	25	1000	44	2·6	76	4·4	6·0	0·4	5·0	0·3	5·8
			2000	73	12·2	52	8·7	28·0	4·8	20·0	3·4	16·9
			4000	102	55·0	32	17·0	120·0	66·0	51·0	28·0	54·0
			6000	127	152·0	22	26·0	228·0	274·0	102·0	122·0	120·0
8-inch B.L.	2000	210	1000	47	2·0	104	4·4	2·0	0·1	2·0	0·1	4·2
			2000	87	8·7	87	8·7	8·0	0·8	9·0	0·9	10·0
			4000	150	44·0	60	17·0	44·0	13·0	40·0	12·0	29·0
			6000	200	113·0	46	26·0	112·0	63·0	79·0	45·0	57·0
12-inch B.L.	1900	714	1000	50	2·3	96	4·4	1·5	0·1	1·5	0·1	4·5
			2000	94	9·6	85	8·7	6·0	0·6	6·0	0·6	10·2
			4000	175	45·0	67	17·0	32·0	8·0	22·0	6·0	26·0
			6000	243	119·0	54	26·0	76·0	37·0	50·0	24·0	49·0
8-inch R.M.L.	1400	180	1000	58	5·1	50	4·4	5·5	0·5	2·0	0·2	8·8
			2000	104	22·5	40	8·7	23·0	5·0	8·0	1·7	22·5
			4000	156	90·0	30	17·0	101·0	58·0	33·0	19·0	58·0
			6000	191	215·0	23	26·0	198·0	224·0	54·0	62·0	113·0

EXPLANATION.—(a.) A correction in the range must be made if the muzzle velocity is above or below the average, from change in the strength of the powder, or from the employment of gun with a worn bore.

Let us take a definite case, that of the 4-inch gun, whose average M.V. is 1900 f.s., which we will suppose is changed to 1850 f.s.; find the decrease in range at a range of 2000 yards, take $\frac{d^2}{w} = 0{\cdot}64$. Assume that the projectile with the changed M.V. (1850 f.s.) just traverses the range: if the gun is fired again with the *same angle of departure*, but if the projectile has the *average* M.V. (1900 f.s.), the range will be increased, and this increase is the correction required. It will be necessary to find the angle of departure: for brevity denote Table VI of $\frac{d^2}{w}t$, p. 283, Table VII of $\frac{d^2}{w}s$, p. 288, and Table VIII of $\frac{d^2}{w}\delta$, p. 293, by (t), (s), and (δ) respectively. The steps of the calculation, which are somewhat intricate, are as follows:—

(1.) With the changed M.V. (1850 f.s.) find the remaining velocity at the end of the range (2000 yards) by means of the (s) table; it is 1087·5 f.s.

(2.) The tabulated value for the time at half the time of flight is $\frac{T_V + T_v}{2}$ or $\frac{T_{1850} + T_{1087\cdot5}}{2}$, corresponding from the (t) table to a velocity of 1353·5 f.s.: this is very approximately the (horizontal) velocity v_0 at the vertex.

(3.) By means of the (δ) table find the change in inclination of the projectile while the velocity changes from M.V. to v_0 (from 1850 f.s. to 1353·5 f.s.); it is 1·6249°. This is the angle of departure sought for.

We now turn to the average M.V.

(4.) Taking the same angle of departure with the average M.V. (1900 f.s.) by means of (δ) table, find the remaining velocity at the vertex v_0. It is 1374 f.s.

TABLE XV.

Variations in Range and Vertical Accuracy—(*continued*).

(5.) The time to change from M.V. to v_0 (from 1900 f.s. to 1374 f.s.) is very approximately half the time of flight; as before, $\frac{T_V + T_v}{2} = T_{v_0}$, whence $T_v = 2T_{v_0} - T_V$, and by (t) table we find final velocity v is 1094 f.s.

(6.) Knowing the M.V. (1900 f.s.), and the final velocity (1094 f.s.), find the range from the (s) table; it is 2073 yards.

(7.) The difference between this range (2073 yards) and the first (2000 yards) is the correction (73 yards) required.

To find the variation in height on a vertical target from the same cause, multiply the difference in range by the tangent of the angle of descent. For the 4-inch gun the angle of descent is 3° 13′ at 2000 yards; consequently this correction becomes 73 × 3 tan 3° 13′ = 12·2 feet.

And generally the variation in height = difference in range × tan. angle of descent: thus the second columns of (b), (c), and (d) are obtained from their first column; they are respectively for the 4-inch gun at 2000 yards—

$$52 \times 3 \tan 3^\circ 13' = 8{\cdot}7 \text{ feet}$$
$$28 \times 3 \tan 3^\circ 13' = 4{\cdot}8 \;\;\text{,,}$$
$$20 \times 3 \tan 3^\circ 13' = 3{\cdot}4 \;\;\text{,,}$$

And similar the last column (e) is

$$100 \times 3 \tan 3^\circ 13' = 16{\cdot}9 \;\;\text{,,}$$

(b.) This correction is given in Part I, Chapter XV, p. 190, and needs no further explanation.

(c.) The correction for wind is made according to Colonel Maitland's method (*vide* Pro. R.A.I., vol. viii, p. 343).

Let us again take a definite example, the same 4-inch projectile at the same range (2000 yards) as before, and suppose half a gale (50 f.s.) blowing down the range with the projectile. Had the direction of the wind been oblique at an angle θ to the line of fire, the resolved velocity of the wind in direction of the range would be 50 cos θ f.s.

The steps of the calculation are as follows:—

(1.) Find the final velocity in still air with M.V. (1900 f.s.) from the (s) table at 2000 yards; it is 1112 f.s. Employ these values of the muzzle and final velocities in the (t) table to find the time of flight; it is 4·0769 seconds.
This time of flight will be the same in still air or in a wind, as the vertical component of velocity is not affected, gravity bringing the projectile to the ground after the same time in each case.

(2.) With this time of flight, supposing the M.V. to be real M.V. − velocity of the wind (1900 − 50 = 1850 f.s.), employ the (t) table and find the final velocity; it is 1097 f.s. Employ these values of muzzle and final velocities (1850 f.s. and 1097 f.s.) to find the range; it is 1960 yards: this is the range *in* the wind, in the same way as a boat rowed in a current ranges a certain distance in a given time from an object moving at the speed of the current; it is *not* the *actual* range with regard to stationary objects.

(3.) In order to find the *actual* range when the wind is blowing, the distance moved by the wind in the time of flight (4·0769 × 50 = 204 feet = 68 yards) must be added to the range (1960 yards) found in (2), and we have 2028 yards. The difference (28 yards) between this last range and the original one (2000 yards) is the correction required.

(d.) These corrections are due to variations in the density of the air. From Table XI, p. 305, we note that with air two-thirds saturated with moisture and thermometer normal (60° F.), a difference of 1 inch (*i.e.*, to 29 or 31 inches) in the barometer from the standard 30 inches alters the density from what is tabulated as unity to 0·966 or 1·033 respectively, a difference of about 0·033 in each case.

Again, if the barometer remains at the standard 30 inches, and the thermometer varies 16° F. either way (to 76° F. or 44° F.), the density of the air changes to 0·967 or 1·033 respectively, a difference of 0·033 in each case, which is identical with the other difference, previously obtained when the barometric pressure was varied.

Taking the 4-inch gun as before at 2000 yards range, if the barometer stands at 29 inches, thermometer normal, or if the barometer is at the standard and the thermometer is at 76° F., the density of the air in each case is less than the normal by about the same amount, and the projectile will range further than under normal conditions.

To find the correction to be made: the angle of departure and M.V. must be found from the range tables, and the range must be calculated by Niven's method (*see* p. 161) with a value of $\frac{d^2}{w} = 0{\cdot}64$, and afterwards with a value of $\frac{d^2}{w} = 0{\cdot}64 \times 0{\cdot}937$ (this last quantity being the factor τ from Table XI for the altered density of the air, corresponding to 29 in bar. or 76° F. ther.). The difference (20 yards) found between these two ranges is the correction required.

Cross Wind.

With regard to the effects of cross wind, the assumption that the deviation is directly proportional to the velocity of the wind resolved across the range and to the time of flight is a fair approximation.

By Colonel Maitland's formula (*vide* Pro. R.A.I., vol. viii, p. 348) the deviation in feet is

$$= Ut - \frac{w}{{\cdot}00232438Ag} \log_e \left(\frac{{\cdot}00232438AgUt}{w} + 1 \right)$$

in which U = velocity of the wind resolved across the range in f.s.,
A = longitudinal area of the projectile in square *feet.*

t, w, g have the meaning usually assigned to them. Lieutenant Whistler, U.S.A., has further investigated this method; but exactly correct results cannot be obtained in all cases, as all the necessary data have not been experimentally determined.

Table XVI.

British Ordnance.

(Chiefly founded on the official "List of Service Rifled Ordnance, 1886.")

Ordnance.

Nature.				Interior Dimensions.				Rifling.			
					Chamber.			Twist one turn in			
	Calibre or Pr.	Weight.	Mark.	Length of bore, including chamber.	Diameter.	Length.	Capacity.	Least at breech.	Greatest at muzzle.	Distance at muzzle twist is uniform.	System.*
				ins.	ins.	ins.	cub. ins.	cals.	cals.	ins.	
B.L. Guns (new type).	16·25-in.†	110 tons.	...	487·5	21·12	83·4	29,000	...	...	...	P.P.
	13·5-in.†	67 tons.	II.	405·0	18·0	66·5	17,100	120	30	166·7	P.H.
	12-in.	45 tons.	V.	303·0	16·0	49·0	9,780	120	35	124·5	P.H.
	10-in.	29 tons.	II.	320·0	14·0	54·0	8,370	120	30	166·7	P.H.
	9·2-in.	22 tons.	V.	289·8	12·0	43·8	5,000	120	30	120·4	P.H.
	8-in.	14 tons.	VI.	236·9	10·5	38·0	3,353	120	35	96·1	P.H.
	6-in.	5 tons.	V.	192·0	8·0	31·5	1,500	0	30	6·25	P.H.
	5-in.	40 cwts.	III.	125·0	5·75	19·3	510	120	25	51·8	P.H.
	4-in.	26 cwts.	IV.	108·0	5·3	18·5	417	120	30	43·77	P.H.
	4-in.	13 cwts.	I.	59·25	4·5	8·12	126·5	116	35	11·39	P.H.
	12-pr.	7 cwts.	I.	84·0	3·625	11·2	118	120	28	35·8	P.H.
M.L. Guns.	17·72-in.	100 tons.	I.	363·0	19·7	59·72	16,957	150	50	2·88	P.P.
	16-in.	80 tons.	I.	288·0	18·0	59·6	14,600	0	50	0	P.P.
	12·5-in.	38 tons.	II.	198·0	14·0	41·125	6,000	438	35	0	W.
	12-in.	35 tons.	I.	162·5	Unchambered.			0	35	0	W.
	12-in.	25 tons.	II.	145·0	Unchambered.			100	50	0	W.
	11-in.	25 tons.	II.	145·0	Unchambered.			0	35	0	W.
	10-in.	18 tons.	II.	145·5	Unchambered.			100	40	0	W.
	9-in.	12 tons.	V.	125·0	Unchambered.			0	45	0	W.
	8-in.	9 tons.	III.	118·0	Unchambered.			0	40	0	W.
	7-in.	7 tons.	IV.	126·0	Unchambered.			35	35	All	W.
	6·6-in.	70 cwts	I.	97·5	6·8	21·0	709	100	35	13·2	P.P.
	64-pr.	64 cwts.	III.	97·5	Unchambered.			40	40	All	Pl.
	40-pr.	35 cwts.	II.	104·5	Unchambered.			35	35	All	W.
	25-pr.	18 cwts.	I.	88·0	Unchambered.			35	35	All	W.
	16-pr.	12 cwts.	I.	68·4	Unchambered.			30	30	All	F.M.
	13-pr.	8 cwts.	I.	84·0	3·15	14·13	109	100	30	9·0	P.P.
	9-pr.	6 cwts.	III.	66·0	Unchambered.			30	30	All	F.M.
	2·5-in.	400 lbs.	I.	66·5	2·56	11·07	54	80	30	3·53	P.P.
	7-pr.	200 lbs.	IV.	36·0	Unchambered.			20	20	All	F.
R.B.L. guns (screw).	7-in.	82 cwts.	...	99·5	7·2	16·0	620·0	37	37	All	P.
	40-pr.	35 cwts.	...	106·3	4·96	13·5	257·8	36½	36½	All	P.
	20-pr.	16 cwts.	...	84·0	3·94	12·0	143·0	38	38	All	P.
	12-pr.	8 cwts.	...	61·3	3·2	8·5	66·0	38	38	All	P.
	9-pr.	6 cwts.	...	52·5	3·2	7·0	55·1	38	38	All	P.
Quick Firing Guns.											
Hotchkiss	6-pr.	8 cwts.	I.	89·76	2·713	10·099	46·0	362	29·89	9·98	P.P.
Nordenfelt	6-pr.	6 cwts.	I. and II.	95·0	2·713	10·099	46·0	362	29·98	14·98	P.P.
Hotchkiss	3-pr.	4 cwts.	I.	74·06	2·283	12·48	46·0	25	25	All	P.P.
Machine Guns.											
Nordenfelt, 4 bar.	1-in.	447 lbs.	III.	35·48	...	...	...	35	35	All	H
Gatling, 10 bar.	0·65-in.	787 lbs.	...	33·0	...	...	...	46·1	46·1	All	H
Gardner, 2 bar. ...	0·45-in.	131 lbs.	I.	30·0	...	...	...	48·9	48·9	All	H

* P means Polygroove; P.H., Polygroove, hook section; P.P., Polygroove, plain section; Pl, Plain; W., Woolwich; F., French; F.M., French modified; H., Henry.

† Not yet actually in the Service; all details not yet finally approved

TABLE XVI (*continued*).

British Ordnance.

Ordnance. Nature.	Calibre or Pr.	Charge (full). Weight.	Charge (full). Description.	Projectile. Diameter.	Projectile. Weight.*	Value of $\frac{d^2}{w}$.	Value of $\frac{w}{d^2}$, or the ballistic co-efficient.	Ballistics (with full charges). Muzzle velocity.	Total muzzle energy.	Muzzle energy per ton of gun.	Perforation of wrought iron. At muzzle.	At 1000 yards range.	At 2000 yards range.
		lbs. ozs.		ins.	lbs. ozs.			f.s.	ft. tons.	ft. tons.	ins.	ins.	ins.
B.L. Guns (new type).	16·25in.	900 0	Prism[1], brown	16·25	1800 0	0·147	0·420	2216†	61,290	554·0	36·18	33·72	31·61
	13·5 in.	600 0	Prism[1], brown	13·5	1250 0	0·146	0 508	2000†	34,675	517·6	30·0	28·0	26·1
	12-in.	295 0	Prism[1], brown	12·0	714 0	0·202	0·413	1910	18,060	420·1	22·5	20·6	18·8
	10-in.	250 0	Prism[1], brown	10·0	...	...	...	2100†	15,290	527·2	23·0	21·2	19·5
	9·2-in.	175 0	Prism[1], brown	9·2	380 0	0·238	0·477	2065	11,240	511·1	20·3	18·3	16·4
	8-in.	125 0	Prism[1], brown	8·0	210 0	0·305	0·410	2200†	7,132	508·2	17·4	15·2	13·3
	6-in.	42 0	P.[2]	6·0	100 0	0·36	0·463	1920	2,556	511·2	12·1	10·3	8·7
	5-in.	16 0	S.P.	5·0	50 0	0·500	0·400	1780	1,098	549·0	8·6	6·8	5·4
	4-in.	12 0	S.P.	4·0	25 0	0·640	0·391	1900	626	500·8	7·3	5·4	4·2
	4-in.	3 4	R.L.G.[2]	4·0	25 0	0·640	0·391	1180	241	371·4	...	...	...
	12-pr.	4 0	S.P.	3·0	12 5	0·720	0·463	1710	254	725·7	...	...	...
M.L. Guns.	17·72in.	450 0	Prism[1], black	17·72	1968 0	0·159	0·374	1548	32,710	327·1	24·5	23·2	21·4
	16-in.	450 0	Prism[1], brown	16·0	1684 0	0·152	0·411	1590	29,530	369·1	24·7	23·1	21·3
	12·5 in.	210 0	Prism[2], black	12·5	809 6	0·193	0·415	1575	13,930	366·5	18·4	17·5	15·9
	12-in.	140 0	P[2].	12·0	706 12	0·204	0·409	1390	9,469	250·5	15·9	14·4	13·1
	12-in.	85 0	P[2].	12·0	608 6	0·237	0·352	1292	7,046	281·8	13·5	12·6	11·4
	11-in.	85 0	P.	11·0	543 2	0·223	0·408	1360	7,015	280·6	14·3	12·8	11·5
	10-in.	70 0	P.	10·0	406 0	0·246	0·406	1379	5,356	297·5	12·9	11·6	10·4
	9-in.	50 0	P.	9·0	253 5	0·320	0·347	1440	3,643	303·6	11·3	9·6	8·4
	8-in.	35 0	P.	7·92	174 12	0·359	0·352	1384	2,323	258·1	9·6	8·1	7·0
	7-in.	30 0	P.	6·92	112 1	0·427	0·338	1561	1,895	270·6	9·5	7·7	6·4
	6·6-in.	25 0	P.	6·6	97 12	0·446	0·340	1416	1,358	388·1	...	...	...
	64-pr.	8 0	R.L.G.[2]§	6·28	67 2	0·588	0·271	1260	740	231·3	...	...	...
	40-pr.	7 0	R.L.G.[2]§	4·75	40 14	0·552	0·382	1425	576	329·4	...	...	...
	25-pr.	4 0	R.L.G.[2]§	3·94	25 7	0·605	0·420	1340	320	356·0	...	...	...
	16-pr.	3 0	R.L.G.[2]§	3·54	16 4	0·771	0·366	1355	207	344·7	...	...	...
	13-pr.	3 2	R.L.G.[2]	3·0	13 1	0·689	0·484	1560	220	540·8	...	...	...
	9-pr.	1 12	R.L.G.[2]§	2·94	9 2	0·946	0·359	1390	122	407·6	...	...	...
	2·5-in.	1 8	R.L.G.[2]	2·5	7 0	0·893	0·448	1440	96	538·4	...	...	...
	7-pr.	0 12	R.F.G.[2]‖	2·94	7 5	1·181	0·288	950	46	513·1	...	...	...
R.B.L. guns (screw).	7-in.	11 0	R.L.G.[2]	7·0	109 0	0·450	0·318	1100	915	223·1	...	...	...
	40-pr.	5 0	R.L.G.[2]§	4·75	40 2	0·562	0·374	1180	388	221·4	...	...	...
	20-pr.	2 8	R.L.G.[2]§	3·75	21 13	0·645	0·414	1130	193	241·5	...	...	...
	12-pr.	1 8	R.L.G.[2]§	3·0	11 4	0·800	0·417	1239	118	294·5	...	...	...
	9-pr.	1 2	R.L.G.[2]§	3·0	8 8	1·059	0·315	1055	66	218·8	...	...	...
Quick Firing Guns.											Mild steel plate.		
Hotchkiss ...	6-pr.	1 15¼	C[2] (Sevran Levry)	2·236	6 0	0·844	0·527	1820	137·8	344·5	3·2	2·0	...
Nordenfelt ...	6-pr.	2 2‡	Pigou and Wilks	2·23	6 0	0·844	0·527	1860	143·9	479·6	3·3	2·1	...
Hotchkiss ...	3-pr.	1 1⅞	C[2] (Sevran Levry)	1·85	3 8	0·978	0·553	2000	97·1	485·5	2·9	1·8	...
Machine Guns.		grains.			oz. grs.								
Nordenfelt ...	1-in.	625	...	1·0	7¼ 0	2·207	0·453	1464	...	...	⅝-in. at 200 yards.		
Gatling ...	0·65-in.	270	...	0·65	0 1425	2·109	0·730	...	...	...	Not known.		
Gardner ...	0·45-in.	85	...	0·45	0 480	2·952	0·751	...	...	...	Same as M.-H. bullet.		

* For the higher natures the weight of projectile given is for Palliser shot; for the lower natures it is for filled common shell.
† Estimated. ‡ Or Prism[3] § Or R.L.G., or L.G. ‖ Or F.G.
NOTE.—This list does not include all British rifled guns; howitzers are omitted altogether.

Table XVII.

Foreign Ordnance.

(Shortened from Captain Orde Browne's Table in Lord Brassey's Naval Annual, 1886.)

Ordnance.

Nation.	Calibre.	Corresponding calibre.	M.L. or B.L.	Weight, including breech gear.	Length of rifled portion of bore.	Length of chamber.	Twist of rifling, one turn in
	cm.	ins.		tons.	ins.	ins.	calibres.
Austria	30·5	12·01	B.L.	47·8	314·8	69·9	...
	28	11·02	B.L.	27·1	169·8	37·4	45
	26	10·24	B.L.	21·7	148·4	46·1	70
	15	5·87	B.L.	4·69	149·6	35·4	25
	12	4·72	B.L.	2·25	...	...	25
	9	3·43	...	0·48	57·5	16·5	45
Denmark	35·5	13·98	B.L.	51·2	...	...	45
	30·5	12·01	B.L.	35·4	...	...	45
	25·42	10·01	M.L.	20·0	...	...	45
France	37	14·66	B.L.	71	...	...	...
	34	13·39	B.L.	51·8	...	...	...
	27	10·80	B.L.	27·85	...	...	4°
	10	3·94	B.L.	1·24	...	...	7°
	9	3·54	B.L.	0·59	...	...	7°
Germany	30·5	12·01	B.L.	35·4	181·9	45·3	45
	26	10·33	B.L.	21·7	149·8	44·7	50
	24	9·45	B.L.	18·7	201·6	53·5	25
	15	5·87	B.L.	4·04	128·5	31·1	25
	10·5	3·96	B.L.	1·15	113·6	19·5	25
	8·7	3·43	B.L.	0·44	62·7	10·7	40
Holland	28	11·02	B.L.	27·21	170·8	36·4	...
	18	7·0	M.L.	7·17	95·5	15·5	...
	17	6·80	B.L.	5·51	112·7	36·0	...
Italy*	43	17·00	B.L.	104·3	...	...	...
	45	17·70	M.L.	99·9	305·0	56·5	50
	28	11·00	M.L.	25·0	121·0	24·5	35
	25	10·00	M.L.	18·0	120·0	26·0	40
	16	7·00	M.L.	5·12	96·0	21·3	42·5
	7·5	3·00	B.L.	0·29	52·0	10·2	48
Russia	30·5	12·00	B.L.	42·7	...	...	...
	28·0	11·00	B.L.	28·2	158·0	50·4	...
	22·86	9·00	B.L.	15·0	124·0	28·5	60
	15·24	6·00	B.L.	4·03	98·0	22·2	68
	10·67	4·20	B.L.	0·60	61·5	10·5	40
	8·71	3·43	B.L.	0·45	62·6	10·7	40
Spain	20	7·87	B.L.	10·8	156·9	40·0	40
	16	6·34	B.L.	6·1	...	...	40
	15·24	6·00	B.L.	4·0	126·9	29·7	45
Sweden and Norway	26	10·24	B.L.	21·7	160·4	34·1	45
	21·59	8·50	M.L.	21·4	137·3	38·2	55
	15	5·91	B.L.	3·9	112·4	22·6	45
United States	26·67	10·5	B.L.	29·0	281·8	69·0	30
	20·32	8	B.L.	12·28	199·0	45·5	30
	15·24	6	B.L.	4·58	144·26	39·5	30
	12·70	5	B.L.	2·77	120·75	29·2	30

* Italy has also some 40 c.m. guns of 118 tons of recent manufacture by Krupp.

Note.—This table only gives a few guns of each nation.

Table XVII.

Foreign Ordnance.

Nation.	Calibre.	Ammunition. Weight of full charge.	Ammunition. Weight of projectile.*	Ballistics. Muzzle velocity.	Ballistics. Total muzzle energy.	Ballistics. Perforation of wrought iron at muzzle.	Remarks.
	cm.	lbs.	lbs.	f.s.	ft. tons.	ins.	
Austria	30·5	...	1003·1	...	...	...	Krupp.
	28	123·5	557·8	1568	9,513	17·0	Krupp.
	26	70·5	395·7	1404	5,409	13·2	Krupp.
	15	38·8	86·0	1969	2,312	11·7	Krupp.
	12	19·8	57·3	1738	1,201	9·4	Krupp.
	9	3·31	14·2	1470	198	...	Uchatius.
Denmark	35·5	253·5	1157·4	1772	21,580	22·9	Krupp.
	30·5	158·7	725·3	1673	12,200	18·56	Krupp.
	25·42	69·4	402·3	1388	5,376	13·3	10-inch Armstrong.
France	37	546·0	1180·0	1955	31,272	27·4	Pattern 1884. For description of later patterns, see Manual of Naval Gunnery, 1886.
	34	357·1	925·9	...	24,868	25·51	Pattern 1881.
	27	165·3	476·2	1736	9,942	17·8	Pattern 1875.
	10	10·1	30·9	1673	600	...	Pattern 1875.
	9	3·62	17·6	1493	272	...	
Germany	30·5	202·8	725·3	1718	14,740	20·5	Krupp.
	26	105·8	412·3	1588	7,211	15·4	Krupp.
	24	149·9	474·0	1657	9,024	18·1	Krupp.
	15	33·1	112·4	1624	2,055	11·0	Krupp.
	10·5	8·8	39·7	1526	641	...	Krupp.
	8·7	3·3	14·9	1545	247	...	Krupp.
Holland	28	121·3	560·0	1558	9,423	17·0	Krupp.
	18	30·0	114·6	1558	1,929	9·7	Armstrong.
	17	27·6	132·3	1558	2,226	10·5	Krupp.
Italy	43	899·5	1799·0	2018	50,810	32·3	Armstrong, new pattern.
	45	507·1	2001·8	1614	36,178	26·3	Armstrong.
	28	94·8	560·0	1329	6,859	14·2	Armstrong.
	25	77·2	451·9	1388	6,035	14·1	Armstrong.
	16	19·8	101·4	1303	1,195	7·5	Old pattern.
	7·5	1·9	9·17	1335	113	...	
Russia	30·5	282·2	758·4	1959	19,880	24·2	Obuchoff.
	28·0	132·2	562·2	1516	8,999	16·6	Obuchoff,
	22·86	47·0	275·6	1463	3,701	11·7	Obuchoff.
	15·24	18·1	86·0	1463	1,276	8·4	Obuchoff.
	10·67	4·5	27·6	...	...	...	Long 9-pr. Field.
	8·71	3·1	15·2	...	...	...	Long 4-pr. Field.
Spain	20	61·7	183·0	...	...	...	
	16	71·7	132·3	2051	3,860	14·6	
	15·24	38·8	80·0	2100	2,446	11·9	6-inch Armstrong.
Sweden and Norway	26	99·2	463·0	1575	7,966	16·2	Krupp.
	21·59	121·3	447·5	1555	7,502	15·6	8·5-inch Armstrong.
	15	18·7	86·0	1575	1,479	9·2	Krupp.
United States	ins.						
	10·5	275·0	550·0	2100	16,813	23·7	Under construction.
	8	125·0	250·0	2150	8,011	18·7	
	6	50·0	100·0	2000	2,773	12·7	
	5	30·0	60·0	1950	1,582	10·5	Finished.

* For the heavier natures the weights are those of armour-piercing projectiles: for lighter pieces they are those of empty common shell.

TABLE XVIII.

Conversion of Measures.

(Chiefly based on data contained in Col. Noble's Useful Tables.)

Length.

Metric to English.

Mètres.	Yards.	Feet.	Inches.
1	1·0936	3·2809	39·37
2	2·1873	6·5618	78·74
3	3·2809	9·8427	118·11
4	4·3745	13·1236	157·48
5	5·4682	16·4045	196·85
6	6·5618	19·6854	236·22
7	7·6554	22·9663	275·60
8	8·7491	26·2472	314·97
9	9·8427	29·5281	354·34

English to Metric.

Yds.	Mètres.	Ft.	Mètres.	Ins.	Centimètres.
1	0·91438	1	0·30479	1	2·5400
2	1·82877	2	0·60959	2	5·0799
3	2·74315	3	0·91438	3	7·6199
4	3·65753	4	1·21918	4	10·1598
5	4·57192	5	1·52397	5	12·6998
6	5·48630	6	1·82877	6	15·2397
7	6·40068	7	2·13356	7	17·7797
8	7·31507	8	2·43836	8	20·3196
9	8·22945	9	2·74315	9	22·8596

Metric Table of Length.

Millimètres.
10 = 1 centimètre.
100 = 1 décimètre.
1000 = 1 mètre.
Mètres.
10 = 1 décamètre.
100 = 1 hectomètre.
1000 = 1 kilomètre.

EXPLANATION.—To convert any number from one measure to the other, take the values of the different multiples of 10 by shifting the position of the decimal point, and add together. Thus, find the number

of yards in 2354 mètres (see cols. 1 and 2.)	of feet in 12·4 mètres (see cols. 1 and 3).	of inches in 30·5 centimètres (see cols. 1 and 4). Note, 1 m. = 100 cm.	of mètres in 1026 yards (see cols. 5 and 6).	of mètres in 1742 feet (see cols. 7 and 8).	of centimètres in 17·72 ins. (see cols. 9 and 10).
metres. yards.	metres. feet.	cms. inches.	yards. metres.	feet. metres.	inches. cms.
2000=2187·3	10 =32·809	30·0=11·811	1000=914·38	1000=304·79	10 =25·400
300= 328·09	2 = 6·562	·5= 0·197	20= 18·29	700=213·36	7 =17·780
50= 54·68	0·4= 1·312		6= 5·49	40= 12·19	0·7 = 1·778
4= 4·37				2= 0·61	0·02= 0·051
. 2354=2574·44	∴ 12·4=40·683	∴ 30·5=12·008	∴ 1026=938·16	∴ 1742=530·95	∴ 17·72=45·009

NOTE.—If a table of conversion is not at hand, an approximation to the English equivalent in inches of a distance measured in centimètres may be obtained by multiplying by 0·4: thus 30·5 cm. multiply by 0·4, and we have 12·2 inches; the real equivalent as shown above is 12·008 inches.

Weight.

Metric to English.

Kilogrammes.	Tons.	Pounds Avoirdupois.	Grains Troy.
1	·000984	2·2046	15432·3
2	·001968	4·4092	30864·7
3	·002953	6·6139	46297·0
4	·003937	8·8185	61729·4
5	·004921	11·0231	77161·7
6	·005905	13·2277	92594·1
7	·006889	15·4323	108026·4
8	·007874	17·6370	123458·8
9	·008858	19·8416	138891·1

English to Metric.

Tons.	Metric tons or milliers.	Pounds Avoirdupois.	Kilogrammes.	Grains. Troy.	Grammes.
1	1·016	1	0·4536	1	·0648
2	2·032	2	0·9072	2	·1296
3	3·048	3	1·3608	3	·1944
4	4·064	4	1·8144	4	·2592
5	5·080	5	2·2680	5	·3240
6	6·096	6	2·7216	6	·3888
7	7·112	7	3·1751	7	·4536
8	8·128	8	3·6287	8	·5184
9	9·144	9	4·0823	9	·5832

Metric Table of Weight.

Milligrammes.
10 = 1 centigramme.
100 = 1 décigramme.
1000 = 1 gramme.
Grammes.
10 = 1 décagramme.
100 = 1 hectogramme.
1000 = 1 kilogramme.
Kilogrammes.
10 = 1 myriagramme.
100 = 1 quintal.
1000 = 1 millier, or metric ton.

EXPLANATION.—To convert any number from one measure to the other, take the values of the different multiples of 10 by shifting the position of the decimal point and add together. Thus, find the number

of tons in 35 milliers (see cols. 1 and 2).	of pounds in 56·3 kgms. (see cols. 1 and 3).	of grains in 120 grammes (see cols. 1 and 4).	of milliers in 38 tons (see cols. 5 and 6).	of kilogrammes in 68 pounds (see cols. 7 and 8).	of grammes in 85 grains (see cols. 9 and 10).
milliers. tons.	kgms. lbs.	grammes. grains.	tons. milliers.	lbs. kgs.	grains. grammes.
30 = 29·53	50 =110·231	100 = 1543·23	30 = 30·48	60 = 27·216	80 = 5·184
5 = 4·92	6 = 13·228	20 = 308·65	8 = 8·13	8 = 3·629	5 = 0·324
	0·3= 0·661				
∴ 35 = 34·45	∴ 56·3=124·120	∴ 120 = 1851·88	∴ 38 = 38·61	∴ 68 = 30·845	∴ 85 5·508

NOTE.—7000 grains troy = 1 pound avoirdupois.

TABLE XVIII (*continued*).

Pressure.

Metric and Atmospheric to English.

Kilogrammes per sq. cm.	Pounds per square inch.	Tons per square inch.	Atmospheres	Pounds per square inch.	Tons per square inch.
1	14·223	·00635	1	14·7	·00656
2	28·446	·01270	2	29·4	·01313
3	42·668	·01905	3	44·1	·01969
4	56·891	·02540	4	58·8	·02625
5	71·114	·03175	5	73·5	·03281
6	85·337	·03810	6	88·2	·03938
7	99·560	·04445	7	102·9	·04594
8	113·783	·05080	8	117·6	·05250
9	128·005	·05715	9	132·3	·05906

English to Metric and Atmospheric.

Pounds per square inch.	Kilogrammes per sq. cm.	Atmospheres.	Tons per square inch.	Kilogrammes per sq. cm.	Atmospheres.
1	·07031	·068	1	157·49	152·38
2	·14062	·136	2	314·99	304·76
3	·21093	·204	3	472·48	457·14
4	·28124	·272	4	629·97	609·52
5	·35155	·340	5	787·47	761·91
6	·42186	·408	6	944·96	914·29
7	·49217	·476	7	1102·45	1066·67
8	·56248	·544	8	1259·95	1219·05
9	·63279	·612	9	1417·44	1371·43

EXPLANATION.—To convert any number from one measure to the other, take the values of the different multiples of 10 by shifting the position of the decimal point, and add together. Thus, find the number

of pounds per square inch in 32·1 kilogrammes per square centimètre (see cols. 1 and 2).

kgs. per sq. cm.		lbs. per sq. in.
30	=	426·68
2	=	28·45
0·1	=	1·42
∴ 32·1	=	456·55

of tons per square inch in 3210 kilogrammes per square centimètre (see cols. 1 and 3).

kgs. per sq. cm.		tons per sq. in.
3000	=	19·05
200	=	1·27
10	=	0·06
∴ 3210	=	20·38

of tons per square inch in 3254 atmospheres (see cols. 4 and 6).

atmospheres.		tons per sq. in.
3000	=	19·69
200	=	1·31
50	=	0·33
4	=	0·03
∴ 3254	=	21·36

of kilogrammes per square centimètre in 15 lbs. on the square inch (see cols. 7 and 8).

lbs. per sq. in.		kgs. per sq. cm.
10	=	0·7031
5	=	0·3516
∴ 15	=	1·0547

of kilogrammes per square centimètre in 18·3 tons per square inch (see cols. 10 and 11).

tons per sq. in.		kgs. per sq. cm.
10	=	1574·9
8	=	1259·95
0·3	=	47·25
∴ 18·3	=	2882·1

of atmospheres in 14·6 tons per square inch (see cols. 10 and 12).

tons per sq. in.		atmospheres.
10	=	1523·8
4	=	609·5
0·6	=	91·4
∴ 14·6	=	2224·7

Energy.

Metric to English.

Mètre-tons.	Foot-tons.
1	3·2291
2	6·4581
3	9·6872
4	12·9162
5	16·1453
6	19·3743
7	22·6034
8	25·8324
9	29·0615

English to Metric.

Foot-tons.	Mètre-tons.
1	0·3097
2	0·6194
3	0·9291
4	1·2388
5	1·5484
6	1·8581
7	2·1678
8	2·4775
9	2·7872

EXPLANATION.—To convert any number from one measure to the other, take the values of the different multiples of 10 by shifting the position of the decimal point, and add together. Thus, find the number

of foot-tons in 4367 mètre-tons (see cols. 1 and 2).

mètre-tons.		foot-tons.
4000	=	12916·2
300	=	968·72
60	=	193·74
7	=	22·60
∴ 4367	=	14101·26

of mètre-tons in 3592 foot-tons (see cols. 3 and 4).

foot-tons.		mètre-tons.
3000	=	929·1
500	=	154·84
90	=	27·87
2	=	0·62
∴ 3592	=	1112·43

NOTE.—1000 mètre-tons is called a dinamode in Italy.

The Metrical System.

In the metrical system, the mètre is intended to be (and is very nearly) the ten millionth part of the distance on the surface of the earth from the equator to the pole; the cube of a decimètre, or of the tenth part of a mètre, is the unit of capacity, and is called a litre, and the weight of a litre of distilled water at its greatest density is the unit of weight called a kilogram.

Table XIX.

Four Figure Logarithms.

(A.) Logarithms of Numbers.

No.	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
											Fourth Figure.								
10	0000	0043	0086	0128	0170	0212	0253	0294	0334	0374	4	8	12	17	21	25	29	33	37
11	0414	0453	0492	0531	0569	0607	0645	0682	0719	0755	4	8	11	15	19	23	26	30	34
12	0792	0828	0864	0899	0934	0969	1004	1038	1072	1106	3	7	10	14	17	21	24	28	31
13	1139	1173	1206	1239	1271	1303	1335	1367	1399	1430	3	6	10	13	16	19	23	26	29
14	1461	1492	1523	1553	1584	1614	1644	1673	1703	1732	3	6	9	12	15	18	21	24	27
15	1761	1790	1818	1847	1875	1903	1931	1959	1987	2014	3	6	8	11	14	17	20	22	25
16	2041	2068	2095	2122	2148	2175	2201	2227	2253	2279	3	5	8	11	13	16	18	21	24
17	2304	2330	2355	2380	2405	2430	2455	2480	2504	2529	2	5	7	10	12	15	17	20	22
18	2553	2577	2601	2625	2648	2672	2695	2718	2742	2765	2	5	7	9	12	14	16	19	21
19	2788	2810	2833	2856	2878	2900	2923	2945	2967	2989	2	4	7	9	11	13	16	18	20
20	3010	3032	3054	3075	3096	3118	3139	3160	3181	3201	2	4	6	8	11	13	15	17	19
21	3222	3243	3263	3284	3304	3324	3345	3365	3385	3404	2	4	6	8	10	12	14	16	18
22	3424	3444	3464	3483	3502	3522	3541	3560	3579	3598	2	4	6	8	10	12	14	15	17
23	3617	3636	3655	3674	3692	3711	3729	3747	3766	3784	2	4	6	7	9	11	13	15	17
24	3802	3820	3838	3856	3874	3892	3909	3927	3945	3962	2	4	5	7	9	11	12	14	16
25	3979	3997	4014	4031	4048	4065	4082	4099	4116	4133	2	3	5	7	9	10	12	14	15
26	4150	4166	4183	4200	4216	4232	4249	4265	4281	4298	2	3	5	7	8	10	11	13	15
27	4314	4330	4346	4362	4378	4393	4409	4425	4440	4456	2	3	5	6	8	9	11	13	14
28	4472	4487	4502	4518	4533	4548	4564	4579	4594	4609	2	3	5	6	8	9	11	12	14
29	4624	4639	4654	4669	4683	4698	4713	4728	4742	4757	1	3	4	6	7	9	10	12	13
30	4771	4786	4800	4814	4829	4843	4857	4871	4886	4900	1	3	4	6	7	9	10	11	13
31	4914	4928	4942	4955	4969	4983	4997	5011	5024	5038	1	3	4	6	7	8	10	11	12
32	5051	5065	5079	5092	5105	5119	5132	5145	5159	5172	1	3	4	5	7	8	9	11	12
33	5185	5198	5211	5224	5237	5250	5263	5276	5289	5302	1	3	4	5	6	8	9	10	12
34	5315	5328	5340	5353	5366	5378	5391	5403	5416	5428	1	3	4	5	6	8	9	10	11
35	5441	5453	5465	5478	5490	5502	5514	5527	5539	5551	1	2	4	5	6	7	9	10	11
36	5563	5575	5587	5599	5611	5623	5635	5647	5658	5670	1	2	4	5	6	7	8	10	11
37	5682	5694	5705	5717	5729	5740	5752	5763	5775	5786	1	2	3	5	6	7	8	9	10
38	5798	5809	5821	5832	5843	5855	5866	5877	5888	5899	1	2	3	5	6	7	8	9	10
39	5911	5922	5933	5944	5955	5966	5977	5988	5999	6010	1	2	3	4	6	7	8	9	10
40	6021	6031	6042	6053	6064	6075	6085	6096	6107	6117	1	2	3	4	5	6	7	9	10
41	6128	6138	6149	6160	6170	6180	6191	6201	6212	6222	1	2	3	4	5	6	7	8	9
42	6232	6243	6253	6263	6274	6284	6294	6304	6314	6325	1	2	3	4	5	6	7	8	9
43	6335	6345	6355	6365	6375	6385	6395	6405	6415	6425	1	2	3	4	5	6	7	8	9
44	6435	6444	6454	6464	6474	6484	6493	6503	6513	6522	1	2	3	4	5	6	7	8	9
45	6532	6542	6551	6561	6571	6580	6590	6599	6609	6618	1	2	3	4	5	6	7	8	9
46	6628	6637	6646	6656	6665	6675	6684	6693	6702	6712	1	2	3	4	5	6	7	7	8
47	6721	6730	6739	6749	6758	6767	6776	6785	6794	6803	1	2	3	4	5	5	6	7	8
48	6812	6821	6830	6839	6848	6857	6866	6875	6884	6893	1	2	3	4	4	5	6	7	8
49	6902	6911	6920	6928	6937	6946	6955	6964	6972	6981	1	2	3	4	4	5	6	7	8
50	6990	6998	7007	7016	7024	7033	7042	7050	7059	7067	1	2	3	3	4	5	6	7	8
51	7076	7084	7093	7101	7110	7118	7126	7135	7143	7152	1	2	3	3	4	5	6	7	8
52	7160	7168	7177	7185	7193	7202	7210	7218	7226	7235	1	2	2	3	4	5	6	7	7
53	7243	7251	7259	7267	7275	7284	7292	7300	7308	7316	1	2	2	3	4	5	6	6	7
54	7324	7332	7340	7348	7356	7364	7372	7380	7388	7396	1	2	2	3	4	5	5	6	7
55	7404	7412	7419	7427	7435	7443	7451	7459	7466	7474	1	2	2	3	4	5	5	6	7
56	7482	7490	7497	7505	7513	7520	7528	7536	7543	7551	1	2	2	3	4	5	5	6	7
57	7559	7566	7574	7582	7589	7597	7604	7612	7619	7627	1	2	2	3	4	5	5	6	7
58	7634	7642	7649	7657	7664	7672	7679	7686	7694	7701	1	1	2	3	4	4	5	6	7
59	7709	7716	7723	7731	7738	7745	7752	7760	7767	7774	1	1	2	3	4	4	5	6	7
60	7782	7789	7796	7803	7810	7818	7825	7832	7839	7846	1	1	2	3	4	4	5	6	6
61	7853	7860	7868	7875	7882	7889	7896	7903	7910	7917	1	1	2	3	4	4	5	6	6
62	7924	7931	7938	7945	7952	7959	7966	7973	7980	7987	1	1	2	3	3	4	5	6	6
63	7993	8000	8007	8014	8021	8028	8035	8041	8048	8055	1	1	2	3	3	4	5	5	6
64	8062	8069	8075	8082	8089	8096	8102	8109	8116	8122	1	1	2	3	3	4	5	5	6
65	8129	8136	8142	8149	8156	8162	8169	8176	8182	8189	1	1	2	3	3	4	5	5	6
66	8195	8202	8209	8215	8222	8228	8235	8241	8248	8254	1	1	2	3	3	4	5	5	6
67	8261	8267	8274	8280	8287	8293	8299	8306	8312	8319	1	1	2	3	3	4	5	5	6
68	8325	8331	8338	8344	8351	8357	8363	8370	8376	8382	1	1	2	3	3	4	4	5	6
69	8388	8395	8401	8407	8414	8420	8426	8432	8439	8445	1	1	2	3	3	4	4	5	6
70	8451	8457	8463	8470	8476	8482	8488	8494	8500	8506	1	1	2	2	3	4	4	5	6
71	8513	8519	8525	8531	8537	8543	8549	8555	8561	8567	1	1	2	2	3	4	4	5	5
72	8573	8579	8585	8591	8597	8603	8609	8615	8621	8627	1	1	2	2	3	4	4	5	5
73	8633	8639	8645	8651	8657	8663	8669	8675	8681	8686	1	1	2	2	3	4	4	5	5
74	8692	8698	8704	8710	8716	8722	8727	8733	8739	8745	1	1	2	2	3	4	4	5	5
75	8751	8756	8762	8768	8774	8779	8785	8791	8797	8802	1	1	2	2	3	3	4	5	5
76	8808	8814	8820	8825	8831	8837	8842	8848	8854	8859	1	1	2	2	3	3	4	5	5
77	8865	8871	8876	8882	8887	8893	8899	8904	8910	8915	1	1	2	2	3	3	4	4	5
78	8921	8927	8932	8938	8943	8949	8954	8960	8965	8971	1	1	2	2	3	3	4	4	5

Table XIX.

Four-figure Logarithms—*continued.*

(A.) Logarithms of Numbers.

No.	0	1	2	3	4	5	6	7	8	9	Fourth Figure. 1	2	3	4	5	6	7	8	9
79	8976	8982	8987	8993	8998	9004	9009	9015	9020	9025	1	1	2	2	3	3	4	4	5
80	9031	9036	9042	9047	9053	9058	9063	9069	9074	9079	1	1	2	2	3	3	4	4	5
81	9085	9090	9096	9101	9106	9112	9117	9122	9128	9133	1	1	2	2	3	3	4	4	5
82	9138	9143	9149	9154	9159	9165	9170	9175	9180	9186	1	1	2	2	3	3	4	4	5
83	9191	9196	9201	9206	9212	9217	9222	9227	9232	9238	1	1	2	2	3	3	4	4	5
84	9243	9248	9253	9258	9263	9269	9274	9279	9284	9289	1	1	2	2	3	3	4	4	5
85	9294	9299	9304	9309	9315	9320	9325	9330	9335	9340	1	1	2	2	3	3	4	4	5
86	9345	9350	9355	9360	9365	9370	9375	9380	9385	9390	1	1	2	2	3	3	4	4	5
87	9395	9400	9405	9410	9415	9420	9425	9430	9435	9440	0	1	1	2	2	3	3	4	4
88	9445	9450	9455	9460	9465	9469	9474	9479	9484	9489	0	1	1	2	2	3	3	4	4
89	9494	9499	9504	9509	9513	9518	9523	9528	9533	9538	0	1	1	2	2	3	3	4	4
90	9542	9547	9552	9557	9562	9566	9571	9576	9581	9586	0	1	1	2	2	3	3	4	4
91	9590	9595	9600	9605	9609	9614	9619	9624	9628	9633	0	1	1	2	2	3	3	4	4
92	9638	9643	9647	9652	9657	9661	9666	9671	9675	9680	0	1	1	2	2	3	3	4	4
93	9685	9689	9694	9699	9703	9708	9713	9717	9722	9727	0	1	1	2	2	3	3	4	4
94	9731	9736	9741	9745	9750	9754	9759	9763	9768	9773	0	1	1	2	2	3	3	4	4
95	9777	9782	9786	9791	9795	9800	9805	9809	9814	9818	0	1	1	2	2	3	3	4	4
96	9823	9827	9832	9836	9841	9845	9850	9854	9859	9863	0	1	1	2	2	3	3	4	4
97	9868	9872	9877	9881	9886	9890	9894	9899	9903	9908	0	1	1	2	2	3	3	4	4
98	9912	9917	9921	9926	9930	9934	9939	9943	9948	9952	0	1	1	2	2	3	3	4	4
99	9956	9961	9965	9969	9974	9978	9983	9987	9991	9996	0	1	1	2	2	3	3	3	4

If $g = 32{\cdot}1908$ and $\pi = 3{\cdot}14159$, $\log g = 1{\cdot}5077318$, $\log \pi = 0{\cdot}4971496$, and $\log 2g \times 2240 = 5{\cdot}1590098$.

Explanation by an Example:—Required the logarithm of 256·8. Find the first two figures of the number (in this case 25) in the "No." column and look from them horizontally. Cast the eye down the column headed 6 (which is the third figure in our number), this intersects the horizontal through 25 in ·4082 as the logarithm of 256. To find how much to add for the additional 8, look down the column 8 at "Fourth figure," and ·0014 is indicated (in the horizontal of 25). This must be added to the logarithm already obtained, and we have ·4082 + ·0014 = ·4096. As the index of the logarithm is always one less than the number of integral figures of the number, the complete logarithm of 256·8 is 2·4096.

Remember that the logarithms of 1, 3, 8, &c., are the same as the logarithms 10, 30, 80, &c.

When it is required to find the number corresponding to a logarithm, it is found by the converse of the plan just described. Search for that nearest quantity which is *less* among the columns containing four figures in order to determine the first three figures of the number; the last figure can be obtained by taking the difference between the original logarithms and this tabulated quantity, and noting the heading of the column of "Fourth figure" in the same horizontal which contains the quantity most nearly agreeing with this difference: thus find the number corresponding to the logarithm 2·4096. Disregarding the index for the present we find the nearest tabulated quantity which is less is ·4082, corresponding to a number 256: the difference ·4096 − ·4082 = ·0014. Carry the eye along the same horizontal, and note under which column of "Fourth figure" 14 is found; it is under column headed 8, which is consequently the last figure, and the number becomes 2568. As the index of the logarithm is 2; there are 2 + 1 = 3 integers in the completed number, which is consequently 256·8.

(B.) Logarithms of Sines, Tangents, and Secants.

The formation of the table on the next pages needs a few explanatory words; it is a somewhat difficult matter to arrange a table of four-figure logarithms of sines, tangents, secants, &c., reading to minutes with fair accuracy in the small compass of two pages because many of the differences are irregular; however, by the method of arrangement which has been adopted, an error of 0·0007 is never exceeded; in many instances it is very much less, and in most cases there is no error at all, except the inevitable one of four figures only giving an approximation.

The differences always read *downwards,* thus 0·0177 is the *actual* difference between logarithms of sines 0° 24′ and 0° 25′, and also between cosines 89° 36′ and 89° 35′, it is *not* the difference between logarithms of sines 0° 23′ and 0° 24′, or between cosines 89° 37′ and 89° 36′.

Further down in the table where the tabulated quantities differ by more than two minutes, viz., at 0° 30′ and onwards, the difference given is the *best average* to employ, so that the maximum possible error may be as small as possible. Opposite 0° 30′ the difference for sines and cosecs is given as 0·0137; but the *actual* difference between the logarithms of sines 0° 30′ and 0° 31′ (0·0142) is not given in the table.

The reason for the employment of the average difference in preference to the actual difference between the logarithms of sines 0° 30′ and 0° 31′ will be evident from the results which they respectively give as the logarithms of sines 0° 31′, 0° 32′, 0° 33′, and 0° 34′—

Log. sin.	Employing diff. 0·0137. log.	error.	Employing diff. 0·0142. log.	error.	True log.
0° 31′	7·9408 + 0·0137 = 7·9545	− 0·0006	7·9408 + 0·0142 = 7·9550	− 0·0001	7·9551
0° 32′	7·9408 + 2(0·0137) = 7·9682	− 0·0007	7·9408 + 2(0·0142) = 7·9692	+ 0·0003	7·9689
0° 33′	7·9408 + 3(0·0137) = 7·9819	− 0·0003	7·9408 + 3(0·0142) = 7·9834	+ 0·0012	7·9822
0° 34′	7·9408 + 4(0·0137) = 7·9956	+ 0·0004	7·9408 + 4(0·0142) = 7·9976	+ 0·0024	7·9952

from which it appears that the difference 0·0142 gives a maximum possible error (0·0024) which is more than three times as great as the largest possible error afforded by the use of the *average* difference: this illustration is the worst one which can be given of the accuracy of the table, as below 0° 30′, or above 89° 30′, every logarithm is either recorded, or it can be found exactly by differencing; and as the angles increase above 0° 30′, or decrease below 89° 30′, the differences become more regular and the maximum error sinks rapidly.

In consequence of always reading the differences downwards, a slight complication ensues in the employment of the table with angles *greater* than 45°, that the nearest tabulated angle which is *greater* than the given one must be selected, and then the difference must be subtracted or added.

Practically, however, this difficulty hardly ever occurs in the subject of gunnery, as the angles dealt with, being almost exclusively those connected with the inclination of the trajectory, are always less than 45°.

Table XIX.

Four-figure Logarithms—*continued.*

(B.) Logarithms of Sines, Tangents, and Secants.

Angle.	Sine.	Diff. for 1′.	Cosec.	Tan.	Diff. for 1′.	Cotan.	Secant.	Diff. for 1′.	Cosine.	...
0° 0′	Infin. neg.	Infin.	Infinite.	Infin. neg.	Infin.	Infinite.	10·0000	0	10·0000	90° 0′
0 1	6·4637	3010	13·5363	6·4637	3010	15·5363	10·0000	0	10·0000	89 59
0 2	7·7648	1761	13·2352	6·7648	1761	13·2352	10·0000	0	10·0000	89 58
0 3	6·9408	1249	13·0592	6·9408	1249	13·0592	10·0000	0	10·0000	89 57
0 4	7·0658	969	12·9342	7·0658	969	12·9342	10·0000	0	10·0000	89 56
0 5	7·1627	792	12·8373	7·1627	792	12·8373	10·0000	0	10·0000	89 55
0 6	7·2419	669	12·7581	7·2419	669	12·7581	10·0000	0	10·0000	89 54
0 8	7·3668	512	12·6332	7·3668	512	12·6332	10·0000	0	10·0000	89 52
0 10	7·4637	414	12·5363	7·4637	414	12·5363	10·0000	0	10·0000	89 50
0 12	7·5429	348	12·4571	7·5429	348	12·4571	10·0000	0	10·0000	89 48
0 14	7·6099	300	12·3901	7·6099	300	12·3901	10·0000	0	10·0000	89 46
0 16	7·6678	263	12·3322	7·6678	263	12·3322	10·0000	0	10·0000	89 44
0 18	7·7190	235	12·2810	7·7190	235	12·2810	10·0000	0	10·0000	89 42
0 20	7·7648	212	12·2352	7·7648	212	12·2352	10·0000	0	10·0000	89 40
0 22	7·8061	193	12·1939	7·8062	193	12·1938	10·0000	0	10·0000	89 38
0 24	7·8439	177	12·1561	7·8439	177	12·1561	10·0000	0	10·0000	89 36
0 26	7·8787	164	12·1213	7·8787	164	12·1213	10·0000	0	10·0000	89 34
0 28	7·9109	152	12·0891	7·9109	152	12·0891	10·0000	0	10·0000	89 32
0 30	7·9408	137	12·0592	7·9409	137	12·0591	10·0000	0	10·0000	89 30
0 35	8·0078	118	11·9922	8·0078	118	11·9922	10·0000	0	10·0000	89 25
0 40	8·0658	104	11·9342	8·0658	104	11·9342	10·0000	0	10·0000	89 20
0 45	8·1169	93	11·8831	8·1170	93	11·8830	10·0000	0	10·0000	89 15
0 50	8·1627	84	11·8373	8·1627	84	11·8373	10·0000	0	10·0000	89 10
0 55	8·2041	77	11·7959	8·2041	77	11·7959	10·0001	0	9·9999	89 5
1 0	8·2419	70	11·7581	8·2419	70	11·7581	10·0001	0	9·9999	89 0
1 5	8·2766	65	11·7234	8·2767	65	11·7233	10·0001	0	9·9999	88 55
1 10	8·3088	60	11·6912	8·3089	60	11·6911	10·0001	0	9·9999	88 50
1 15	8·3388	56	11·6612	8·3389	56	11·6611	10·0001	0	9·9999	88 45
1 20	8·3668	53	11·6332	8·3669	53	11·6331	10·0001	0	9·9999	88 40
1 25	8·3931	50	11·6069	8·3932	50	11·6068	10·0001	0	9·9999	88 35
1 30	8·4179	46	11·5821	8·4181	46	11·5819	10·0001	0	9·9999	88 30
1 40	8·4637	42	11·5363	8·4638	42	11·5362	10·0002	0	9·9998	88 20
1 50	8·5050	38	11·4950	8·5053	38	11·4947	10·0002	0	9·9998	88 10
2 0	8·5428	35	11·4572	8·5431	35	11·4569	10·0003	0	9·9997	88 0
2 10	8·5776	32	11·4224	8·5779	32	11·4221	10·0003	0	9·9997	87 50
2 20	8·6097	30	11·3903	8·6101	30	11·3899	10·0004	0	9·9996	87 40
2 30	8·6397	28	11·3603	8·6401	28	11·3599	10·0004	0	9·9996	87 30
2 40	8·6677	26	11·3323	8·6682	26	11·3318	10·0005	0	9·9995	87 20
2 50	8·6940	25	11·3060	8·6945	25	11·3055	10·0005	0	9·9995	87 10
3 0	8·7188	24	11·2812	8·7194	24	11·2806	10·0006	0	9·9994	87 0
3 10	8·7423	22	11·2577	8·7429	22	11·2571	10·0007	0	9·9993	86 50
3 20	8·7645	21	11·2355	8·7652	21	11·2348	10·0007	0	9·9993	86 40
3 30	8·7857	20	11·2143	8·7865	20	11·2135	10·0008	0	9·9992	86 30
3 40	8·8059	19	11·1941	8·8067	19	11·1933	10·0009	0	9·9991	86 20
3 50	8·8251	18	11·1749	8·8261	18	11·1739	10·0010	0	9·9990	86 10
4 0	8·8436	18	11·1564	8·8446	18	11·1554	10·0011	0	9·9989	86 0
4 10	8·8613	17	11·1387	8·8624	17	11·1376	10·0012	0	9·9989	85 50
4 20	8·8783	16	11·1217	8·8795	16	11·1205	10·0011	0	9·9988	85 40
4 30	8·8946	16	11·1054	8·8960	16	11·1040	10·0013	0	9·9987	85 30
4 40	8·9104	15	11·0896	8·9118	15	11·0882	10·0014	0	9·9986	85 20
4 50	8·9256	15	11·0744	8·9272	15	11·0728	10·0015	0	9·9985	85 10
5 0	8·9403	14	11·0597	8·9420	14	11·0580	10·0017	0	9·9983	85 0
5 10	8·9545	14	11·0455	8·9563	14	11·0437	10·0018	0	9·9982	84 50
5 20	8·9682	13	11·0318	8·9701	13	11·0299	10·0019	0	9·9981	84 40
5 30	8·9816	13	11·0184	8·9836	13	11·0164	10·0020	0	9·9980	84 30
5 40	8·9945	13	11·0055	8·9966	13	11·0034	10·0021	0	9·9979	84 20
5 50	9·0070	12	10·9930	9·0093	12	10·9907	10·0023	0	9·9977	84 10
6 0	9·0192	12	10·9808	9·0216	12	10·9784	10·0024	0	9·9976	84 0
6 10	9·0311	12	10·9689	9·0336	12	10·9664	10·0025	0	9·9975	83 50
6 20	9·0426	11	10·9574	9·0453	11	10·9547	10·0027	0	9·9973	83 40
6 30	9·0539	11	10·9461	9·0567	11	10·9433	10·0028	0	9·9972	83 30
6 40	9·0648	11	10·9352	9·0678	11	10·9322	10·0029	0	9·9971	83 20
6 50	9·0755	10	10·9245	9·0786	11	10·9214	10·0031	0	9·9969	83 10
7 0	9·0859	10	10·9141	9·0891	10	10·9109	10·0032	0	9·9968	83 0
7 10	9·0961	10	10·9039	9·0995	10	10·9005	10·0034	0	9·9966	82 50
7 20	9·1060	10	10·8940	9·1096	10	10·8904	10·0036	0	9·9964	82 40
...	Cosine.	Diff. for 1′.	Secant.	Cotan.	Diff. for 1′.	Tan.	Cosec.	Diff. for 1′.	Sine	Angle.

TABLE XIX.

Four-figure Logarithms—*continued.*

(B.) Logarithms of Sines, Tangents, and Secants.

Angle.	Sine.	Diff. for 1′.	Cosec.	Tan.	Diff. for 1′.	Cotan.	Secant.	Diff. for 1′.	Cosine.	...
7° 30′	9·1157	10	10·8843	9·1194	10	10·8806	10·0037	0	9·9963	82° 30
7 40	9·1252	9	10·8748	9·1291	9	10·8709	10·0039	0	9·9961	82 20
7 50	9·1345	9	10·8655	9·1385	9	10·8615	10·0041	0	9·9959	82 10
8 0	9·1436	9	10·8564	9·1478	9	10·8522	10·0042	0	9·9958	82 0
8 10	9·1525	9	10·8475	9·1569	9	10·8431	10·0044	0	9·9956	81 50
8 20	9·1612	9	10·8388	9·1658	9	10·8342	10·0046	0	9·9954	81 40
		Diff. for 10′.			Diff. for 10′.			Diff. for 10′.		
8 30	9·1697	83	10·8303	9·1745	85	10·8255	10·0048	2	9·9952	81 30
9 0	9·1943	78	10·8057	9·1997	80	10·8003	10·0054	2	9·9946	81 0
9 30	9·2176	74	10·7824	9·2236	76	10·7764	10·0060	2	9·9940	80 30
10 0	9·2397	70	10·7603	9·2463	72	10·7537	10·0066	2	9·9934	80 0
10 30	9·2606	67	10·7394	9·2680	69	10·7320	10·0073	2	9·9927	79 30
11 0	9·2806	64	10·7194	9·2887	66	10·7113	10·0081	2	9·9919	79 0
11 30	9·2997	61	10·7003	9·3085	63	10·6915	10·0088	3	9·9912	78 30
12 0	9·3179	58	10·6821	9·3275	61	10·6725	10·0096	3	9·9904	78 0
12 30	9·3353	56	10·6647	9·3458	59	10·6542	10·0104	3	9·9896	77 30
13 0	9·3521	54	10·6479	9·3634	57	10·6366	10·0113	3	9·9887	77 0
13 30	9·3682	52	10·6318	9·3804	55	10·6196	10·0122	3	9·9878	76 30
14 0	9·3837	50	10·6163	9·3968	53	10·6032	10·0131	3	9·9869	76 0
14 30	9·3986	48	10·6014	9·4127	51	10·5873	10·0141	3	9·9859	75 30
15 0	9·4130	46	10·5870	9·4281	50	10·5719	10·0151	3	9·9849	75 0
15 30	9·4269	45	10·5731	9·4430	48	10·5570	10·0161	3	9·9839	74 30
16 0	9·4403	43	10·5597	9·4575	47	10·5425	10·0172	4	9·9828	74 0
17 0	9·4659	40	10·5341	9·4853	44	10·5147	10·0194	4	9·9806	73 0
18 0	9·4900	38	10·5100	9·5118	42	10·4882	10·0218	4	9·9782	72 0
19 0	9·5126	36	10·4874	9·5370	40	10·4630	10·0243	4	9·9757	71 0
20 0	9·5341	34	10·4659	9·5611	39	10·4389	10·0270	5	9·9730	70 0
21 0	9·5543	32	10·4457	9·5842	37	10·4158	10·0298	5	9·9702	69 0
22 0	9·5736	31	10·4264	9·6064	36	10·3936	10·0328	5	9·9672	68 0
23 0	9·5919	29	10·4081	9·6279	35	10·3721	10·0360	6	9·9640	67 0
24 0	9·6093	28	10·3907	9·6486	34	10·3514	10·0393	6	9·9607	66 0
25 0	9·6259	27	10·3741	9·6687	33	10·3313	10·0427	6	9·9573	65 0
26 0	9·6418	25	10·3582	9·6882	32	10·3118	10·0463	6	9·9537	64 0
27 0	9·6570	24	10·3430	9·7072	31	10·2928	10·0501	7	9·9499	63 0
28 0	9·6716	23	10·3284	9·7257	30	10·2743	10·0541	7	9·9459	62 0
29 0	9·6856	22	10·3144	9·7438	29	10·2562	10·0582	7	9·9418	61 0
30 0	9·6990	21	10·3010	9·7614	29	10·2386	10·0625	7	9·9375	60 0
31 0	9·7118	21	10·2882	9·7788	28	10·2212	10·0669	8	9·9331	59 0
32 0	9·7242	20	10·2758	9·7958	28	10·2042	10·0716	8	9·9284	58 0
33 0	9·7361	19	10·2639	9·8125	28	10·1875	10·0764	8	9·9236	57 0
34 0	9·7476	19	10·2524	9·8290	27	10·1710	10·0814	9	9·9186	56 0
35 0	9·7586	18	10·2414	9·8452	27	10·1548	10·0866	9	9·9134	55 0
36 0	9·7692	17	10·2308	9·8613	26	10·1387	10·0920	9	9·9080	54 0
37 0	9·7795	16	10·2205	9·8771	26	10·1229	10·0977	10	9·9023	53 0
38 0	9·7893	16	10·2107	9·8928	26	10·1072	10·1035	10	9·8965	52 0
39 0	9·7989	15	10·2011	9·9084	26	10·0916	10·1095	10	9·8905	51 0
40 0	9·8081	15	10·1919	9·9238	26	10·0762	10·1157	11	9·8843	50 0
41 0	9·8169	14	10·1831	9·9392	25	10·0608	10·1222	11	9·8778	49 0
42 0	9·8255	14	10·1745	9·9544	25	10·0456	10·1289	12	9·8711	48 0
43 0	9·8338	13	10·1662	9·9697	25	10·0303	10·1359	12	9·8641	47 0
44 0	9·8418	13	10·1582	9·9848	25	10·0152	10·1431	12	9·8569	46 0
45 0	9·8495	12	10·1505	10·0000	25	10·0000	10·1505	13	9·8495	45 0
...	Cosine.	Diff. for 10′.	Secant.	Cotan.	Diff. for 10′.	Tan.	Cosec.	Diff. for 10′.	Sine.	Angle.

EXPLANATION.—The differences are given for each minute from 0 up to 8° 29′, and from 81° 31′ up to 90°; between 8° 30′ and 81° 30′ the differences are for 10 minutes.

(A.) To find the logarithms of Sines, Tangents, Secants, &c.

Below 45° seek for the nearest tabulated angle which is *less*; but above 45° take the nearest angle, which is *greater* than the given one: thus—

Find log Sin 3° 24′.	Find log Cot. 10° 13′.	Find log Sec. 88° 57′	Find log Cosec. 59° 23′
log Sin 3° 20′ = 8·7645	log Cot. 10° = 10·7537	log Sec. 89° = 11·7581	log Cosec. 60° = 10·0625
Diff. for 1′ is 21	Diff. for 10′ is 72	Diff. for 1′ is 70	Diff. for 10′ is 7
∴ for 4 it is 4 × 21 = +·0084	∴ for 13 it is 1·3 × 72 = −·0094	∴ for 3′ it is 3 × 70 = −·0210	∴ for 37 it is 3·7 × 7 = +·0026
∴ log Sin 3° 24′ = 8·7729	∴ log 10° 13′ = 10·7443	∴ log Sec. 88° 57′ = 11·7371	∴ log Cosec. 59° 23′ = 10·0651

(B.) To find the angle when the logarithm of the trigonometrical function is given, the ordinary simpler rule is always allowable.

Seek for the nearest tabulated number which is *less*, and *add* for the difference in the case of sines, tangents, and secants, but *subtract* for cosecs, cotangents, and cosines.

Find the angle for logarithmic tan 8·5330	Find the angle for logarithmic Cos. 8·5545
From table logarithmic tan 8·5053 = 1° 50′	From table logarithmic Cos. 8·5428 = 88° 0′
Diff. between these is 277; for 1′ it is 38	Diff. between these is 117; for 1′ it is 35
∴ for 277 it is 277 ÷ 38 = + 7′	∴ for 117 it is 117 ÷ 35 = −3′
∴ logarithmic tan 8·5330 = 1° 57′	∴ logarithmic Cos. 8·5545 = 87° 57′

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